Threshold Resummation for Squarks, Gluinos, and Stops at the LHC

Christoph Julian Borschensky

Westfälische Wilhelms-Universität Münster

in collaboration with Wim Beenakker, Michael Krämer, Anna Kulesza, Eric Laenen, Vincent Theeuwes, Silja Thewes

Forschungsseminar QFT

Münster, 29. Juni 2015

institut für theoretische physik

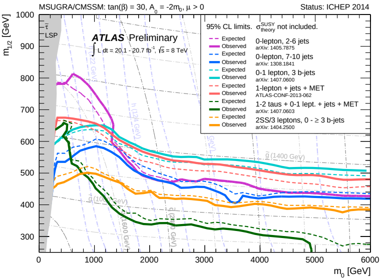
:-)

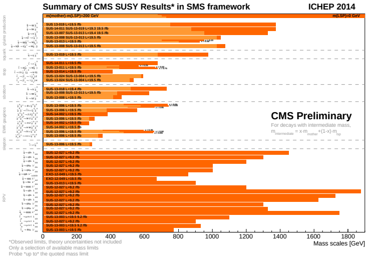
Outline

SUSY particle production at the LHC

Main sparticle production processes:

$$pp \;\rightarrow\; \tilde{g}\tilde{g} \;,\; \tilde{q}\tilde{q}^* \;,\; \tilde{q}\tilde{g} \;,\; \tilde{q}\tilde{q} \;,\; \tilde{t}_1\tilde{t}_1^*$$





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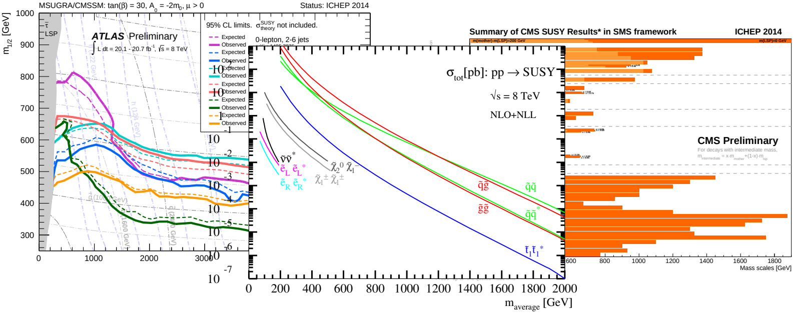

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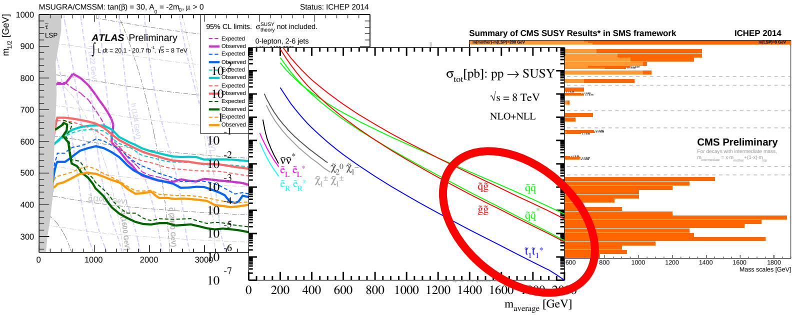

⇒ Cross sections needed at high precision for experimental searches

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Wait... SUSY?

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Supersymmetry (SUSY) connects bosons to fermions and vice versa

→ New particles that "only" differ in their spin quantum number

→ Resolves the **hierarchy problem** (= large radiative corrections to masses of scalar particles)

→ Extends the Standard Model **beyond its limits**

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... just break it **softly**, i.e. keep it renormalisable

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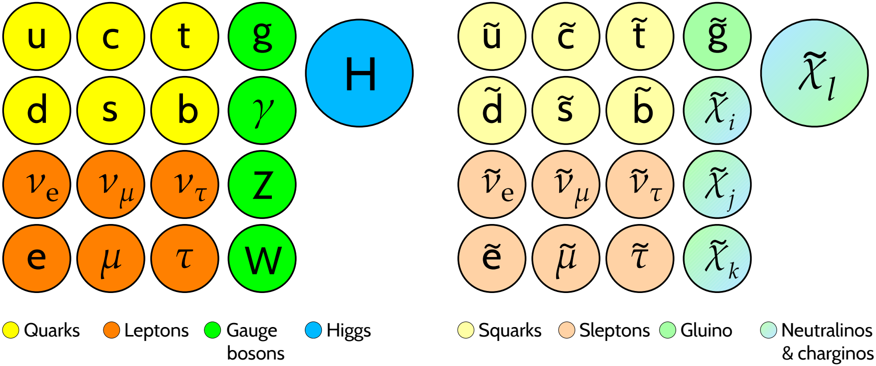
Standard Model particles **Supersymmetric partners**



(“Reality”: broken symmetry, heavy SUSY particles, R-parity, ...)

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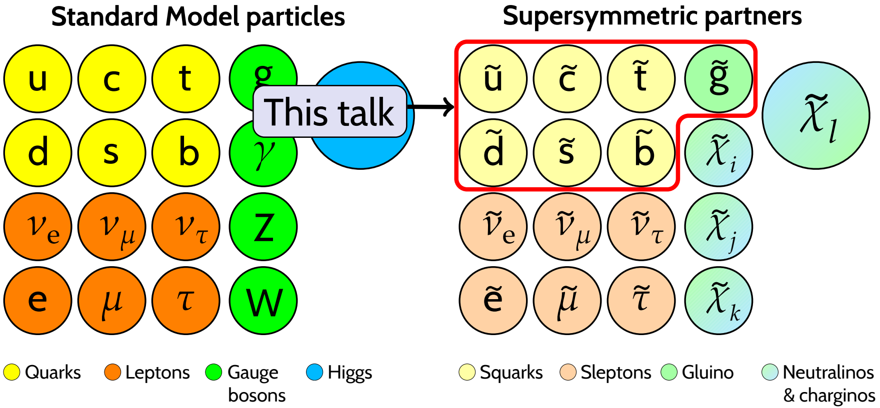

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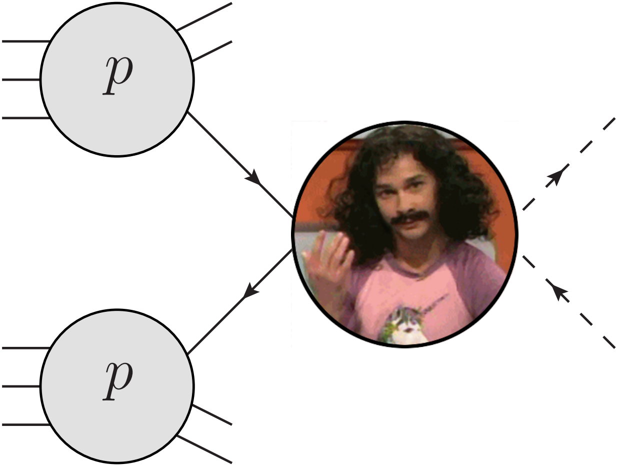

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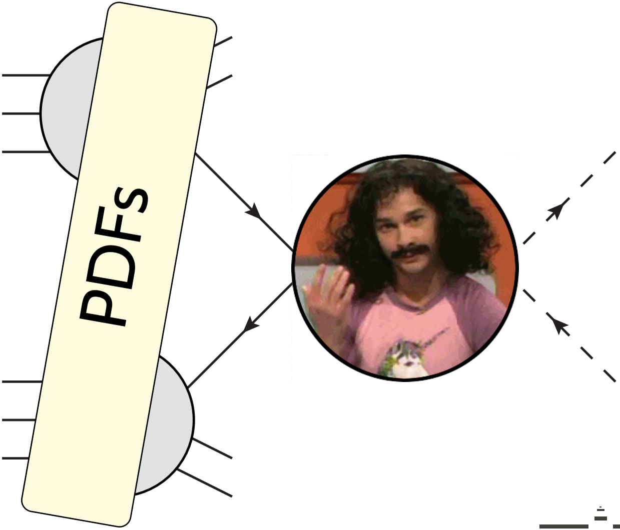

Particle production at the LHC

Particle production at the LHC

Particle production at the LHC, factorisation

Total hadronic cross section:

$$\sigma_{\text{hadr.}}(S, m, \mu_R, \mu_F) = \sum_{i,j} \int_0^1 \mathrm{d}x_1\, f_i(x_1, \mu_F) \int_0^1 \mathrm{d}x_2\, f_j(x_2, \mu_F) \times \hat{\sigma}_{ij\to kl}(\hat{s} = x_1 x_2 S, m, \mu_R, \mu_F)\theta(\hat{s} - 4m^2)$$

with

- $f_i(x, \mu_F)$: parton distribution function
- m: average mass of the produced particles
- $\sqrt{S}$: centre-of-mass energy of the collider
- μ_R, μ_F: renormalisation and factorisation scales
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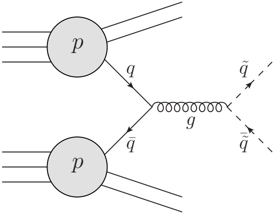

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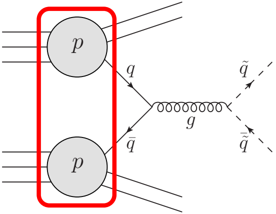

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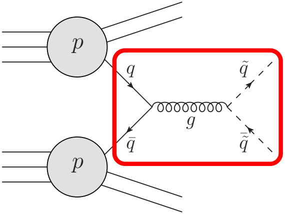

Particle production close to threshold

Heavy SUSY particles $\Rightarrow$ production in the **threshold limit** $\sqrt{\hat{s}} \to 2m$:

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$\Rightarrow$ **Real radiation processes are soft**

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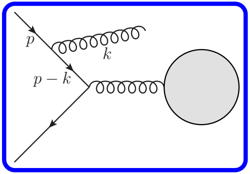
Soft-collinear gluon emission

Real gluon emission from massless quarks:



$$: \mathscr{M} \sim \frac{1}{(p-k)^2} = -\frac{1}{2p \cdot k} = -\frac{1}{2E_g E_q (1 - \cos\vartheta_{qg})}$$

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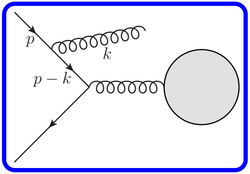

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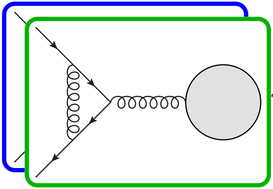

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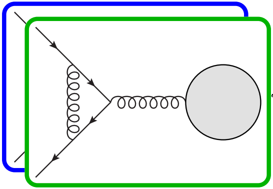

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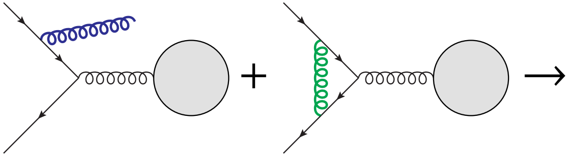
Remainder after cancellation of IR divergencies:



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Soft & collinear gluons

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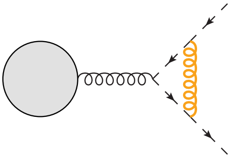

$\longrightarrow$ $\boxed{\sim \alpha_s^n/\beta^n}$

Coulomb gluons

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Enhanced partonic cross sections close to threshold:

- Soft & collinear gluons: $\boxed{\alpha_s^n \ln^m \beta^2 \sim 1}$
- Coulomb gluons: $\boxed{\alpha_s^n/\beta^n \sim 1}$

$\Rightarrow$ Endangering the perturbative series

$\Rightarrow$ Systematic treatment of these terms required

From factorisation to resummation

Basis for resummation of soft-collinear gluons: factorisation between **hard** and **soft** parts

- → Factorisation of matrix elements and phase space
- → **Matrix element**: factorises automatically in soft-collinear limit ✓
- → **Phase space**: momentum-conservation causes entanglement ✗

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Conclusion: resummation does not work.

Thank you for your attention.

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Mellin transform

Basis for resummation of soft-collinear gluons: factorisation between **hard** and **soft** parts

→ Factorisation of matrix elements and phase space

In **Mellin-moment space**, the entanglement vanishes (momentum-conserving delta function turns into an exponential function)

Mellin transform: $$\boxed{\tilde{f}(N) := \int_0^\infty \mathrm{d}x\, x^{N-1} f(x)}$$ with the Mellin moment N

Let's apply this to our cross section!

Mellin transform

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$$= \sum_{i,j} \int_0^1 \mathrm{d}x_1\, f_i(x_1) \int_0^1 \mathrm{d}x_2\, f_j(x_2) \int_0^\infty \mathrm{d}\rho\, \rho^{N-1} \hat{\sigma}_{ij\to kl}(\hat{\rho}) \theta(\hat{s} - 4m^2)$$

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$$= \sum_{i,j} \underbrace{\left[\int_0^1 \mathrm{d}x_1\, x_1^N f_i(x_1)\right]}_{=:\ \tilde{f}_i(N+1)} \underbrace{\left[\int_0^1 \mathrm{d}x_2\, x_2^N f_j(x_2)\right]}_{=:\ \tilde{f}_j(N+1)} \underbrace{\left[\int_0^1 \mathrm{d}\hat{\rho}\, \hat{\rho}^{N-1}\hat{\sigma}_{ij\to kl}(\hat{\rho})\right]}_{=:\ \tilde{\sigma}_{ij\to kl}(N)}$$

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$$= \sum_{i,j} \tilde{f}_i(N+1) \times \tilde{f}_j(N+1) \times \tilde{\sigma}_{ij\to kl}(N)$$

In $\tilde{\sigma}_{ij\to kl}(N)$, hard and soft parts now fully factorised

→ Additionally, convolution with PDFs turns into a simple product

Treating large logarithms

Threshold logarithms in Mellin-moment space (threshold limit: $\beta \to 0 \;\hat{=}\; N \to \infty$):

$$\boxed{\ln\beta^2 \xrightarrow{\text{Mellin}} \ln N \equiv L}$$

(neglect subleading terms $\mathscr{O}(1/N)$)

Reordering of the perturbative series in α_s and L (schematically):

$$\tilde{\sigma} \sim \tilde{\sigma}^{(0)} \Big[1 + \alpha_s \left(L^2 + L + 1\right) + \alpha_s^2 \left(L^4 + L^3 + L^2 + L + 1\right) + \ldots \Big]$$

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Summation of all these terms → exponential function (g_1, g_2, g_3 known):

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(Precision level: LL

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Reordering of the perturbative series in α_s **and** L (schematically):

$$\tilde{\sigma} \sim \tilde{\sigma}^{(0)} \Big[1 + \alpha_s \left(L^2 + L + 1\right) + \alpha_s^2 \left(L^4 + L^3 + L^2 + L + 1\right) + \dots \Big]$$

LO $\sim \alpha_s^2$

Summation of all these terms → exponential function (g_1, g_2, g_3 known):

$$\boxed{\tilde{\sigma} \sim \tilde{\sigma}^{(0)} \times C(\alpha_s) \exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \dots \right]}$$

[Kodaira, Trentadue '82][Sterman '87][Catani, D'Emilio, Trentadue '88][Catani, Trentadue '89][Kidonakis, Sterman '96][Kidonakis, Oderda, Sterman '98][Contopanagos, Laenen, Sterman '96][Catani, de Florian, Grazzini '01][Moch, Vermaseren, Vogt '04][Beneke, Falgari, Schwinn '09][Czakon, Mitov, Sterman '09][Ferroglia, Neubert, Pecjak, Yang '09] ...

(Precision level: LL + NLL + NNLL)

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Treating large logarithms

Threshold logarithms in **Mellin-moment space** (threshold limit: $\beta \to 0 \;\hat{=}\; N \to \infty$):

$$\boxed{\ln \beta^2 \xrightarrow{\text{Mellin}} \ln N \equiv L}$$

(neglect subleading terms $\mathcal{O}(1/N)$)

Reordering of the perturbative series in α_s and L (schematically):

$$\tilde{\sigma} \sim \tilde{\sigma}^{(0)} \Big[1 + \boxed{\alpha_s \left(L^2 + L + 1\right)} + \boxed{\alpha_s^2 \left(L^4 + L^3 + L^2 + L + 1\right)} + \ldots \Big]$$

LO $\sim \alpha_s^2$

$\sim$ NLO

$\sim$ NNLO

Summation of all these terms $\to$ exponential function (g_1, g_2, g_3 known):

$$\boxed{\tilde{\sigma} \sim \tilde{\sigma}^{(0)} \times C(\alpha_s) \exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right]}$$

[Kodaira, Trentadue '82][Sterman '87][Catani, D'Emilio, Trentadue '88][Catani, Trentadue '89][Kidonakis, Sterman '96][Kidonakis, Oderda, Sterman '98][Contopanagos, Laenen, Sterman '96][Catani, de Florian, Grazzini '01][Moch, Vermaseren, Vogt '04][Beneke, Falgari, Schwinn '09][Czakon, Mitov, Sterman '09][Ferroglia, Neubert, Pecjak, Yang '09] ...

(Precision level: LL + NLL + NNLL)

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Factorised resummation formula

$$\tilde{\sigma}_{ij\to kl}(N) = \sum_{\text{colours } I} H_{ij\to kl,I}(N,\mu) \times \Delta_i(N,\mu)\Delta_j(N,\mu)S_{ij\to kl,I}(N,\mu)$$

with

- process dependent **hard part** $H_{ij\to kl,I}(N,\mu)$
- process independent **soft-collinear radiation factors** $\Delta_{i/j}(N,\mu)$
- **soft wide-angle radiation factor** $S_{ij\to kl,I}(N,\mu)$

Factorised resummation formula

$$\tilde{\sigma}_{ij\to kl}(N) = \sum_{\text{colours } I} H_{ij\to kl,I}(N,\mu) \times \Delta_i(N,\mu)\Delta_j(N,\mu)S_{ij\to kl,I}(N,\mu)$$

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- **soft wide-angle radiation factor** $S_{ij\to kl,I}(N,\mu)$

Obtain exponentiated forms of $\Delta_{i/j}(N,\mu)$ and $S_{ij\to kl,I}(N,\mu)$ from **renormalisation group equations**:

→ Assume that the (physical) cross section is independent of the (unphysical) factorisation scale

→ Scale dependence cancels out between the constitutents

Factorised resummation formula

$$\tilde{\sigma}_{ij\to kl}(N) = \sum_{\text{colours } I} H_{ij\to kl,I}(N,\mu) \times \Delta_i(N,\mu)\Delta_j(N,\mu)S_{ij\to kl,I}(N,\mu)$$

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- process dependent **hard part** $H_{ij\to kl,I}(N,\mu)$
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- **soft wide-angle radiation factor** $S_{ij\to kl,I}(N,\mu)$

Obtain exponentiated forms of $\Delta_{i/j}(N,\mu)$ and $S_{ij\to kl,I}(N,\mu)$ from **renormalisation group equations**:

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\tilde{\sigma}(N) = 0 \longrightarrow \frac{\mathrm{d}\ln H}{\mathrm{d}\ln\mu} + \frac{\mathrm{d}\ln\Delta_i}{\mathrm{d}\ln\mu} + \frac{\mathrm{d}\ln\Delta_j}{\mathrm{d}\ln\mu} + \frac{\mathrm{d}\ln S}{\mathrm{d}\ln\mu} = 0$$

with $\dfrac{\mathrm{d}\ln S}{\mathrm{d}\ln\mu} = -\gamma_S(\mu)$ etc.

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:-)

Outline

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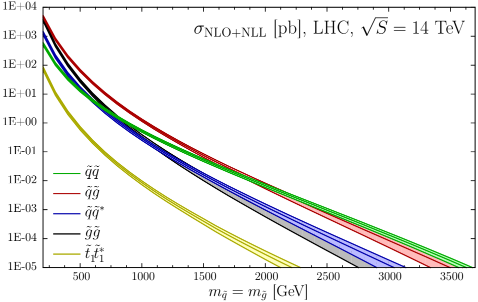
Numerical package: NLL-fast

Code package to compute NLO+NLL cross sections

- $\tilde{g}\tilde{g}$, $\tilde{q}\tilde{q}^*$, $\tilde{q}\tilde{g}$, $\tilde{q}\tilde{q}$, $\tilde{t}\tilde{t}^*$ and decoupling limit
- Including α_s, PDF, and scale variation
- At different LHC energies: $\sqrt{S}$ = **7 TeV**, **8 TeV**, **13 TeV**, 14 TeV, 33 TeV, 100 TeV, or upon request
- Used for current experimental analysis by ATLAS and CMS

[Krämer, Kulesza, van der Leeuw, Mangano, Padhi, Plehn, Portell '12]



NLL-fast

http://pauli.uni-muenster.de/~akule_01/nllwiki

[Kulesza, Motyka '08-'09][Beenakker, Brensing, Kulesza, Laenen, Niessen '09-'10]

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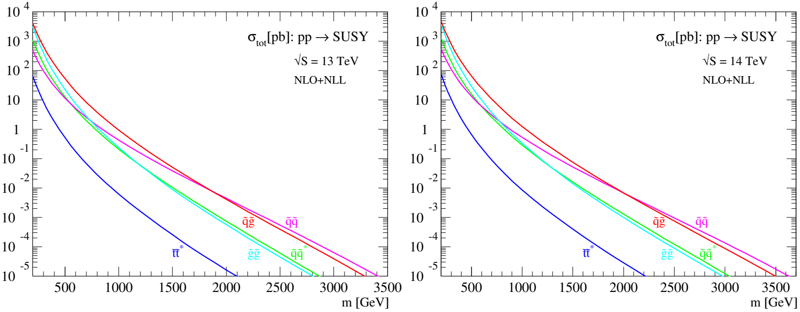
Predictions for future LHC and *pp* collider runs (1)

Cross section predictions for future LHC runs at 13, 14 TeV:

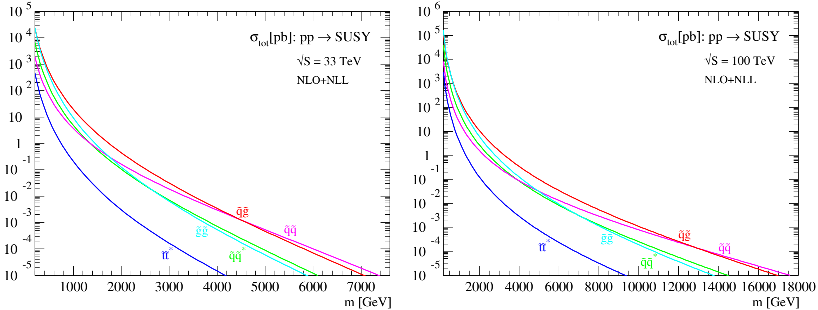


Reference paper for squark and gluino production at future LHC and *pp* collider energies: *[CB, Krämer, Kulesza, Mangano, Padhi, Plehn, Portell; EPJ C74 (2014) 12, 3174]*

See also: `https://twiki.cern.ch/twiki/bin/view/LHCPhysics/SUSYCrossSections`

Predictions for future LHC and *pp* collider runs (1)

Cross section predictions for future *pp* colliders operating at 33, 100 TeV:



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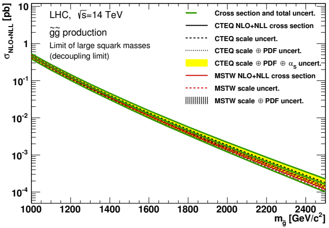
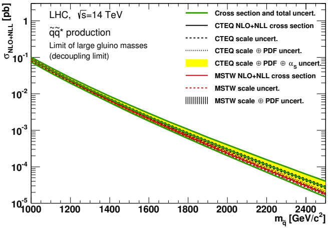
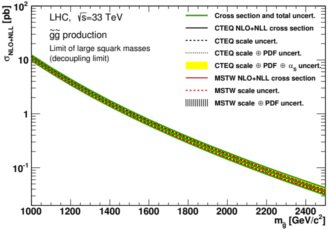
See also: https://twiki.cern.ch/twiki/bin/view/LHCPhysics/SUSYCrossSections

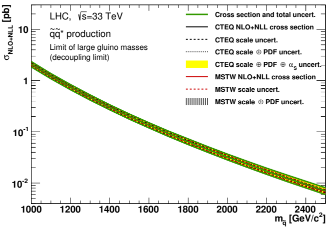
Predictions for future LHC and pp collider runs (2)

Detailed analysis of α_s, PDF, and scale uncertainties for $\tilde{g}\tilde{g}$ and $\tilde{q}\tilde{q}^*$ in the decoupling limit and $\tilde{t}_1\tilde{t}_1^*$ using CTEQ6.6M and MSTW2008 PDF sets:









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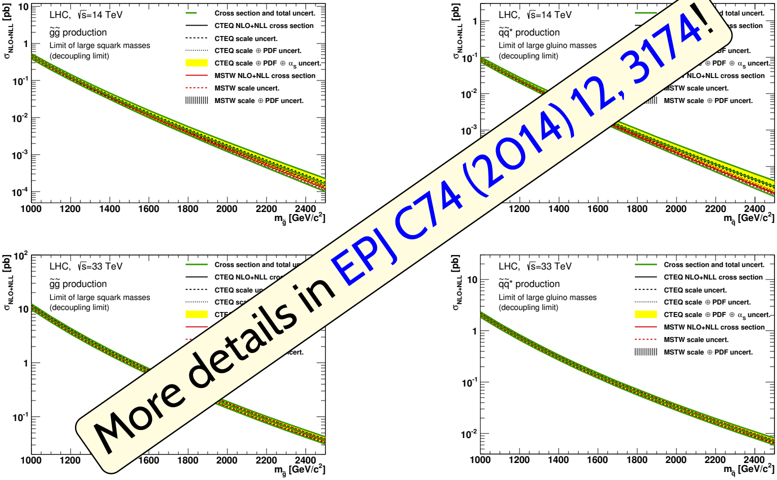

More details in EPJ C74 (2014) 12, 3174!

:-)

Outline

Matching coefficients

Higher order terms of different origin; **split-up close to threshold** *[Beneke, Falgari, Schwinn '09-10]*:

$$\boxed{C(\alpha_s) = C(N, \alpha_s) = \mathscr{C}^{\mathrm{Hard}}(\alpha_s) \times \mathscr{C}^{\mathrm{Coul}}(N, \alpha_s)}$$

- $\mathscr{C}^{\mathrm{Hard}}(\alpha_s)$: **hard matching coefficients** (independent of N)
  - → Calculated from NLO contributions *[Beenakker, Janssen, Lepoeter, Krämer, Kulesza, Laenen, Niessen, Thewes, Van Daal '13][Broggio, Ferroglia, Neubert, Vernazza, Yang '13]*
- $\mathscr{C}^{\mathrm{Coul}}(N, \alpha_s)$: **Coulomb terms** (final state gluon exchange):
  - → Can also be resummed *[Kulesza, Motyka '09][Beneke, Falgari, Schwinn '10][Falgari, Schwinn, Wever '12]*, using non-relativistic methods *[Fadin, Khoze '87][Peskin, Strassler '91][Hagiwara, Yokoya '09][Kauth, Kühn, Marquard, Steinhauser '09-11][Kauth, Kress, Kühn '11]*

Matching coefficients

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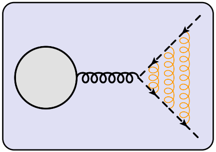

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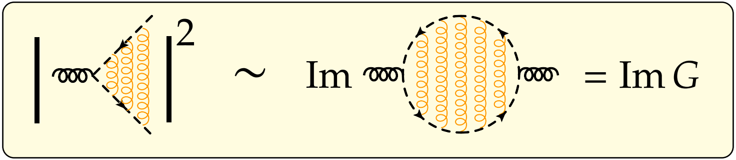
Coulomb Green's function



Calculation of ladder diagrams leads to **non-relativistic Schrödinger equation**:

$$\left\{\left[\frac{(-i\nabla)^2}{2m_{\text{red}}} + V_C(\vec{r})\right] - (E + i\Gamma)\right\} G(\vec{r}, E + i\Gamma) = \delta^{(3)}(\vec{r})$$

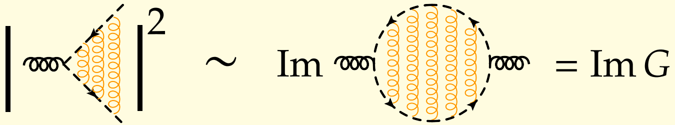
with $E = \sqrt{\hat{s}} - 2m$: energy and Γ: average decay width of the final state particles, m_{red}: reduced mass,

and the **Coulomb potential**:

$$\boxed{V_C(\vec{r}) = -D_{R_\alpha} \frac{\alpha_s}{|\vec{r}|} + \mathcal{O}(\alpha_s^2)}$$

with D_{R_α}: colour factor related to Casimir invariants

Coulomb Green's function



The solution at origin is *[Beneke, Signer, Smirnov '99][Pineda, Signer '06]*:

$$G(\vec{0}, E + i\Gamma) = i\frac{m_{\mathrm{red}}^2}{\pi}v + D_{R_\alpha}\frac{\alpha_s m_{\mathrm{red}}^2}{\pi}\left[g_{\mathrm{LO}} + \frac{\alpha_s}{4\pi}g_{\mathrm{NLO}} + \ldots\right]$$

with g_{LO} (g_{NLO}) contributions from LO (NLO) Coulomb potential ($g_{\mathrm{LO}} \sim$ Sommerfeld factor)

Incorporate into resummation framework (here: $\Gamma = 0$; $\tilde{q}$, $\tilde{g}$ stable):

$$\boxed{\hat{\sigma}^{\mathrm{Coul},res} = \hat{\sigma}^{\mathrm{LO}} \times \frac{\mathrm{Im}\, G(\vec{0}, E)}{\mathrm{Im}\, G^{\mathrm{free}}(\vec{0}, E)}}$$

with the velocity $v = \sqrt{\frac{E + i\Gamma}{2m_{\mathrm{red}}}} \approx \sqrt{\frac{m}{2m_{\mathrm{red}}}}\beta$

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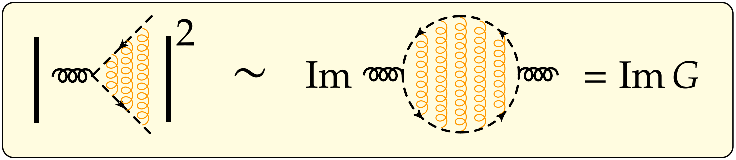
Coulomb Green's function: boundstates



The solution at origin is *[Beneke, Signer, Smirnov '99][Pineda, Signer '06]*:

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with g_{LO} (g_{NLO}) contributions from LO (NLO) Coulomb potential ($g_{\mathrm{LO}} \sim$ Sommerfeld factor)

$G(\vec{0}, E + i\Gamma)$ develops poles **below** threshold → boundstates for attractive Coulomb potential:

$$\boxed{G(\vec{0}, E + i\Gamma) = \sum_n \frac{|\Psi(0)|^2}{E_n - (E + i\Gamma)}}$$

with the wave function for the boundstate system at origin $\psi(0)$

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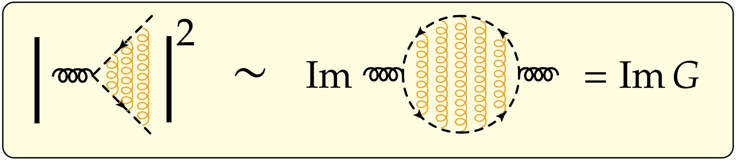
Coulomb Green's function: boundstates



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$G(\vec{0}, E + i\Gamma)$ develops poles **below** threshold … states for attractive Coulomb potential:

$$G(\vec{0}, E \ldots \sum_n \frac{|\Psi(0)|^2}{E_n - (E + i\Gamma)}$$

with the wave functi… boundstate system at origin $\psi(0)$

Implementation still under construction...

Matching to fixed order

Resummed results added to fixed order cross section at NNLO$_{\text{approx.}}$:

$$\boxed{\sigma_{\text{hadr.}}^{\text{NNLO}_{\text{approx.}}} = \sigma_{\text{hadr.}}^{\text{NLO}} + \Delta\sigma_{\text{hadr.}}^{\text{NNLO}_{\text{approx.}}}}$$

(ΔNNLO$_{\text{approx.}}$ consists of dominant terms in β for $\beta \to 0$ for arbitrary colour representations *[Beneke, Czakon, Falgari, Mitov, Schwinn '09]*)

Total resummed cross section:

$$\sigma_{\text{hadr.}}^{\text{NNLL matched}}(\rho) = \sigma_{\text{hadr.}}^{\text{NNLO}_{\text{approx.}}}(\rho) + \sum_{\text{flavours } i,j} \frac{1}{2\pi i} \int_{\text{CT}} \mathrm{d}N\, \rho^{-N} \tilde{f}_i(N+1) \tilde{f}_j(N+1) \times \left[\tilde{\sigma}_{ij\to kl}^{(\text{res.})}(N) - \tilde{\sigma}_{ij\to kl}^{(\text{res.})}(N)\Big|_{\text{NNLO}} \right]$$

→ NNLO matching needed to avoid double counting

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Matching to fixed order

Resummed results added to fixed order cross section at NNLO$_{\text{approx.}}$:

$$\sigma^{\text{NNLO}_{\text{approx.}}}_{\text{hadr.}} = \boxed{\sigma^{\text{NLO}}_{\text{hadr.}}} + \Delta\sigma^{\text{NNLO}_{\text{approx.}}}_{\text{hadr.}}$$

PROSPINO

(ΔNNLO$_{\text{approx.}}$ consists of dominant terms in β for $\beta \to 0$ for arbitrary colour representations *[Beneke, Czakon, Falgari, Mitov, Schwinn '09]*)

Total resummed cross section:

PDFs in Mellin space

$$\sigma^{\text{NNLL matched}}_{\text{hadr.}}(\rho) = \sigma^{\text{NNLO}_{\text{approx.}}}_{\text{hadr.}}(\rho) + \sum_{\text{flavours } i,j} \frac{1}{2\pi i} \int_{\text{CT}} \mathrm{d}N\, \rho^{-N} \boxed{\tilde{f}_i(N+1)\tilde{f}_j(N+1)} \times \left[\tilde{\sigma}^{(\text{res.})}_{ij\to kl}(N) - \tilde{\sigma}^{(\text{res.})}_{ij\to kl}(N)\Big|_{\text{NNLO}} \right]$$

$\rho = 4m^2/S$

$\rightarrow$ NNLO matching needed to avoid double counting

Matching to fixed order

Resummed results added to fixed order cross section at NNLO$_{\text{approx.}}$:

$$\sigma^{\text{NNLO}_{\text{approx.}}}_{\text{hadr.}} = \boxed{\sigma^{\text{NLO}}_{\text{hadr.}}} + \Delta\sigma^{\text{NNLO}_{\text{approx.}}}_{\text{hadr.}}$$

(ΔNNLO$_{\text{approx.}}$ consists of dominant terms in β for $\beta \to 0$ for arbitrary colour representations *[Beneke, Czakon, Falgari, Mitov, Schwinn '09]*)

PROSPINO 2

Total resummed cross section:

PDFs in Mellin space

$$\sigma^{\text{NNLL matched}}_{\text{hadr.}}(\rho) = \sigma^{\text{NNLO}_{\text{approx.}}}_{\text{hadr.}}(\rho) + \sum_{\text{flavours } i,j} \frac{1}{2\pi i} \int_{\text{CT}} \mathrm{d}N\, \rho^{-N} \boxed{\tilde{f}_i(N+1)\tilde{f}_j(N+1)} \times \left[\tilde{\sigma}^{(\text{res.})}_{ij\to kl}(N) - \tilde{\sigma}^{(\text{res.})}_{ij\to kl}(N)\Big|_{\text{NNLO}}\right]$$

$\rho = 4m^2/S$

→ NNLO matching needed to avoid double counting

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Matching to fixed order

Resummed results added to fixed order cross section at NNLO$_{\text{approx.}}$:

$$\sigma_{\text{hadr.}}^{\text{NNLO}_{\text{approx.}}} = \boxed{\sigma_{\text{hadr.}}^{\text{NLO}}} + \Delta\sigma_{\text{hadr.}}^{\text{NNLO}_{\text{approx.}}}$$

(ΔNNLO$_{\text{approx.}}$ consists of dominant terms in β for $\beta \to 0$ for arbitrary colour representations *[Beneke, Czakon, Falgari, Mitov, Schwinn '09]*)

PROSPINO (2)

Total resummed cross section:

PDFs in Mellin space

$$\sigma_{\text{hadr.}}^{\text{NNLL matched}}(\rho) = \sigma_{\text{hadr.}}^{\text{NNLO}_{\text{approx.}}}(\rho) + \sum_{\text{flavours } i,j} \frac{1}{2\pi i} \int_{\text{CT}} dN\, \rho^{-N} \boxed{\tilde{f}_i(N+1)\tilde{f}_j(N+1)} \times \left[\tilde{\sigma}_{ij\to kl}^{(\text{res.})}(N) - \tilde{\sigma}_{ij\to kl}^{(\text{res.})}(N)\Big|_{\text{NNLO}} \right]$$

$\rho = 4m^2/S$

→ NNLO matching needed to avoid double counting

WESTFÄLISCHE WILHELMS-UNIVERSITÄT MÜNSTER

:-)

Outline

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Theoretical status - fixed order

- QCD and SUSY-QCD effects:
  - LO $\sim \mathcal{O}(\alpha_s^2)$

    [Kane, Leveille '82][Harrison, Llewellyn Smith '83][Reya, Roy '85][Dawson, Eichten, Quigg '85][Baer, Tata '85]
  - NLO $\sim \mathcal{O}(\alpha_s^3)$

    [Beenakker, Höpker, Spira, Zerwas '96][Beenakker, Krämer, Plehn, Spira, Zerwas '97]
  - Approximated NNLO for $\tilde{q}\tilde{q}^*$, $\tilde{g}\tilde{g}$, $\tilde{t}\tilde{t}^* \sim \mathcal{O}(\alpha_s^4)$

    [Langenfeld, Moch '09][Langenfeld, Moch, Pfoh '12][Langenfeld '11][Broggio, Ferroglia, Neubert, Vernazza, Yang '13]

Corrections of the order of a few tens of percents

- Electroweak effects: LO $\sim \mathcal{O}(\alpha\alpha_s), \mathcal{O}(\alpha^2)$, and NLO corrections $\sim \mathcal{O}(\alpha\alpha_s^2)$

  [Bozzi, Fuks, Klasen '05][Alan, Cankocak, Demir '07][Bornhauser et al. '07][Hollik, Kollar, Trenkel '07][Hollik, Mirabella '08][Hollik, Mirabella, Trenkel '08][Germer, Hollik, Mirabella, Trenkel '10][Germer, Hollik, Mirabella '11][Mirabella '09]

Corrections of the order of a few percents

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Theoretical status - resummation

- NLO+NLL for all processes, NLO+NNLL for $\tilde{q}\tilde{q}^*$

  [Kulesza, Motyka '08-'09][Beenakker, Brensing, Kulesza, Laenen, Niessen '09-'10][Beenakker, Brensing, Kulesza, Laenen, Niessen '11]

- $\text{NNLO}_{\text{approx.}}$+NNLL for $\tilde{g}\tilde{g}$

  [Pfoh '13]

- NLO+NNLL for $\tilde{t}\tilde{t}^*$

  [Broggio, Ferroglia, Neubert, Vernazza, Yang '13]

- Resummed LO Coulomb corrections for $\tilde{q}\tilde{q}^*$ and $\tilde{g}\tilde{g}$

  [Kulesza, Motyka '09]

- Bound state effects for $\tilde{g}\tilde{g}$ and $\tilde{q}\tilde{g}$

  [Hagiwara, Yokoya '09][Kauth, Kühn, Marquard, Steinhauser '10-'11][Kauth, Kress, Kühn '11]

- SCET, combined Coulomb and soft-gluon resummation up to $\text{NNLO}_{\text{approx.}}$+NNLL for $\tilde{g}\tilde{g}$, $\tilde{q}\tilde{q}^*$, $\tilde{q}\tilde{g}$, $\tilde{q}\tilde{q}$

  [Beneke, Schwinn, Falgari '09-'10][Falgari, Schwinn, Wever '12][Beneke, Falgari, Piclum, Schwinn, Wever '13-'14]

Corrections of the order of ~1-100 percents (wrt. NLO)

Theoretical status - resummation

- NLO+NLL for all processes, NLO+NNLL for $\tilde{q}\tilde{q}^*$

  [Kulesza, Motyka '08-'09][Beenakker, Brensing, Kulesza, Laenen, Niessen '09-'10][Beenakker, Brensing, Kulesza, Laenen, Niessen '11]

- NNLO$_{\text{approx.}}$+NNLL for $\tilde{g}\tilde{g}$

  [Pfoh '13]

Now:

NLO/NNLO$_{\text{approx.}}$+NNLL for $\tilde{g}\tilde{g}$, $\tilde{q}\tilde{q}^*$, $\tilde{q}\tilde{g}$, $\tilde{q}\tilde{q}$, $\tilde{t}\tilde{t}^*$ with Mellin-space resummation

- Bound state effects for $\tilde{g}\tilde{g}$ and $\tilde{q}\tilde{g}$

  [Hagiwara, Yokoya '09][Kauth, Kühn, Marquard, Steinhauser '10-'11][Kauth, Kress, Kühn '11]

- SCET, combined Coulomb and soft-gluon resummation up to NNLO$_{\text{approx.}}$+NNLL for $\tilde{g}\tilde{g}$, $\tilde{q}\tilde{q}^*$, $\tilde{q}\tilde{g}$, $\tilde{q}\tilde{q}$

  [Beneke, Schwinn, Falgari '09-'10][Falgari, Schwinn, Wever '12][Beneke, Falgari, Piclum, Schwinn, Wever '13-'14]

Corrections of the order of ~1-100 percents (wrt. NLO)

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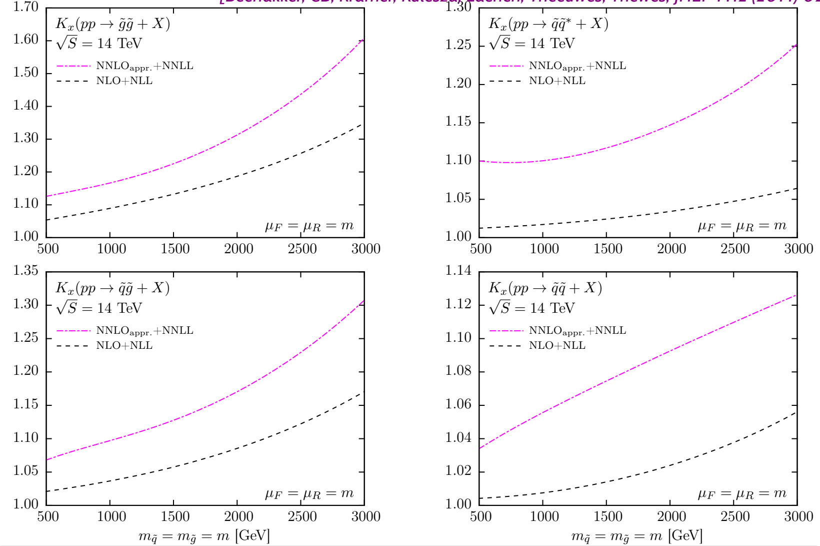
Results: $\tilde{g}\tilde{g}$, $\tilde{q}\tilde{q}^*$, $\tilde{q}\tilde{g}$, $\tilde{q}\tilde{q}$ with soft + Coulomb resum.

[Beenakker, CB, Krämer, Kulesza, Laenen, Theeuwes, Thewes; JHEP 1412 (2014) 023]



$K_x = \sigma^x/\sigma^{\text{NLO}}$; NNLO$_{\text{appr.}}$+NNLL with hard-matching coeffs. and 2-loop Coulomb

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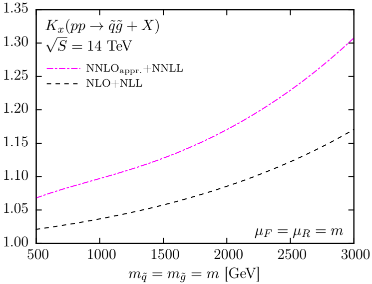

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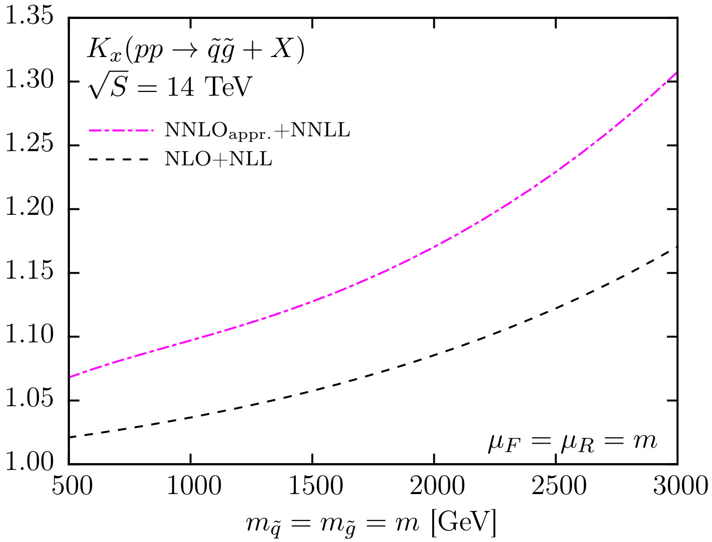

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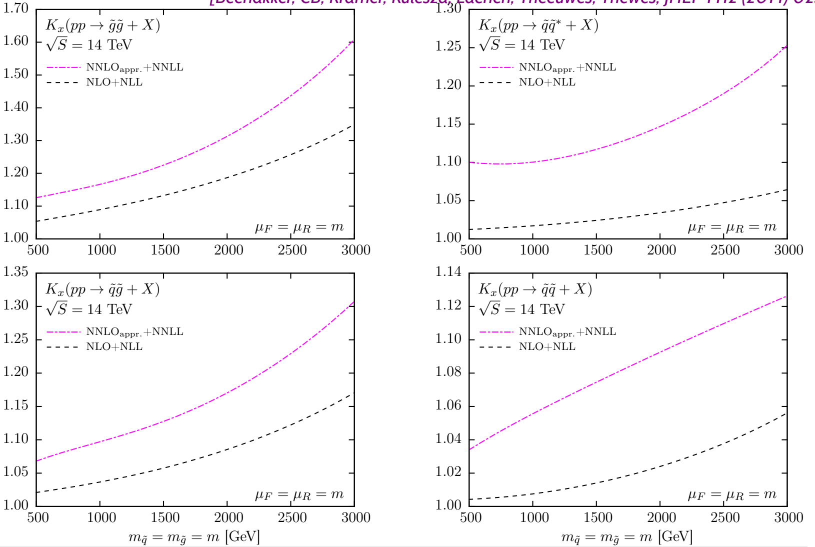

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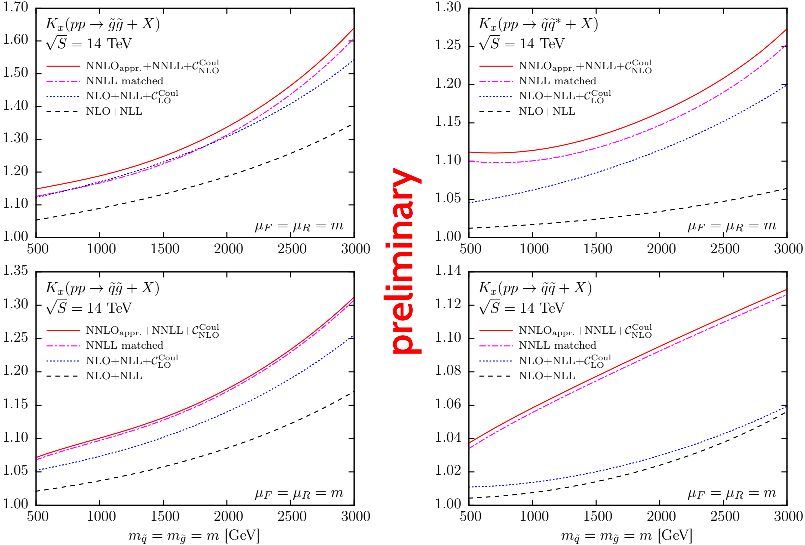

$K_x = \sigma^x/\sigma^{\rm NLO}$; $C^{\rm Coul}_{\rm LO}$ & $C^{\rm Coul}_{\rm NLO}$: resummed Coulomb corrections from LO & NLO potential

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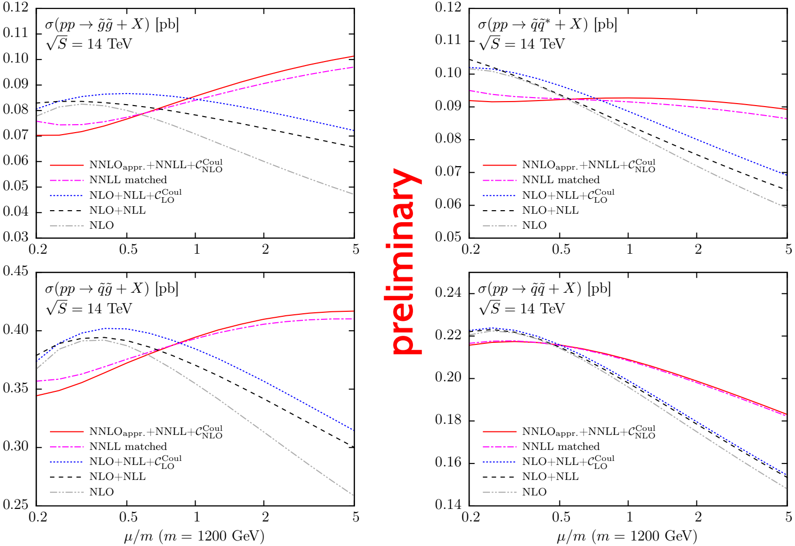

Using MSTW 2008 PDFs (NLO & NNLO); squark and gluino masses set to 1200 GeV

Intermezzo: why are stops different?

Stops in the weak interaction basis do not correspond to physical states

→ From the MSSM Lagrangian, mass terms for the stops can be extracted:

$$\mathscr{L}_{\mathrm{MSSM}} \to \begin{pmatrix} \tilde{t}_L^* & \tilde{t}_R^* \end{pmatrix} \begin{pmatrix} M^2_{\tilde{t},LL} & \Delta_{\tilde{t},LR} \\ \Delta_{\tilde{t},RL} & M^2_{\tilde{t},RR} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}$$

(in the basis of "left-" and "right-handed" stop fields, non-physical basis)

Eigenvalues correspond to the physical masses $m^2_{\tilde{t}_i}$ of the stops ($i = 1, 2$)

→ $\boxed{\Delta_{\tilde{t},LR} = \Delta^*_{\tilde{t},RL} \propto m_t \gg 0}$: $m^2_{\tilde{t}_i}$ contain a mixture of L and R

Intermezzo: why are stops different?

Stops in the weak interaction basis do not correspond to physical states

$\Rightarrow$ Rotate into physical basis:

$$\boxed{\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\tilde{t}} & \sin\theta_{\tilde{t}} \\ -\sin\theta_{\tilde{t}} & \cos\theta_{\tilde{t}} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}}$$

(light-flavoured squark mixing is negligible due to small quark masses)

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Additional parameters in Feynman rules: $\theta_{\tilde{t}}, m_{\tilde{q}}, m_{\tilde{g}}, m_{\tilde{t}_2}$

$\rightarrow$ From benchmark point 40.2.5 *[AbdusSalam, Allanach, Dreiner, Ellis et al.; arXiv: 1109.3859]*

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In many SUSY scenarios, $\tilde{t}_1$ is among the lightest sparticles $\Rightarrow$ easy detection?

Otherwise similar to $\tilde{q}\tilde{q}^*$ production with certain production channels suppressed by very small PDFs

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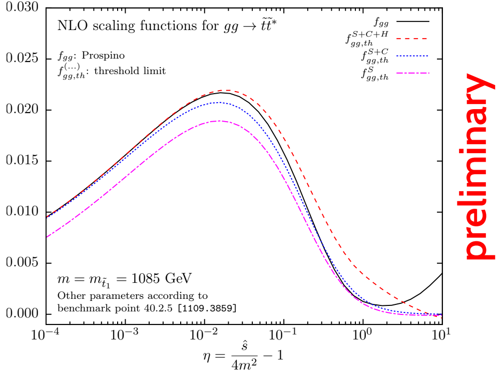
Results: $\tilde{t}_1\tilde{t}_1^*$ production



From:

$$\hat{\sigma}_{ij} = \frac{\alpha_s^2}{m_{\tilde{t}_1}^2}\left\{f_{ij}^B + 4\pi\alpha_s\left[f_{ij} + \bar{f}_{ij}\ln\left(\frac{\mu^2}{m_{\tilde{t}_1}^2}\right)\right]\right\}$$

with:
- S: soft-gluon logarithms
- C: Coulomb terms
- H: hard-matching coefficient *[Broggio et al. '13]*

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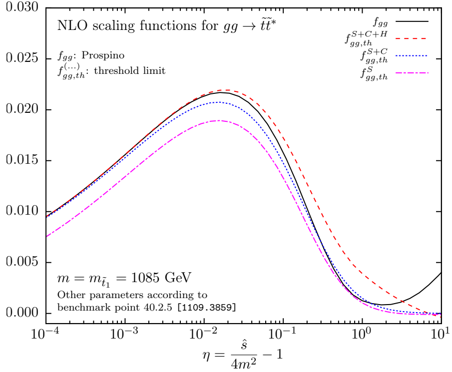

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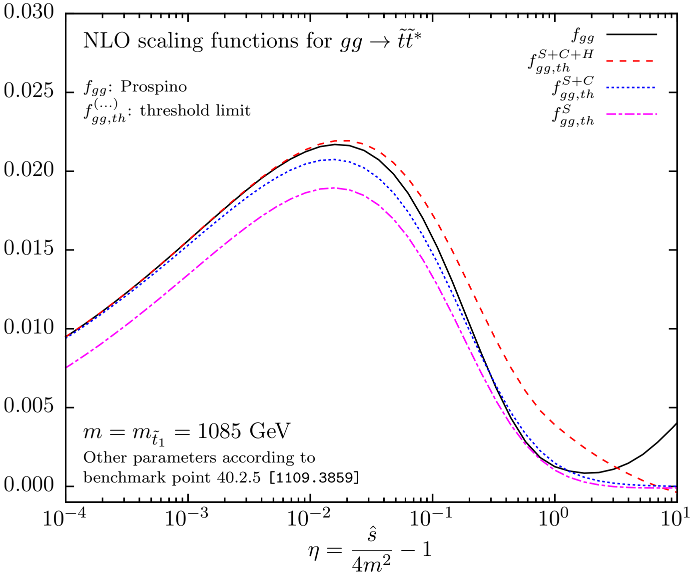

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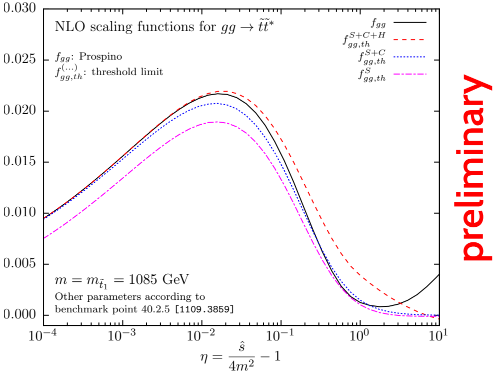

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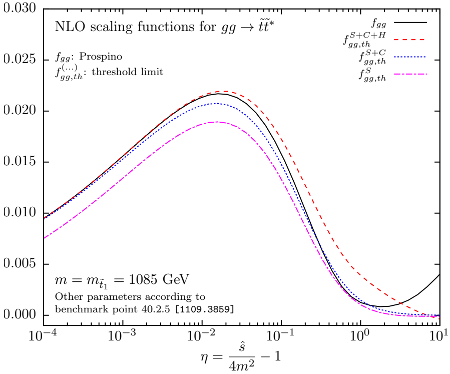
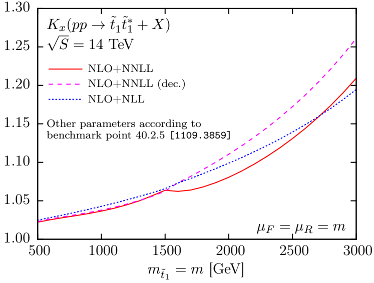
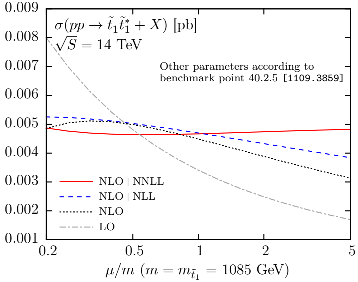


preliminary





Benchmark point 40.2.5

$$\begin{cases} \sin 2\theta_{\tilde{t}} = 0.669 \\ m_{\tilde{q}} = 1496 \text{ GeV} \\ m_{\tilde{g}} = 1493 \text{ GeV} \\ m_{\tilde{t}_2} = 1321 \text{ GeV} \end{cases}$$

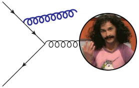
[arXiv: 1109.3859]

Summary

SUSY at the LHC:

- SUSY-QCD processes **dominantly produced** at hadron colliders
- Stops interesting in particular due to **light mass**

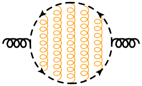
Soft-gluon resummation:



- Higher-order terms produce **large logs** at threshold
- **Factorisation** of contributions in **Mellin-moment space**
- Threshold region **contributing significantly** to total cross section

Coulomb contributions:

- Coulomb exchange important for **slowly moving heavy particles**
- Possibility to resum these terms with **non-relativistic methods**
- Possibility of **boundstate creation**



Conclusions and outlook

Conclusions:

✓ Corrections from **soft-gluon** and **Coulomb effects** can be sizeable
→ Coulomb contributions mainly covered by two-loop terms

✓ $\tilde{q}\tilde{q}$ and $\tilde{q}\tilde{g}$ **largest processes at the LHC** for high $m_{\tilde{q}}$ and $m_{\tilde{g}}$

✓ **Enhancement of the K factor** and **improvement of the theoretical uncertainty**

✓ $\tilde{t}_1\tilde{t}_1^*$ **dependence on additional SUSY parameters increased** at NLO+NNLL with respect to NLO due to hard-matching coefficients

Outlook:

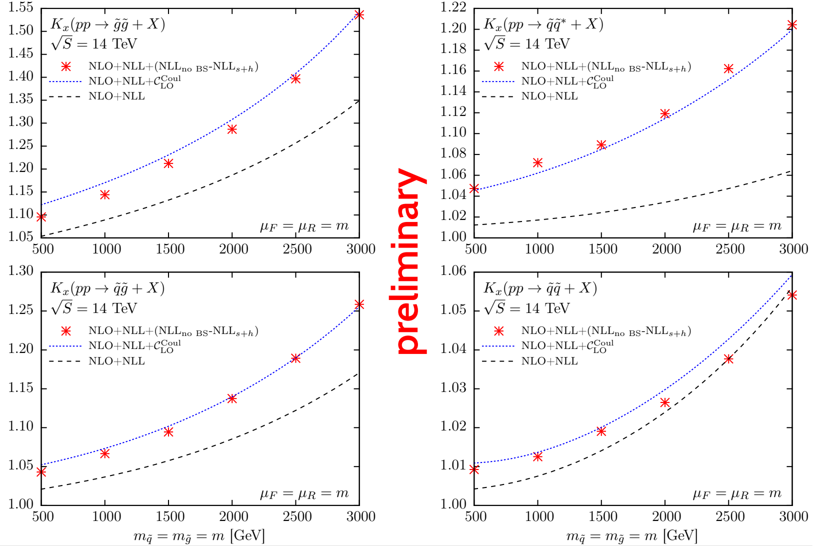
✍ Boundstate corrections

✍ Comparison with SCET resummation method

✍ Public code: NNLL-fast

:-)

Comparison to SCET: NLL with Coulomb resum.



$K_x = \sigma^x/\sigma^{\text{NLO}}$; NLL$_{\text{no BS}}$ & NLL$_{s+h}$ from *[Falgari, Schwinn, Wever; arXiv: 1202.2260]*