

Parton distribution functions for heavy ion collisions @ LHC

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Outline

1. Introduction to PDFs
2. Motivations & review of available nPDFs
3. Details of nuclear CTEQ analysis
4. Future experiments and nPDF outlook

1. Introduction to PDFs

PDF intro

- Scattering - *THE* tool to study inner structure of atoms, nuclei & proton
- Parton distribution functions & the inner structure of the proton
 - direct descendants of Rutherford's experiments

- **Rutherford's scattering**

spin-0 non-relativistic projectile with very heavy target (no recoil)

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Ruth.}} = \frac{(\alpha Z)^2}{4E^2 \sin^4 \frac{\theta}{2}}.$$



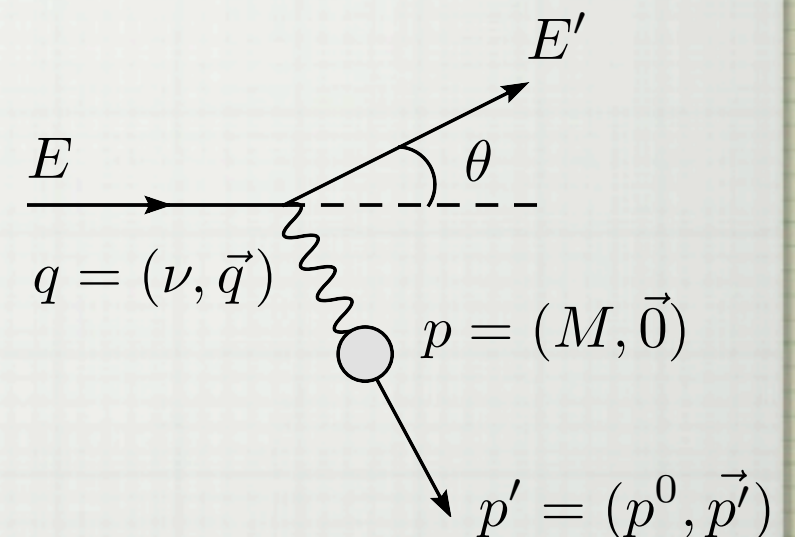
E.Rutherford - nobel prize in chemistry - 1908

- **Mott's scattering**

spin-0 relativistic projectile with recoil of the target

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{(\alpha Z)^2}{4E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{E} \cos^2 \frac{\theta}{2}$$

- **Kinematics of elastic scattering**



PDF intro

- Scattering - *THE* tool to study inner structure of atoms, nuclei & proton
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 - direct descendants of Rutherford's experiments

- **Scattering of electrons on muons**

spin-1/2 relativistic projectile scattering on muons

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{E} \left[\cos^2 \frac{\theta}{2} - \frac{q^2}{2M^2} \sin^2 \frac{\theta}{2} \right]$$

- **Rosenbluth's scattering**

spin-1/2 relativistic projectile scattering on non-point like target with spin-1/2 (proton)

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{E} \left[\left(F_1(q)^2 - \frac{\kappa^2 q^2}{4M^2} F_2(q)^2 \right) \cos^2 \frac{\theta}{2} - \frac{q^2}{2M^2} (F_1(q) + \kappa F_2(q))^2 \sin^2 \frac{\theta}{2} \right]$$

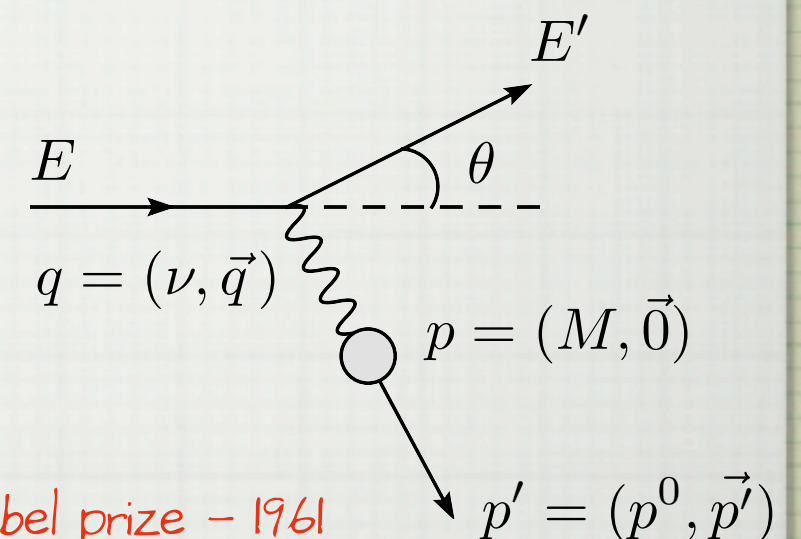


electric form-factor



magnetic form-factor

- **Kinematics of elastic scattering**



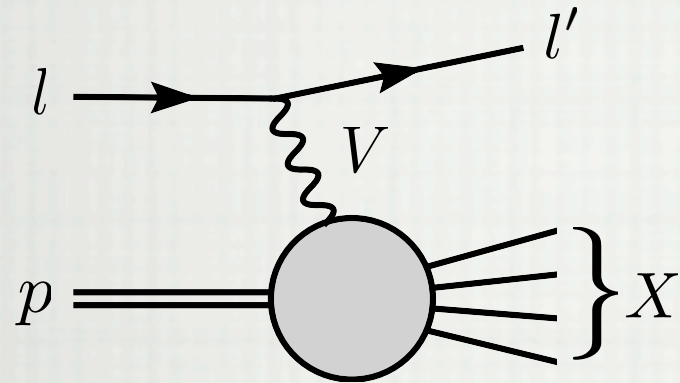
R.Hofstadter - nobel prize - 1961

PDF intro

- Deep Inelastic Scattering



R.Taylor, H.Kendall, J.Friedman – nobel prize – 1990



$$l(k) + p(p) \rightarrow l'(k') + X$$

- Kinematic variables

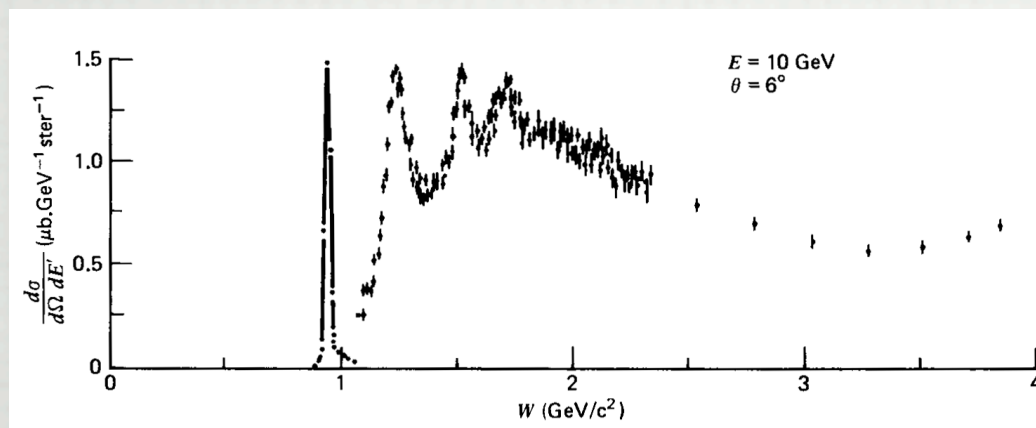
$$q = k - k' \quad x = \frac{Q^2}{2p \cdot q} \quad x \in (0, 1)$$

$$Q^2 = -q^2$$

- Distinguishing between elastic & inelastic scattering

$$W^2 = (p'_1 + p'_2 + \dots + p'_n)^2$$

- if $W^2 = m_p^2$ elastic scattering
- if $W^2 \gg m_p^2$ inelastic scattering



- DIS cross-section & structure functions

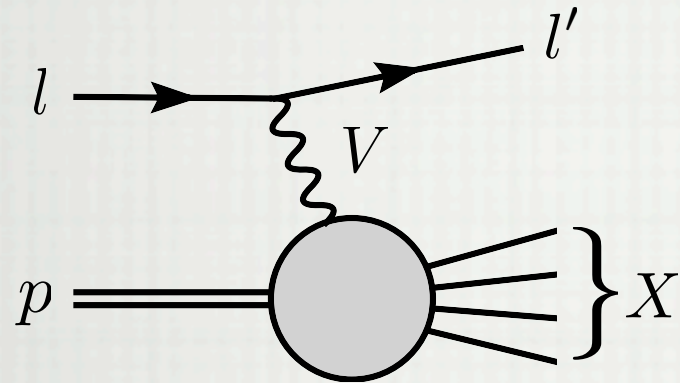
$$\frac{d\sigma}{dE' d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left(\frac{2F_1(x, Q^2)}{M} \sin^2 \frac{\theta}{2} + \frac{F_2(x, Q^2)}{E - E'} \cos^2 \frac{\theta}{2} \right)$$

PDF intro

- Deep Inelastic Scattering



R.Taylor, H.Kendall, J.Friedman – nobel prize – 1990



$$l(k) + p(p) \rightarrow l'(k') + X$$

- Kinematic variables

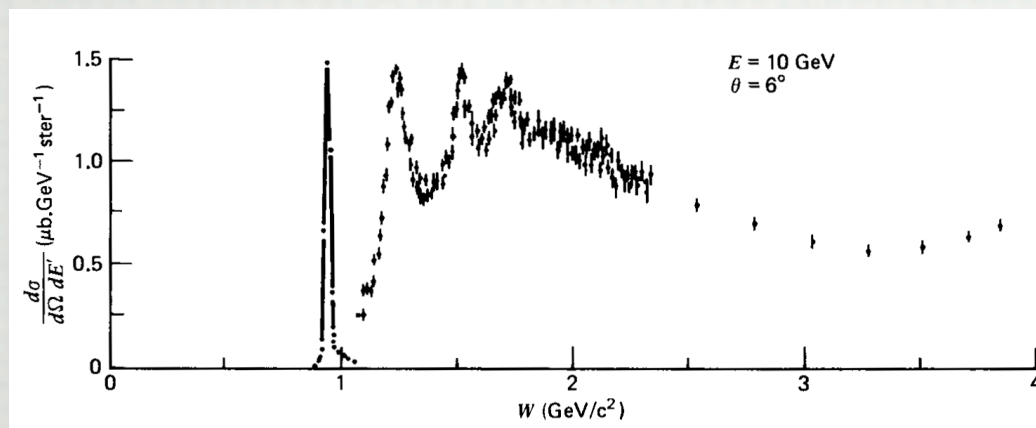
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- Distinguishing between elastic & inelastic scattering

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- Assuming elastic scattering on partons in the proton (in LO)

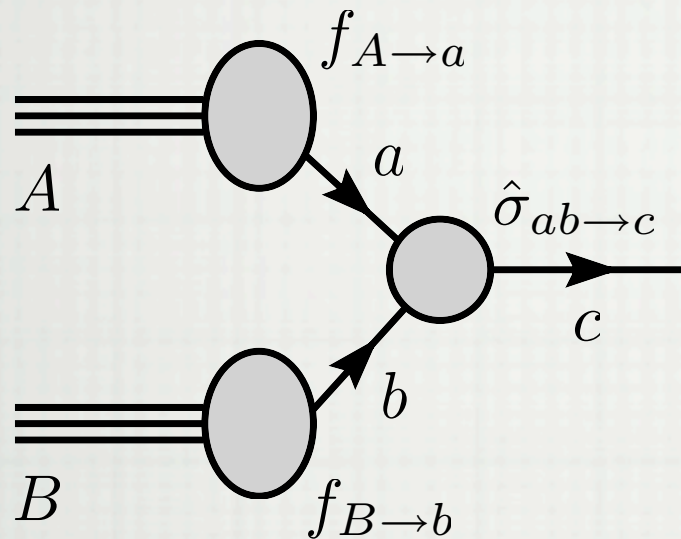
$$F_2(x, \cancel{Q^2}) = \sum_i e_i^2 \int d\xi \xi f_i(\xi) \delta\left(\xi - \frac{Q^2}{2p \cdot q}\right) \quad \text{mom. fraction } \xi = x \text{ kinematic variable}$$



parton distribution function

PDF intro

- Factorization & PDFs



$$\sigma = \int dx_1 dx_2 \underbrace{f_{A \rightarrow a}(x_1, Q^2)}_{\text{from experiment}} \underbrace{f_{B \rightarrow b}(x_2, Q^2)}_{\text{from experiment}} \underbrace{\hat{\sigma}_{ab \rightarrow c}(x_1 x_2 s)}_{\text{from pQCD}}$$

- Parton distribution functions (PDFs)

$$f_{A \rightarrow a}(x, Q^2)$$

- universal, non-perturbative objects
- describe the structure of hadrons
in terms of partons - quarks & gluons
- Q-dependance governed by the DGLAP evolution equations

$$\frac{df_i(x, Q^2)}{d \ln Q^2} = \frac{\alpha_S(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} P_{ij}(y) f_j(x/y, Q^2)$$

- x-dependance determined from a fit to the data

PDF intro

- CTEQ framework to fit PDFs from experimental data

CTEQ6M [hep-ph/0201195](https://arxiv.org/abs/hep-ph/0201195)

- the input scale set to $Q_0 = 1.3 \text{ GeV}$
- parameterisation of the PDFs in x

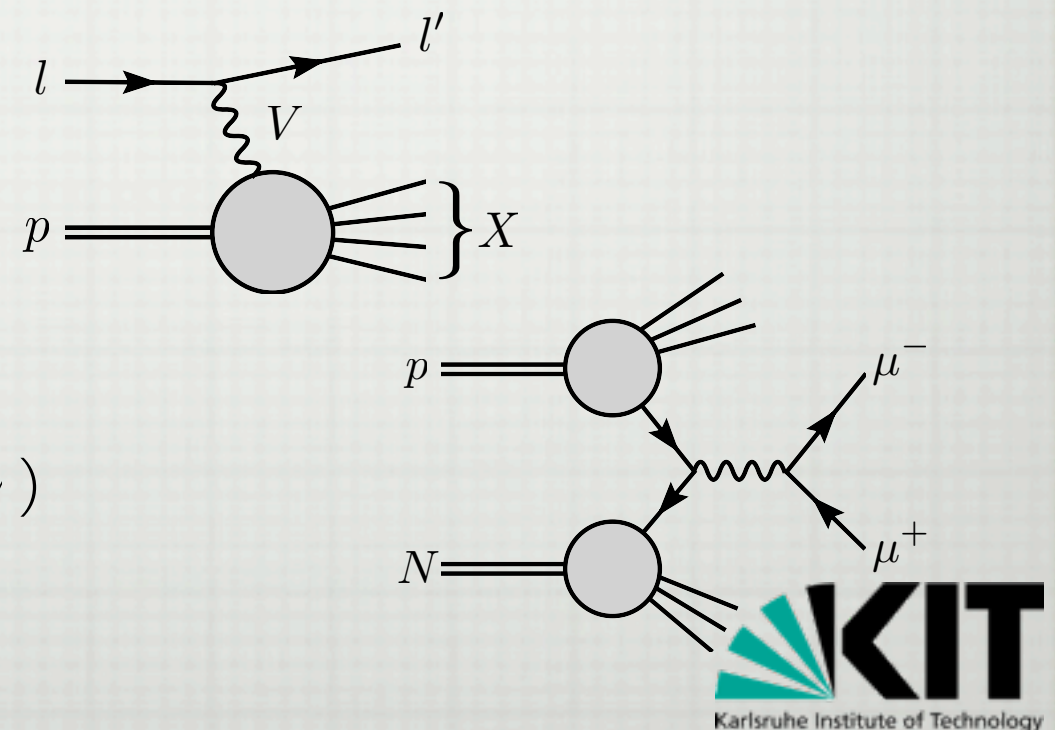
$$x f_k(x, Q_0) = c_0 x^{c_1} (1-x)^{c_2} e^{c_3 x} (1 + e^{c_4 x})^{c_5} \quad k = u_v, d_v, g, \bar{u} + \bar{d}, s, \bar{s}$$

$$\bar{d}(x, Q_0)/\bar{u}(x, Q_0) = c_0 x^{c_1} (1-x)^{c_2} + (1 + c_3 x)(1-x)^{c_4}$$

- make sure # of free parameters not too high - CTEQ approx. 20 free params
- carefully choose data sets & kinematic cuts to constrain free parameters
- perform χ^2 fit to data

- Which data sets are included ?

- Deep Inelastic Scattering ($l^\pm p, l^- d, \nu N, \bar{\nu} N$)
- Neutrino DIS di-muon production
- Drell-Yan & vector boson production (W^\pm, Z^0, γ)
- hadronic jet data



PDF intro

- Some details of PDF determination
 - which data constrain what distribution ?

charged DIS: $F_2(x, \mu^2) = x \sum_i e_q^2 [q_i(x, \mu^2) + \bar{q}_i(x, \mu^2)]$

quark & anti-quark distributions

neutrino DIS:

$$F_2(x, \mu^2) = x \sum_i [q_i(x, \mu^2) + \bar{q}_i(x, \mu^2)]$$

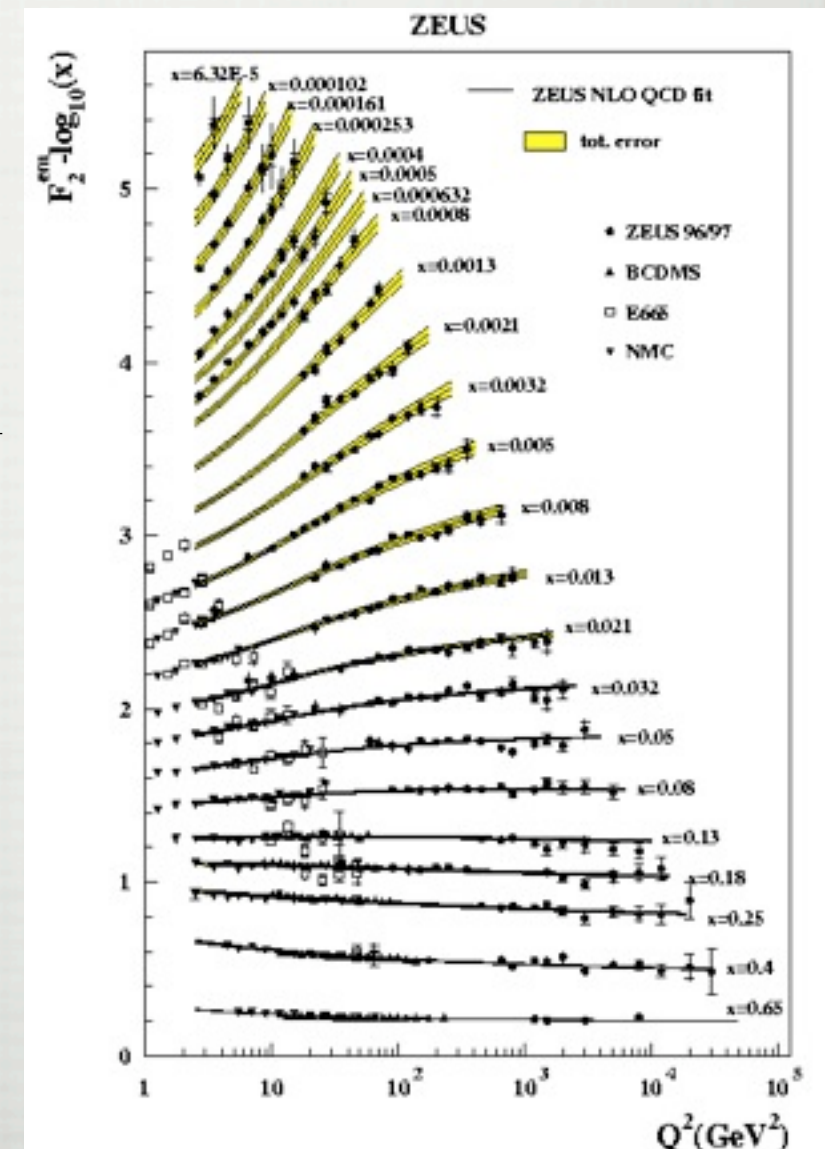
$$xF_3(x, \mu^2) = x \sum_i [q_i(x, \mu^2) - \bar{q}_i(x, \mu^2)]$$

DIS scaling violations & jet data - gluon distribution

- breaking of Bjorken scaling at small x driven by gluon PDF

$$\mu^2 \frac{dF_2}{d\mu^2} = \frac{\alpha_S}{2\pi} \sum_i e_i^2 \int_x^1 \frac{dy}{y} P_{qg}(y) f_g(x/y, \mu^2)$$

- high x behaviour constrained by jet data



2. Nuclear PDFs

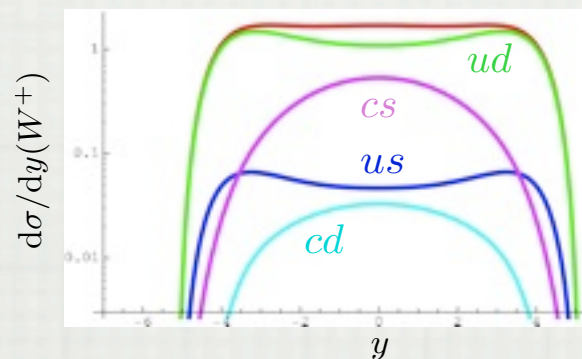
nPDF motivation

- What are nuclear parton density functions (nPDF) ?
 - parton densities for partons in bound proton & neutron

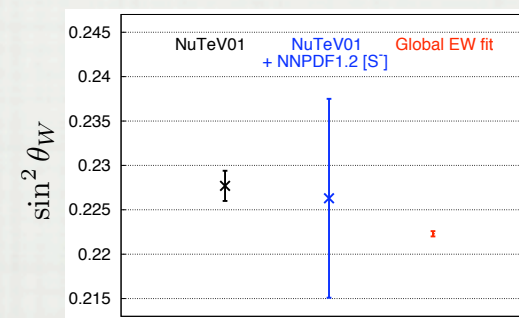
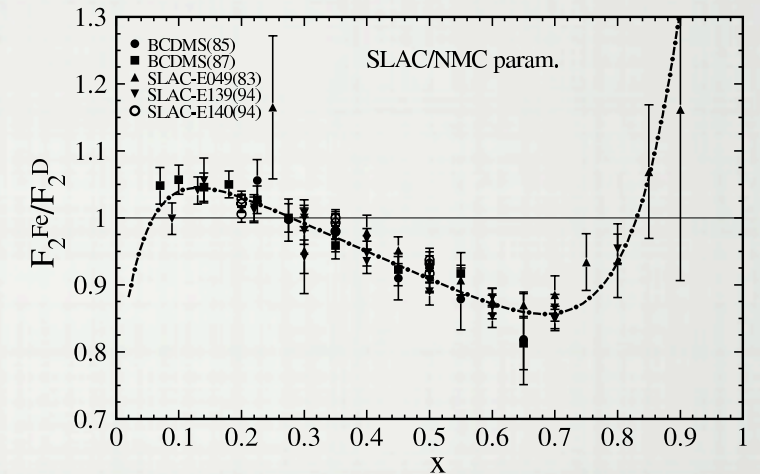
- Where are nuclear parton density functions useful ?

1. Strange quark content of the proton

(anti-)strange PDF from (anti-)neutrino DIS with heavy nuclei - nuclear effects important



W-boson production @ LHC



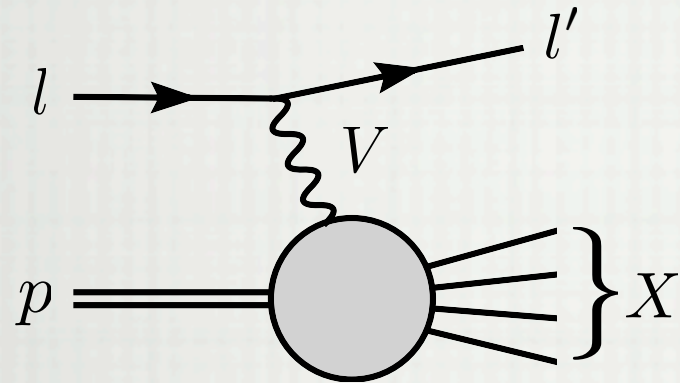
weak mixing angle from
NuTeV experiment

2. Heavy ion collisions @ RHIC, LHC

lead & gold heavy nuclei - nuclear effects in gluon PDF substantial

nPDF motivation

- Deep Inelastic Scattering on nuclei



$$l(k) + N(p_N) \rightarrow l'(k') + X$$

- Kinematic variables of the nucleus

$$q = k - k' \quad Q^2 = -q^2 \quad x_N = \frac{Q^2}{2p_N \cdot q} \quad x_N \in (0, 1)$$

- New kinematic variables for the bound protons

$$p_A = \frac{p_N}{A} \quad \text{average momentum of a nucleon}$$

$$x_A = \frac{Q^2}{2p_A \cdot q} = A x_N \quad x_A \in (0, A)$$

momentum fraction of a single nucleon

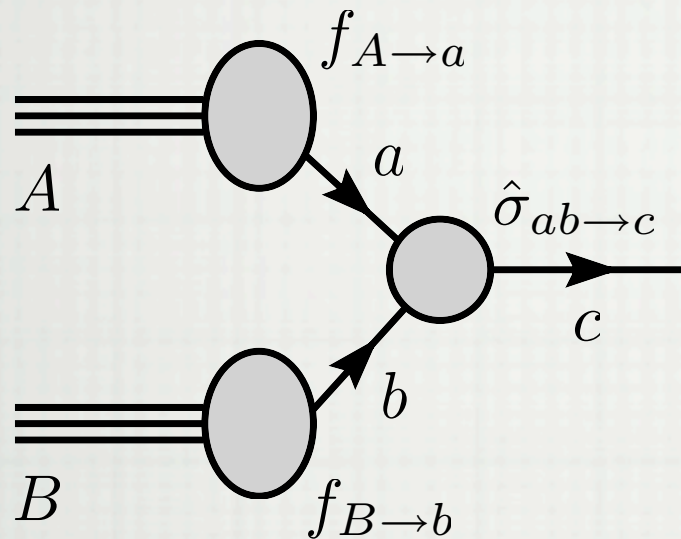
- Distinguishing between elastic & inelastic scattering (assuming interaction only with one nucleon)

$$W^2 = (p'_1 + p'_2 + \dots + p'_n)^2$$

- if $W^2 = m_p^2$ elastic scattering
- if $W^2 \gg m_p^2$ inelastic scattering

nPDF motivation

- Factorization & PDFs



$$\sigma = \int dx_1 dx_2 \underbrace{f_{A \rightarrow a}(x_1, Q^2)}_{\text{from experiment}} \underbrace{f_{B \rightarrow b}(x_2, Q^2)}_{\text{from experiment}} \underbrace{\hat{\sigma}_{ab \rightarrow c}(x_1 x_2 s)}_{\text{from pQCD}}$$

- Parton distribution functions (PDFs)

$$f_{A \rightarrow a}(x, Q^2)$$

- universal, non-perturbative objects
- describe the structure of hadrons
(in terms of partons - quarks & gluons)
- obey DGLAP evolution equations

- Nuclear parton distribution functions (nPDFs)

- distributions of partons bound inside protons & neutrons in nuclei
- simple assumption that nuclear modifications of proton PDF depends on A - # of nucleons

nPDF review

- Review of existing global analyses of nuclear PDF

1. Multiplicative nuclear correction factor

$$f_i^A(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i(x_N, Q_0^2)$$


bound parton density


free parton density

Hirai, Kumano, Nagai [[PRC76\(2007\)065207](#)] arXiv: 0709.0338

Eskola, Paukkunen, Salgado [[JHEP0904\(2009\)065](#)] arXiv: 0902.4154


de Florian, Sassot, Stratmann, Zurita [[PRD85\(2012\)074028](#)] arXiv: 1112.6324

2. Native nuclear PDF

$$f_i^A(x_N, Q_0^2) = f_i(x_N, A, Q_0^2)$$


bound parton density

$$f_i(x_N, Q_0^2) = f_i(x_N, A = 1, Q_0^2)$$


free parton density

nCTEQ [[PRD80\(2009\)094004](#)] arXiv: 0907.2357

nPDF review

- Review of existing global analyses of nuclear PDF

HIRAI, KUMANO, NAGAI'07 [PRC76(2007)065207]
LO, NLO, ERROR PDFS

- uses multiplicative factor

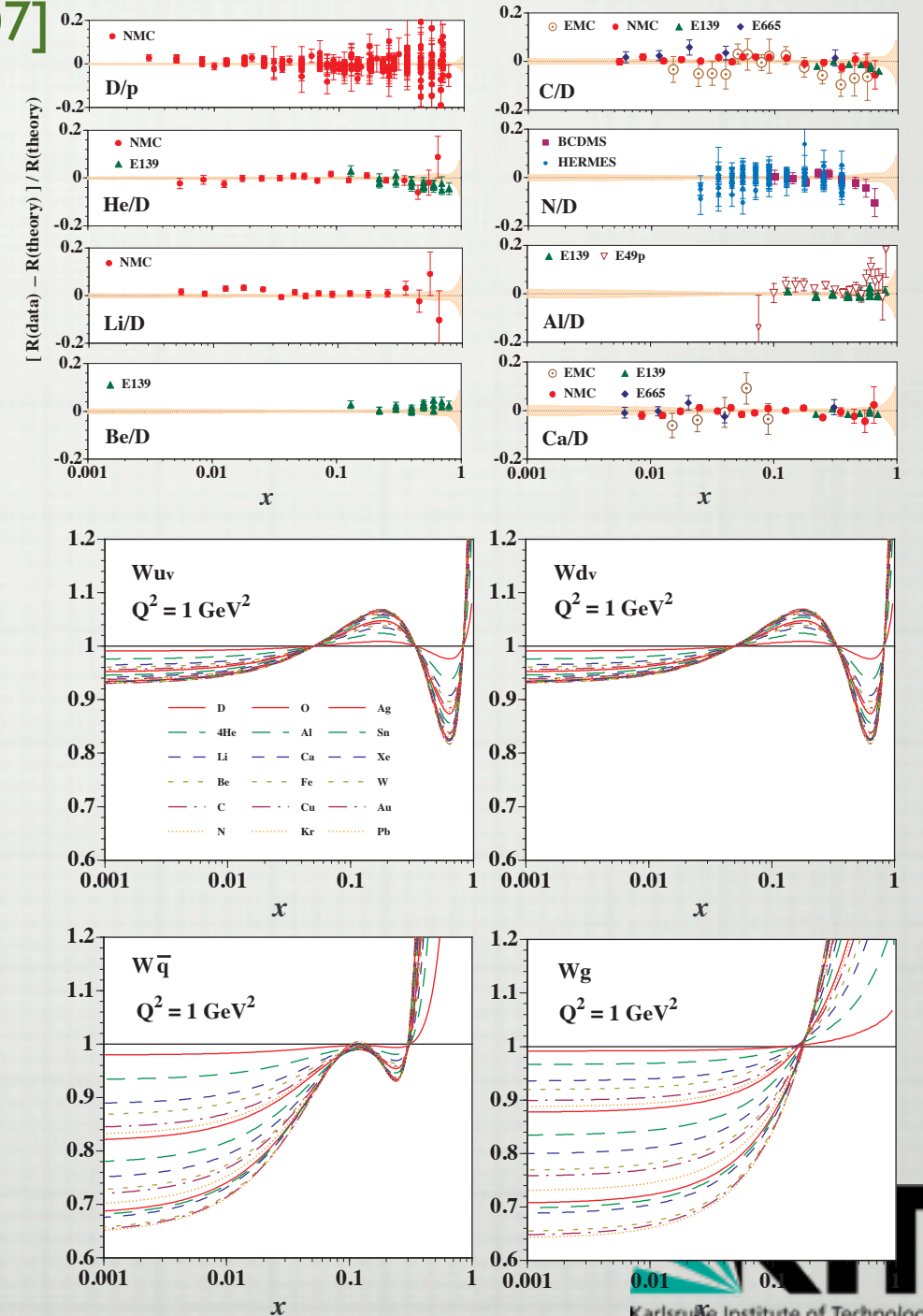
$$f_i^A(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i(x_N, Q_0^2)$$

where proton PDF in MRST 1998 and factor

$$R_i(x, A, Z) = 1 + \left(1 - \frac{1}{A^\alpha}\right) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1 - x)^{\beta_i}}$$

- neglects region $x > 1$
- includes all current DIS & DY data set (same as our analysis - discussed later)
- use Hessian method to produce error PDFs

$$\chi^2_{\text{dof}} = 1.2$$



nPDF review

- Review of existing global analyses of nuclear PDF

ESKOLA, PAUKKUNEN, SALGADO'09 [JHEP0904(2009)065]
LO, NLO, ERROR PDFS

$$\chi^2/\text{dof} = 0.8$$

- uses multiplicative factor

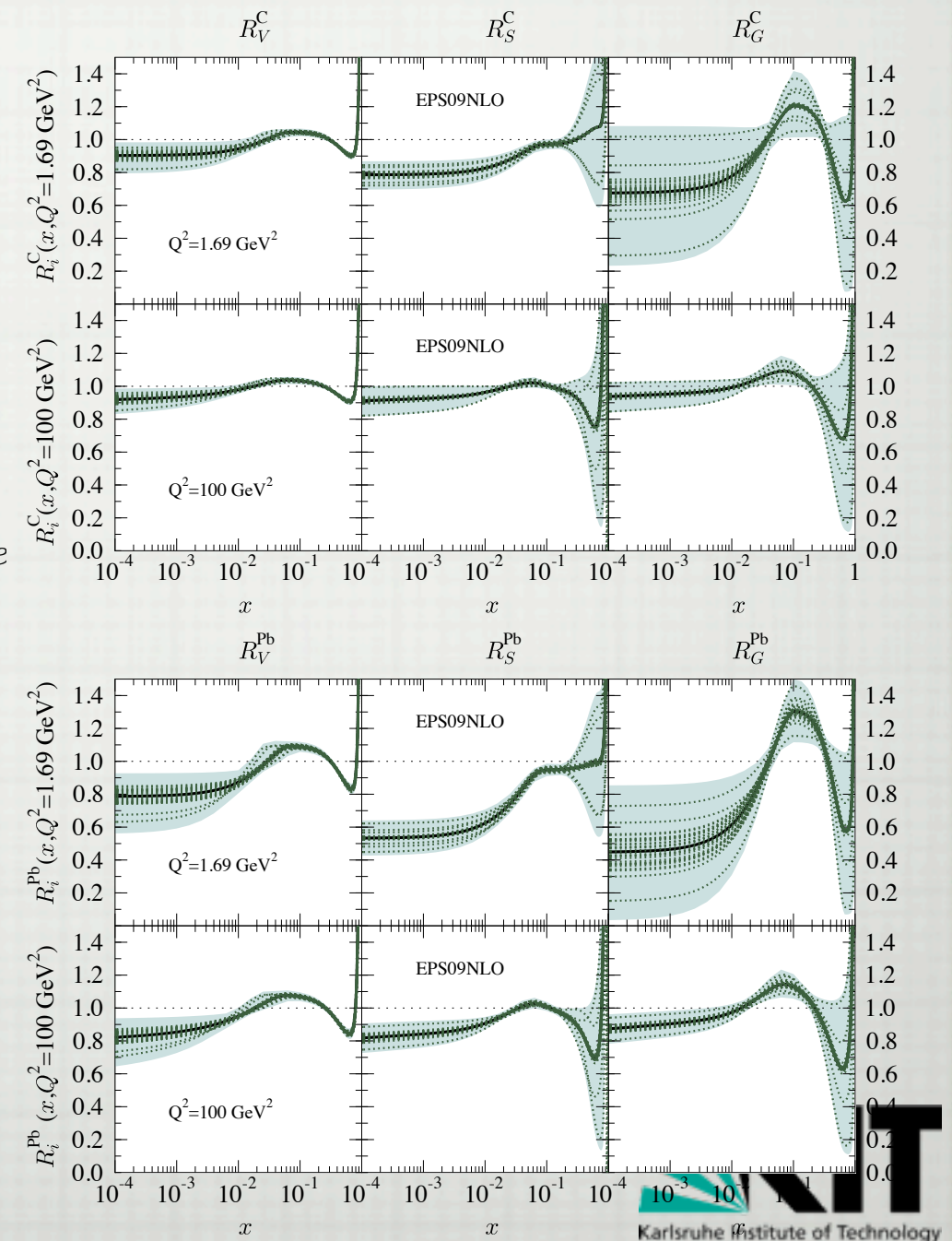
$$f_i^A(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i(x_N, Q_0^2)$$

where proton PDF in CTEQ6.1M and factor is a complicated piecewise defined function

$$R_i(x, A, Z) = \begin{cases} a_0 + (a_1 + a_2 x)(e^{-x} - e^{-x_a}) & x \leq x_a \\ b_0 + b_1 x + b_2 x^2 + b_3 x^3 & x_a \leq x \leq x_e \\ c_0 + (c_1 - c_2 x)(1 - x)^{-\beta} & x_e \leq x \leq 1 \end{cases}$$

with A-dependent parameters

- neglects region $x > 1$
- includes all current DIS & DY data set & π^0 RHIC data to constrain gluon
- use Hessian method to produce error PDFs



nPDF review

- Review of existing global analyses of nuclear PDF

DE FLORIAN, SASSOT, STRATMANN, ZURITA [PRD85(2012)074028]
LO, NLO, ERROR PDFS

- uses multiplicative factor

$$f_i^A(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i(x_N, Q_0^2)$$

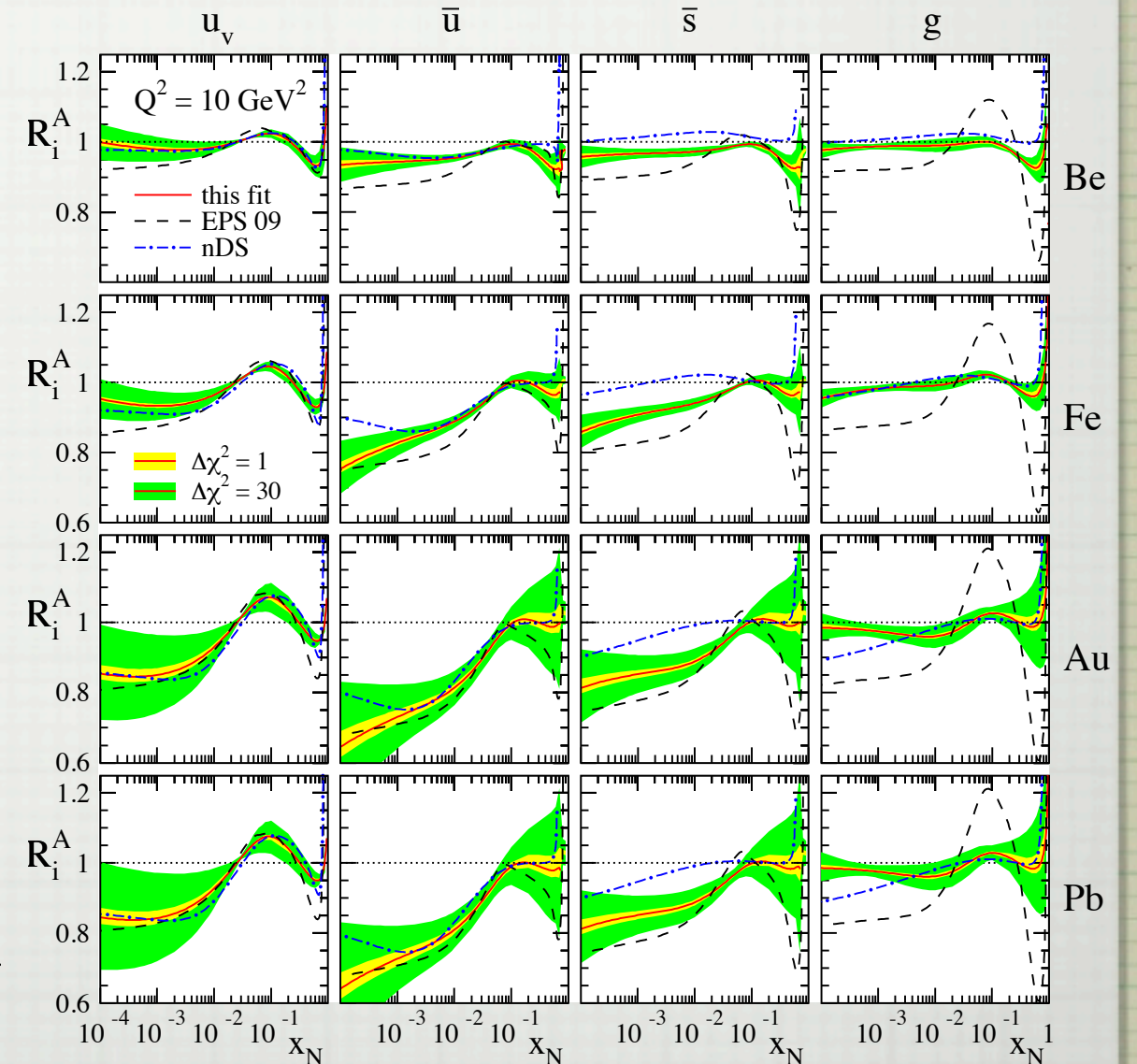
where proton PDF in MSTW08 and factor is a complicated function different for each flavour

$$R_v^A(x, Q_0^2) = \epsilon_1 x^{\alpha_v} (1-x)^{\beta_1} (1 + \epsilon_2 (1-x)^{\beta_2}) \times (1 + a_v (1-x)^{\beta_3})$$

$$R_s^A(x, Q_0^2) = R_v^A(x, Q_0^2) \frac{\epsilon_s}{\epsilon_1} \frac{1 + a_s x^{\alpha_s}}{a_s + 1}$$

$$R_g^A(x, Q_0^2) = R_v^A(x, Q_0^2) \frac{\epsilon_g}{\epsilon_1} \frac{1 + a_g x^{\alpha_g}}{a_g + 1}$$

- includes all current DIS & DY data set & π^0 RHIC data and $F_2^{\nu A}$ from neutrino data
- use Hessian method to produce error PDFs



- CTEQ framework for nuclear PDF - based on CTEQ6M proton fit

- functional form for bound protons same as for free proton PDF (restrict x to $0 < x < 1$)

$$x f_k(x, Q_0) = c_0 x^{c_1} (1 - x)^{c_2} e^{c_3 x} (1 + e^{c_4 x})^{c_5} \quad k = u_v, d_v, g, \bar{u} + \bar{d}, s, \bar{s}$$

$$\bar{d}(x, Q_0)/\bar{u}(x, Q_0) = c_0 x^{c_1} (1 - x)^{c_2} + (1 + c_3 x)(1 - x)^{c_4}$$

- coefficients with A-dependance (reduces to proton for $A=1$)

$$c_k \rightarrow c_k(A) \equiv c_{k,0} + c_{k,1} (1 - A^{-c_{k,2}}), \quad k = \{1, \dots, 5\}$$

- proton coefficients $c_{k,0}$ fixed to special CTEQ6M fit without much of nuclear data

- PDF for a nucleus with A-nucleons out of which Z-protons

$$f_i^{(A,Z)}(x, Q) = \frac{Z}{A} f_i^{p/A}(x, Q) + \frac{A-Z}{A} f_i^{n/A}(x, Q)$$

- Input scale and other input parameters as in CTEQ6M proton analysis

$$Q_0 = m_c = 1.3 \text{ GeV} \quad m_b = 4.5 \text{ GeV} \quad \alpha_s(m_Z) = 0.118$$

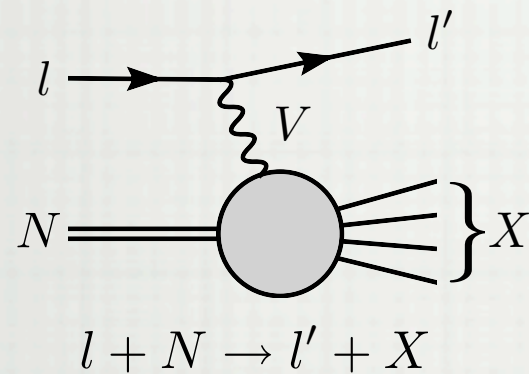
- Kinematic cuts on data

$$Q > 2 \text{ GeV} \quad W > 3.5 \text{ GeV}$$

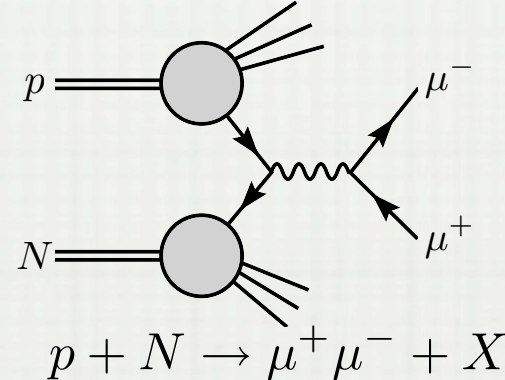
- Experiments included in the analysis

Charged lepton

Deep Inelastic Scattering



Drell-Yan process



CERN BCDMS & EMC & NMC

$N = (\text{D, Al, Be, C, Ca, Cu, Fe, Li, Pb, Sn, W})$

FNAL E-665

$N = (\text{D, C, Ca, Pb, Xe})$

DESY Hermes

$N = (\text{D, He, N, Kr})$

SLAC E-139 & E-049

$N = (\text{D, Ag, Al, Au, Be, C, Ca, Fe, He})$

FNAL E-772 & E-886

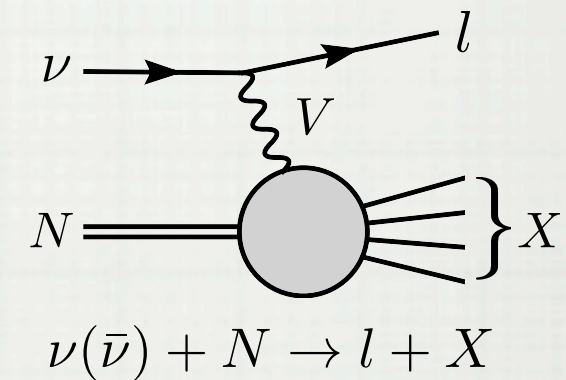
$N = (\text{D, C, Ca, Fe, W})$

1233 data points (708 after cuts)

NOT (YET) INCLUDED

Neutrino

Deep Inelastic Scattering



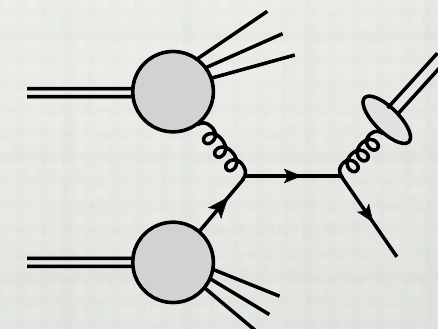
CHORUS

$N = \text{Pb}$

CCFR & NuTeV

$N = \text{Fe}$

Single pion production



RHIC - PHENIX & STAR

$N = \text{Au}$

NPDF fit properties:

- we fit nuclear data with NLO QCD predictions
- we include heavy quark effects (ACOT)
- applied standard CTEQ kinematical cuts $Q > 2\text{GeV}$ & $W > 3.5\text{GeV}$

NPDF fit results:

- 708 (1233) data points after (before) cuts
- 17 free parameters - 691 degrees of freedom
- overall $\chi^2/\text{dof} = 0.87$
- individually for different data subsets
 - for F_2^A/F_2^D $\chi^2/\text{pt} = 0.80$
 - for $F_2^A/F_2^{A'}$ $\chi^2/\text{pt} = 0.51$
 - for $\sigma_{DY}^{pA}/\sigma_{DY}^{pA'}$ $\chi^2/\text{pt} = 0.85$

F_2^A/F_2^D : Observable	Experiment	# data
D	NMC-97	275
He/D	SLAC-E139	18
	NMC-95, re	16
	Hermes	92
	NMC-95	15
Li/D	NMC-95	15
Be/D	SLAC-E139	17
C/D	EMC-88	9
	EMC-90	2
	SLAC-E139	7
	NMC-95, re	16
	NMC-95	15
	FNAL-E665-95	4
	BCDMS-85	9
	Hermes	92
N/D	SLAC-E049	18
Al/D	SLAC-E139	17
	EMC-90	2
Ca/D	SLAC-E139	7
	NMC-95, re	15
Fe/D	FNAL-E665-95	4
	BCDMS-85	6
	BCDMS-87	10
	SLAC-E049	14
Cu/D	SLAC-E139	23
	SLAC-E140	6
	EMC-88	9
	EMC-93(addendum)	10
Kr/D	EMC-93(chariot)	9
	Hermes	84
Ag/D	SLAC-E139	7
Sn/D	EMC-88	8
Xe/D	FNAL-E665-92	4
Au/D	SLAC-E139	18
Pb/D	FNAL-E665-95	4
Total:		862

$F_2^A/F_2^{A'}$: Observable	Experiment	# data
Be/C	NMC-96	15
Al/C	NMC-96	15
Ca/C	NMC-95	20
	NMC-96	15
Fe/C	NMC-95	15
Sn/C	NMC-96	144
Pb/C	NMC-96	15
C/Li	NMC-95	20
Ca/Li	NMC-95	20
Total:		279

$\sigma_{DY}^{pA}/\sigma_{DY}^{pA'}$: Observable	Experiment	# data
C/D	FNAL-E772-90	9
Ca/D	FNAL-E772-90	9
Fe/D	FNAL-E772-90	9
W/D	FNAL-E772-90	9
Fe/Be	FNAL-E866-99	28
W/Be	FNAL-E866-99	28
Total:		92

- NPDF Hessian analysis:



$$\chi^2 = \chi_0^2 + \frac{1}{2} H_{ij} (a_i - a_i^0)(a_j - a_j^0) \quad H_{ij} = \frac{\partial^2 \chi^2}{\partial a_i \partial a_j}$$

- 17 free parameters - 7 gluon parameters
 - 8 valence parameters
 - 2 sea parameters

- Eigenvalues span 10 orders of magnitude  numerical precision required

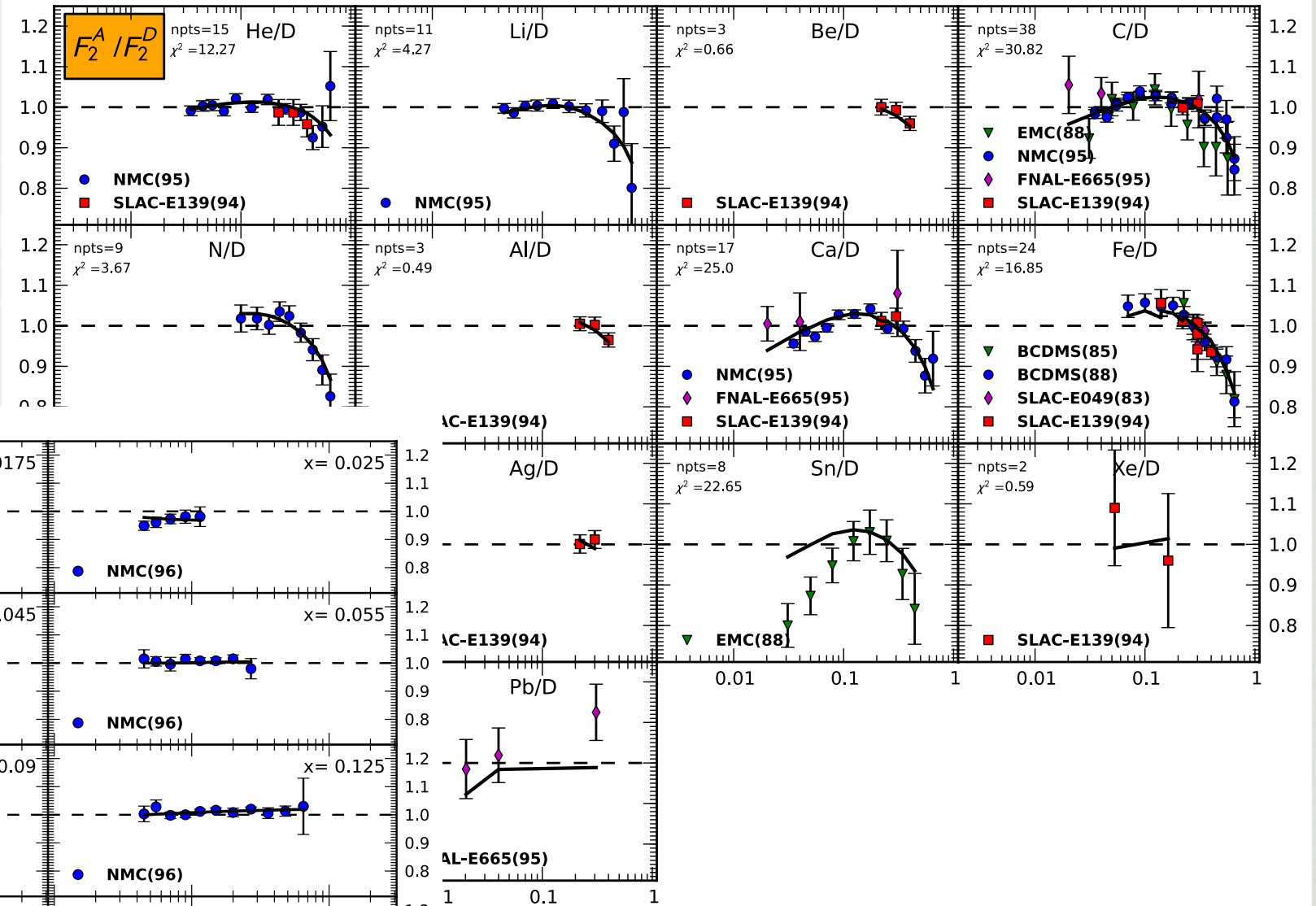
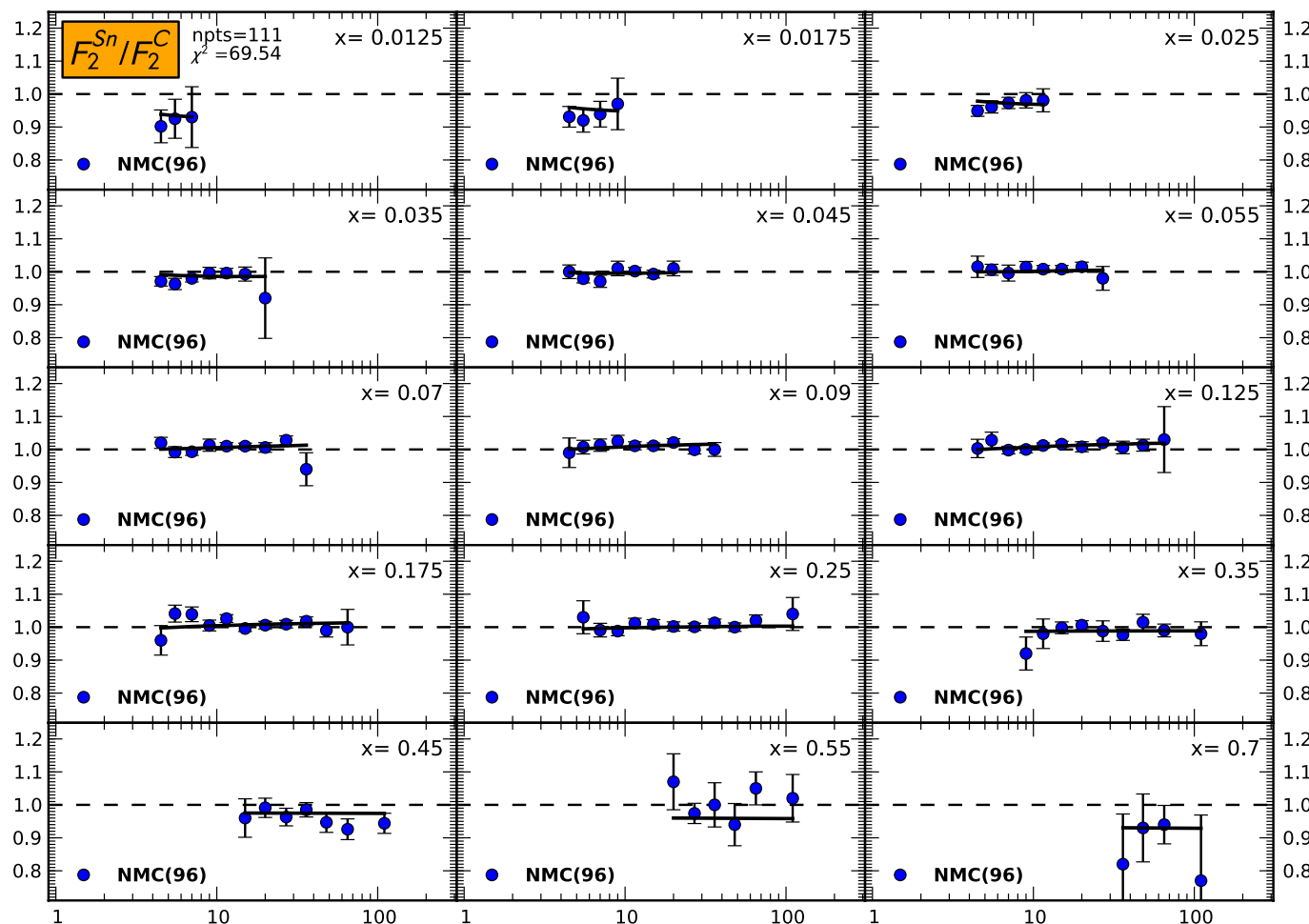
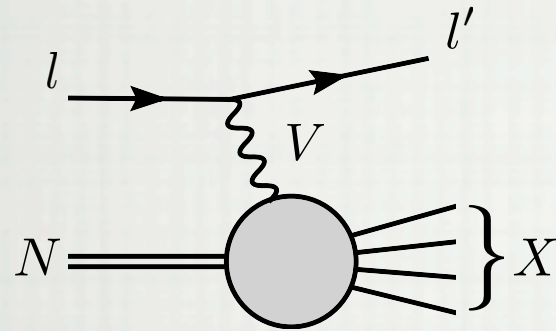
- Use improved derivatives - less sensitive to noise

$$\frac{\partial f}{\partial x} = \frac{f_1 - f_{-1}}{2h} \quad \begin{array}{l} \nearrow \frac{f_1 - f_{-1} + 2(f_2 - f_{-2}) + 3(f_3 - f_{-3})}{28h} \\ \searrow \frac{f_1 - f_{-1} + 2(f_2 - f_{-2}) + 3(f_3 - f_{-3}) + 4(f_4 - f_{-4}) + 5(f_5 - f_{-5})}{110h} \end{array}$$

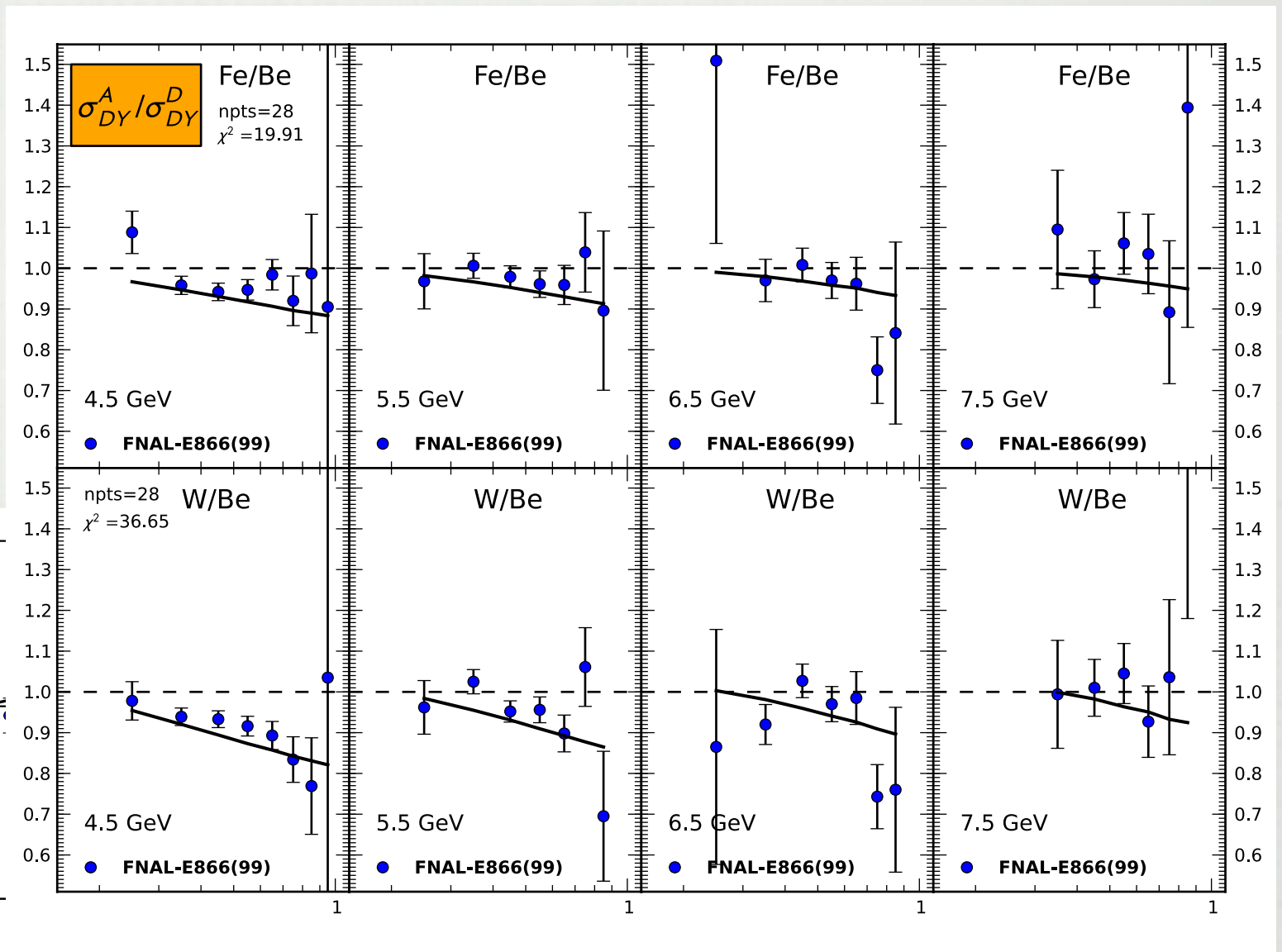
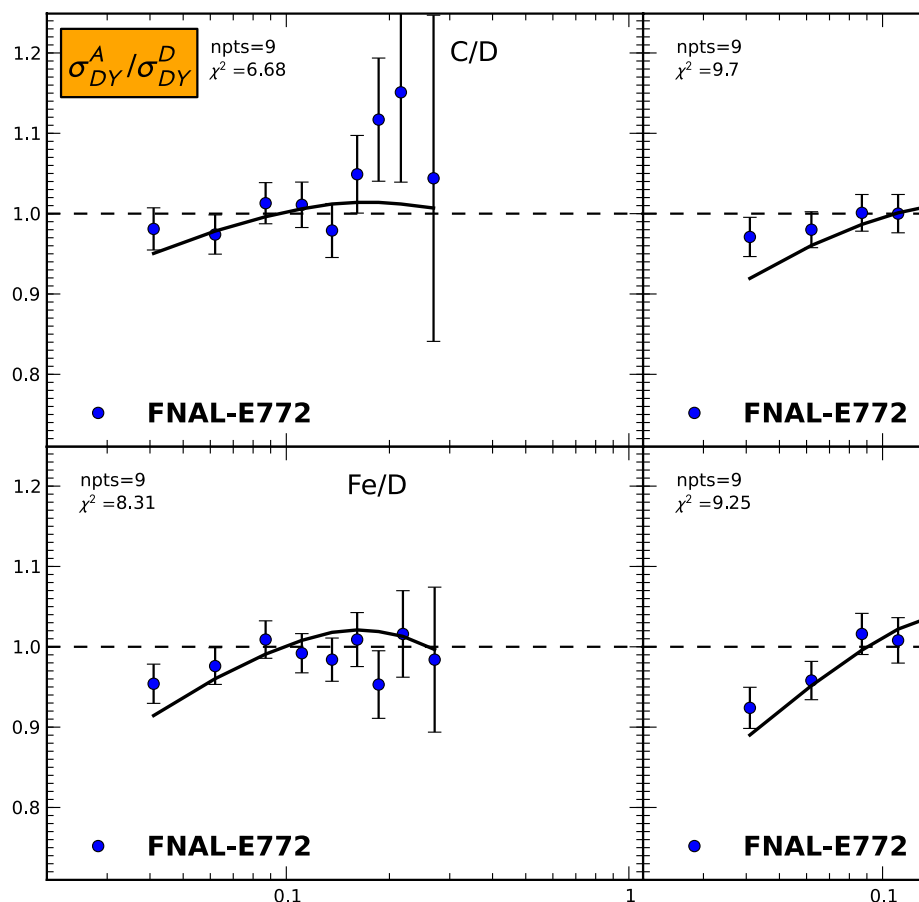
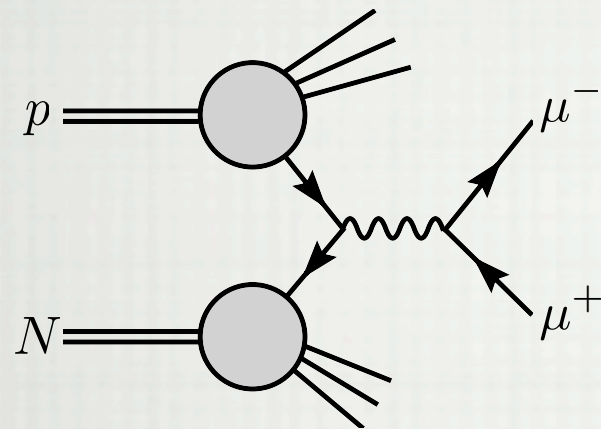
 central differences
  noise robust Lanczos 3, 5-point derivative

- $\Delta\chi^2 = 35$ determined so that every nuclear target is described within 90% C.L.

Deep Inelastic Scattering

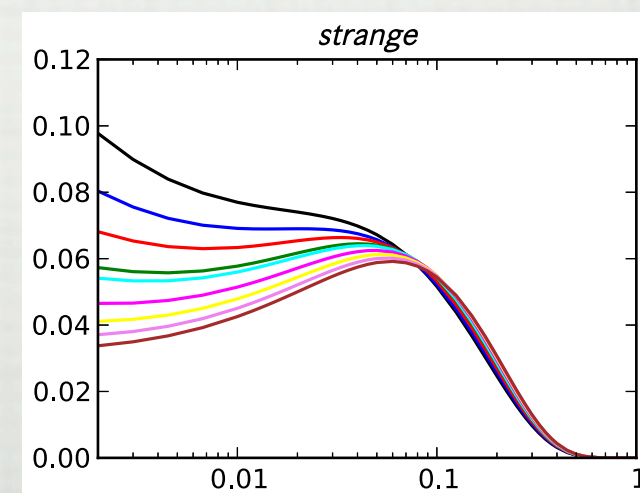
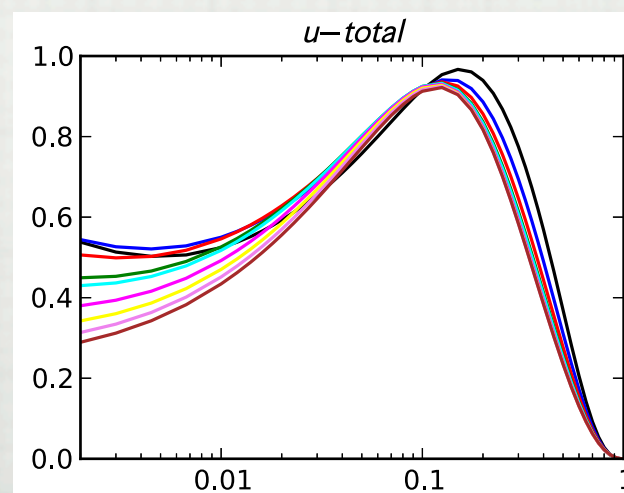
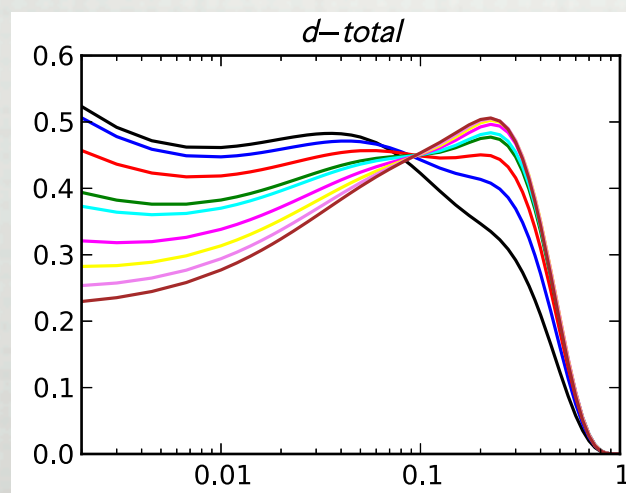
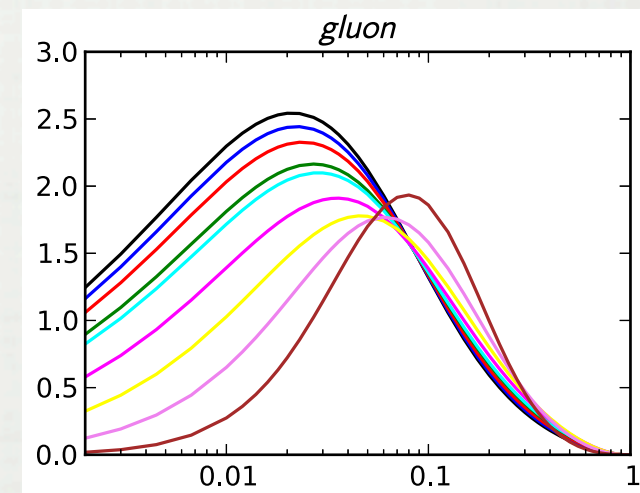
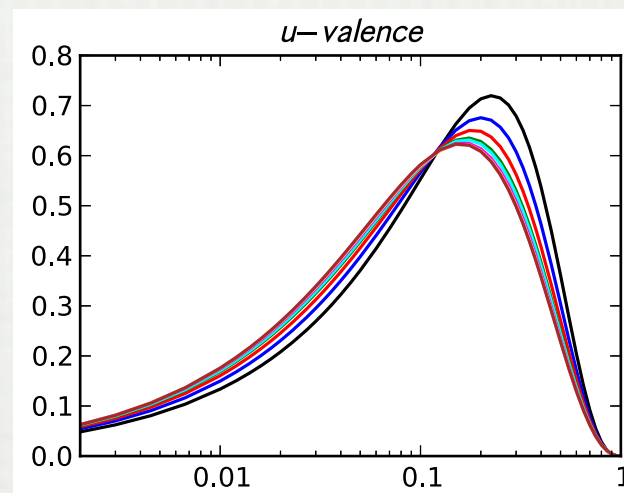
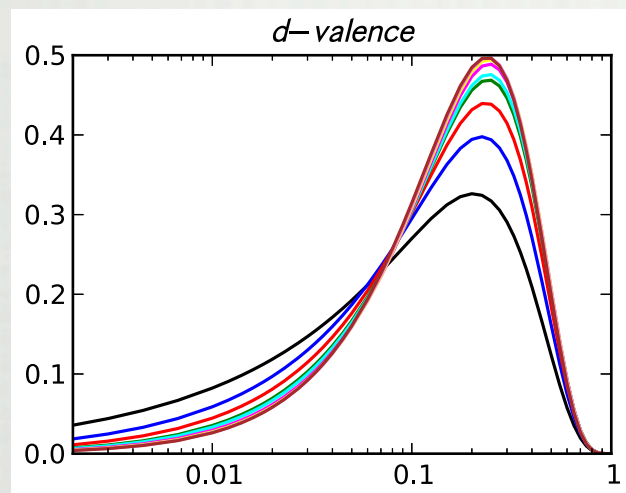


● Drell-Yan process



- Parton density functions for bound partons as a function of x

black
yellow
brown
 $x f_k^A(x, Q)$ for $A = (1, 2, 4, 9, 12, 27, 56, 108, 207)$
red
purple

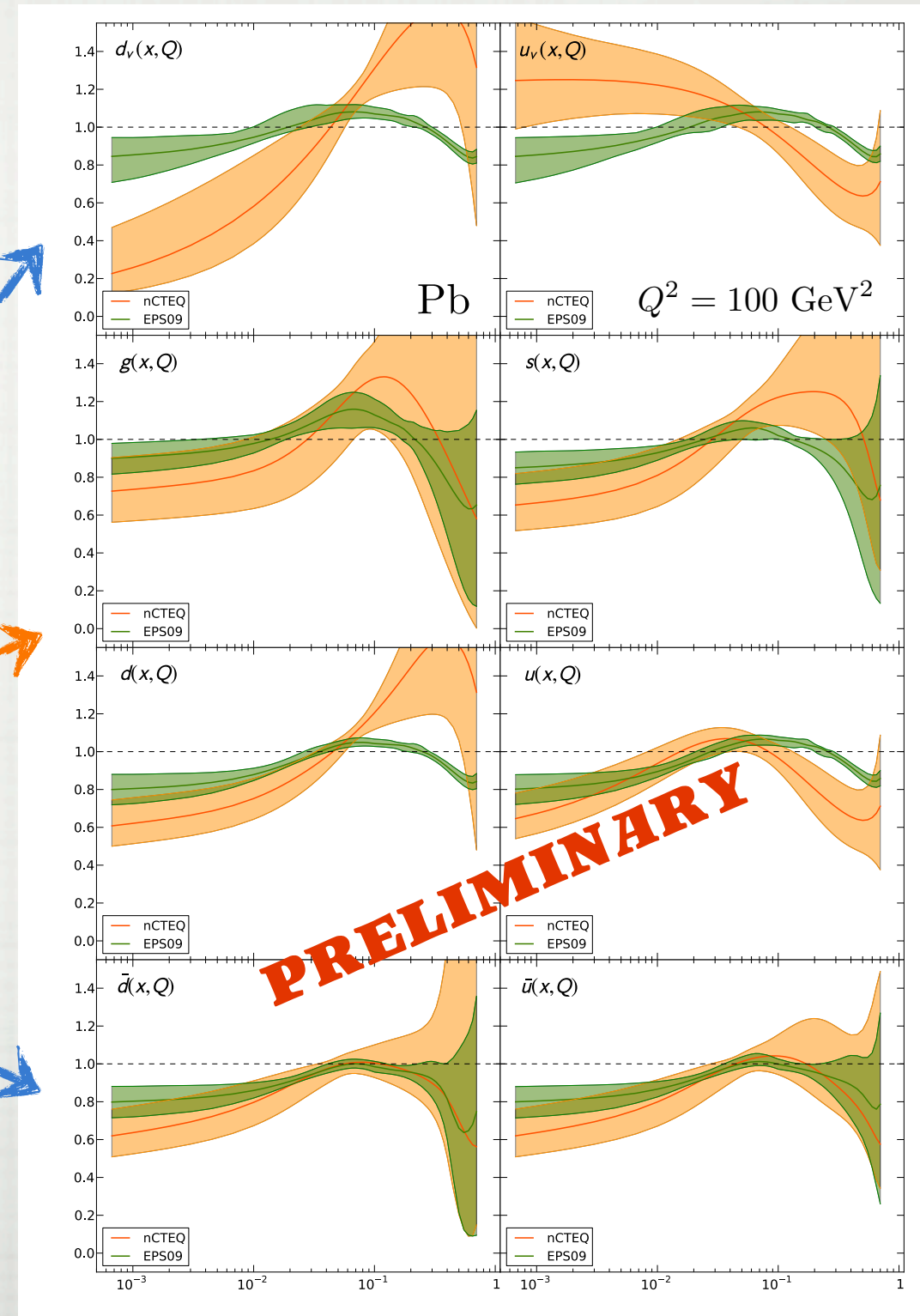


nCTEQ

- nCTEQ nuclear correction factors with uncertainties

$$R_i(\text{Pb}) = \frac{f_i^{\text{Pb}}(x, Q)}{f_i^{\text{p}}(x, Q)} \quad @ \quad Q^2 = 100 \text{ GeV}^2$$

- different solution for d-valence & u-valence compared to EPS09
- larger uncertainty @ gluon nuclear correction factor & bigger low-x suppression
- sea quark nuclear correction factors similar to EPS09
- nuclear correction factors depend largely on underlying proton baseline

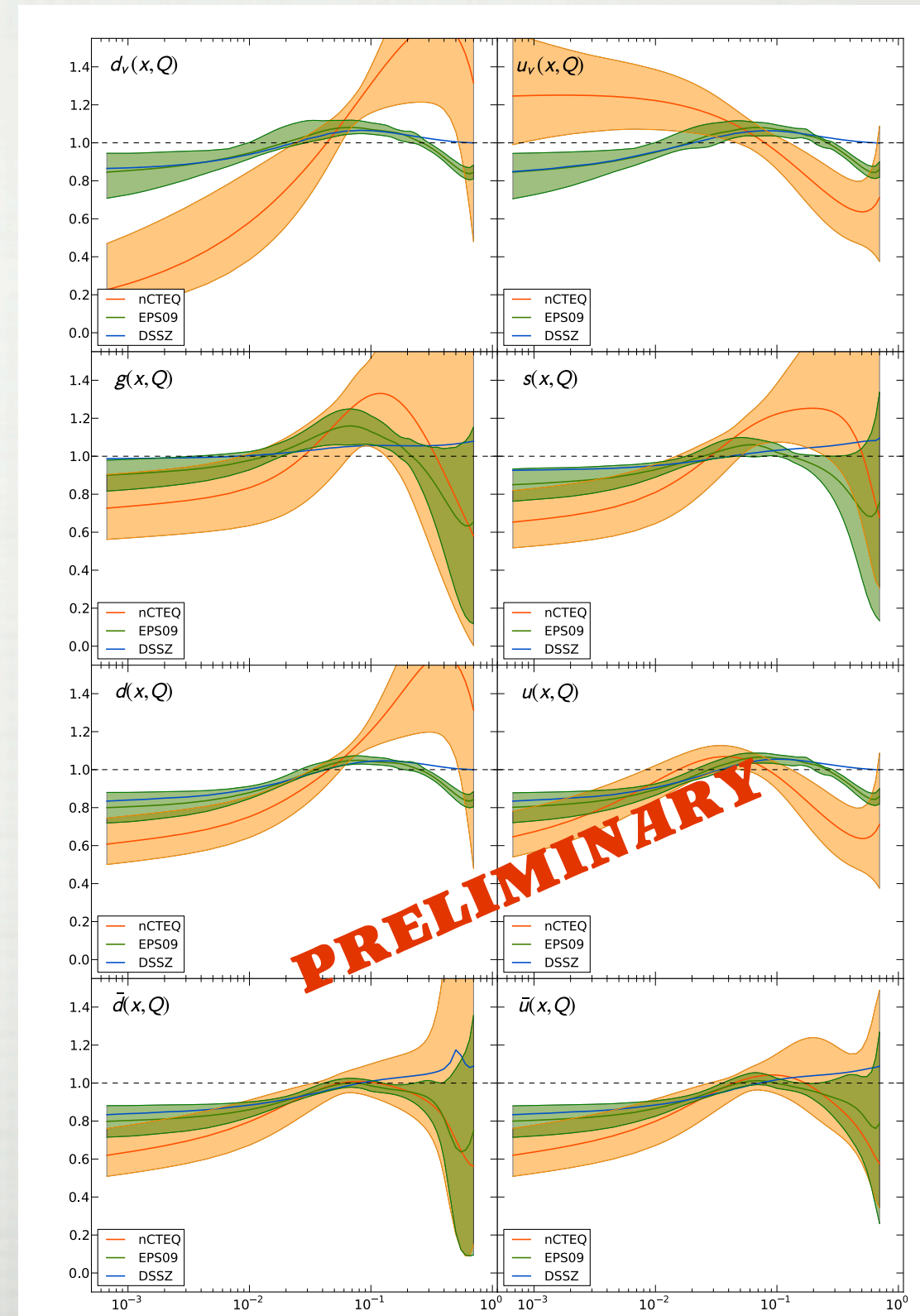


nCTEQ

- nCTEQ nuclear PDFs with uncertainties

$$x f_i^{\text{Pb}}(x, Q) \quad @ \quad Q^2 = 100 \text{ GeV}^2$$

- nCTEQ d-valence & u-valence solution between HKN07 & EPS09
- nCTEQ nuclear uncertainties larger than previous nPDF analyses
- nPDFs not dependant on proton baseline - better agreement between different nPDFs
- Results still very preliminary

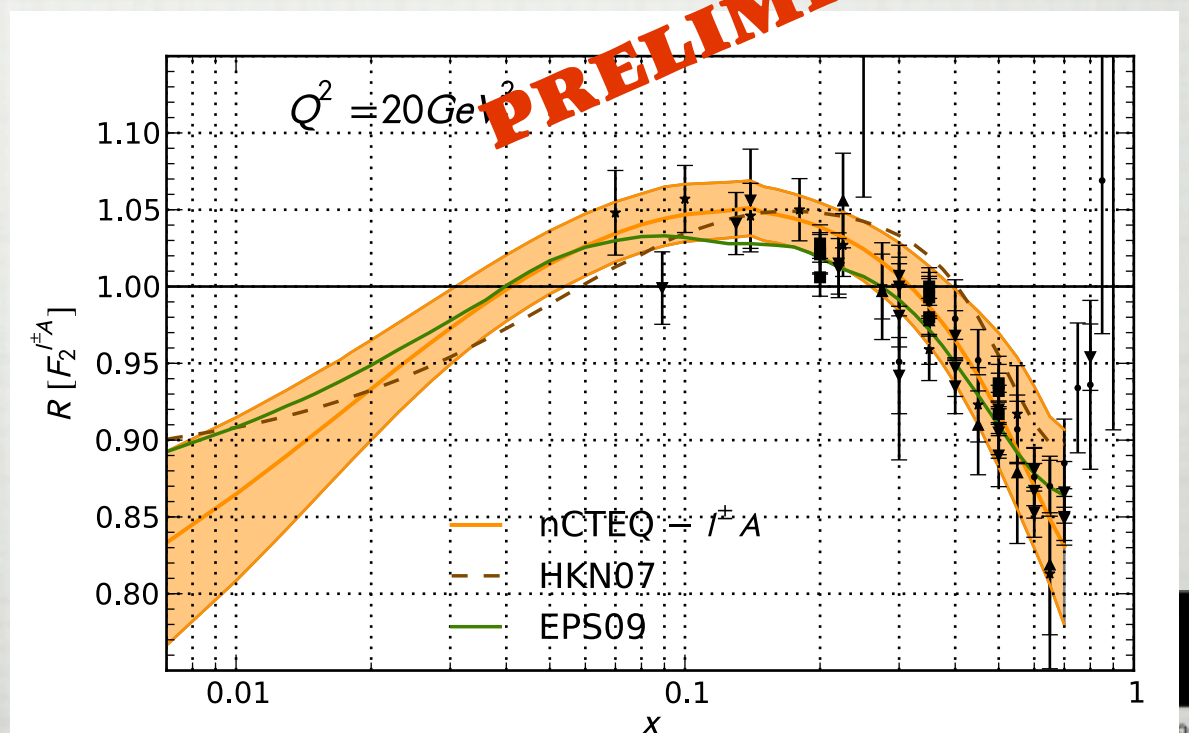
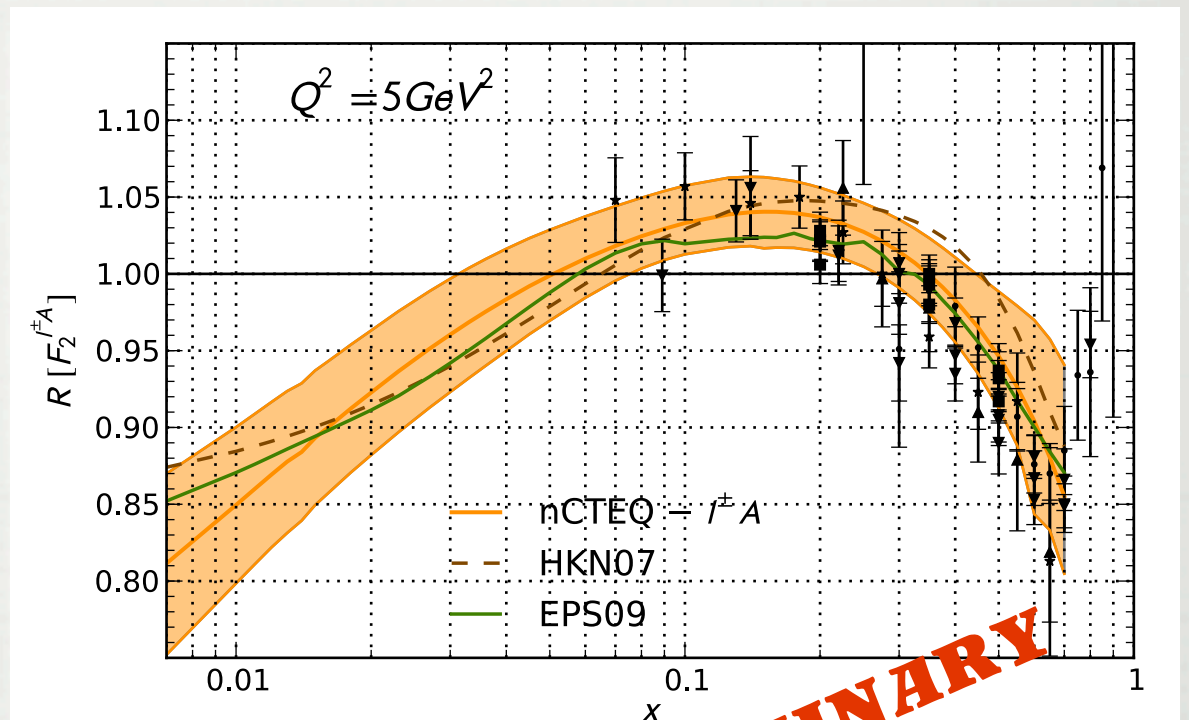


nCTEQ

- nCTEQ structure function ratios with uncertainties

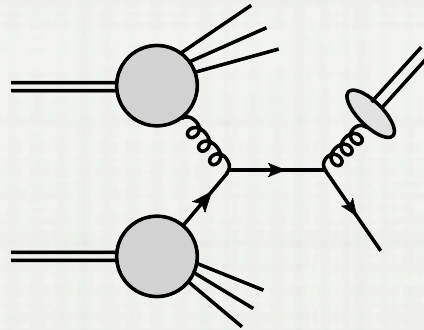
$$R = \frac{F_2^{Fe}(x, Q)}{F_2^D(x, Q)}$$

- Structure function ratios are fitted observables
- Despite different d-valence & u-valence solutions - ratio of structure functions remain very similar
- Good description of data & differences between nCTEQ and other nPDFs appear at low-x where there's no data



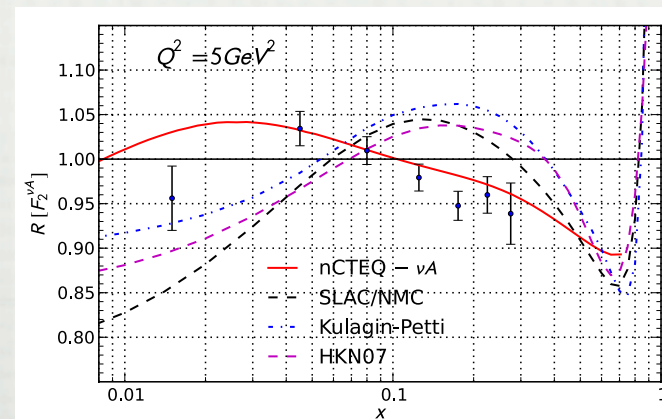
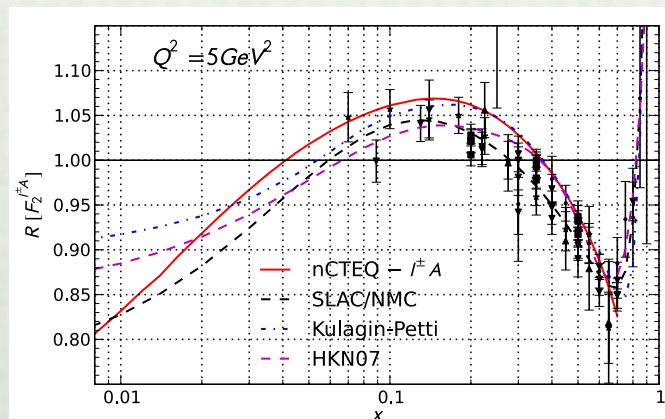
- Why is the nCTEQ analysis still **PRELIMINARY?**

- On-going work - inclusion of single inclusive pion production data from d-Au from RHIC



- more realistic estimate of the nuclear gluon correction factor @ intermediate & high- x

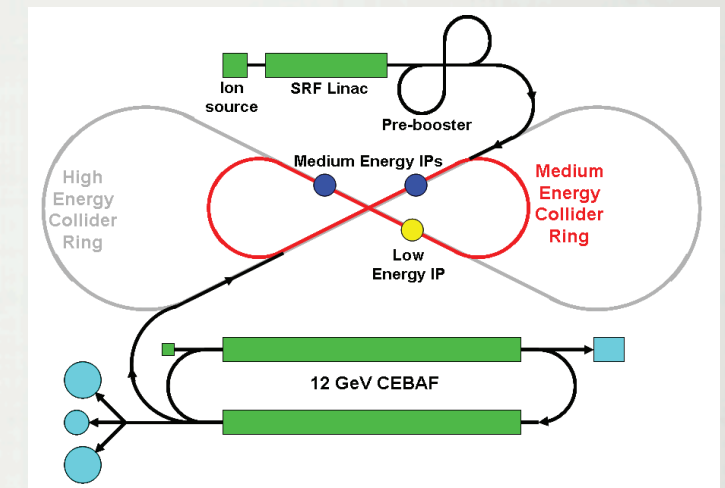
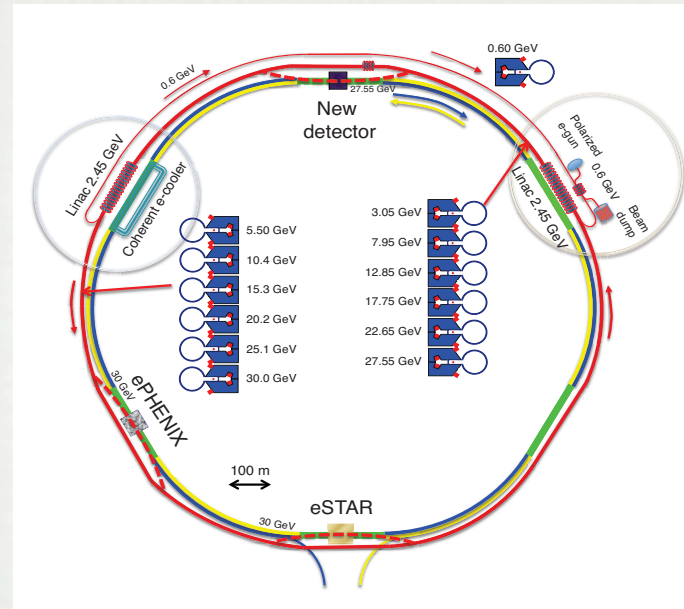
- In discussion - inclusion of neutrino DIS data (inconsistencies within NuTeV data)



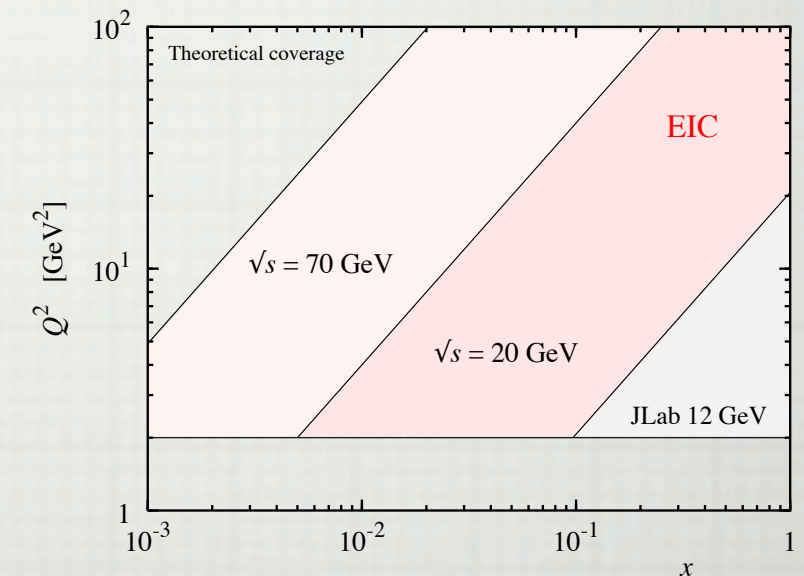
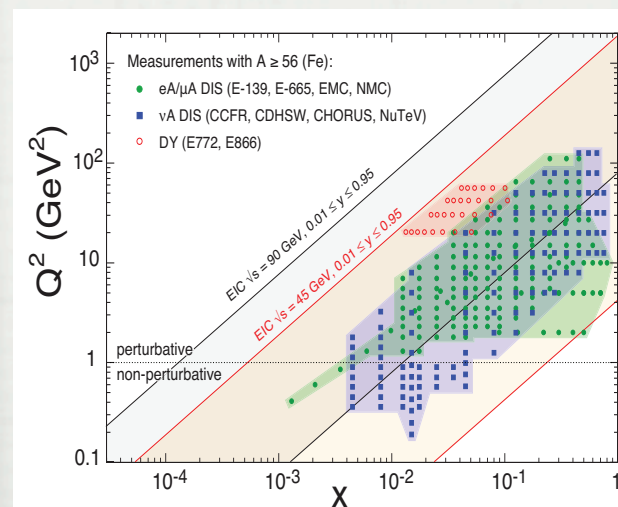
- better flavour separation for nuclear effects

Future experiments

- Electron-Ion Collider (EIC)
 - 2 different proposals - JLab & RHIC
 - multiple nuclear targets
 - reach to small- x

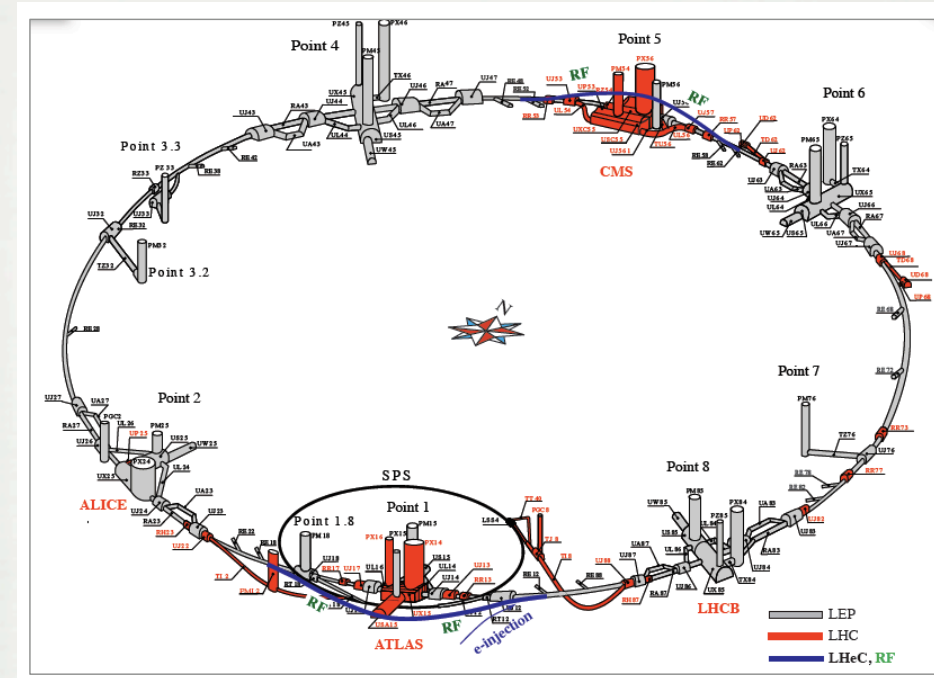


- nPDF requirements on EIC
 - coverage in x - Q^2 plane (small- x)
 - precision (e.g. for gluon PDF)
 - # nuclei

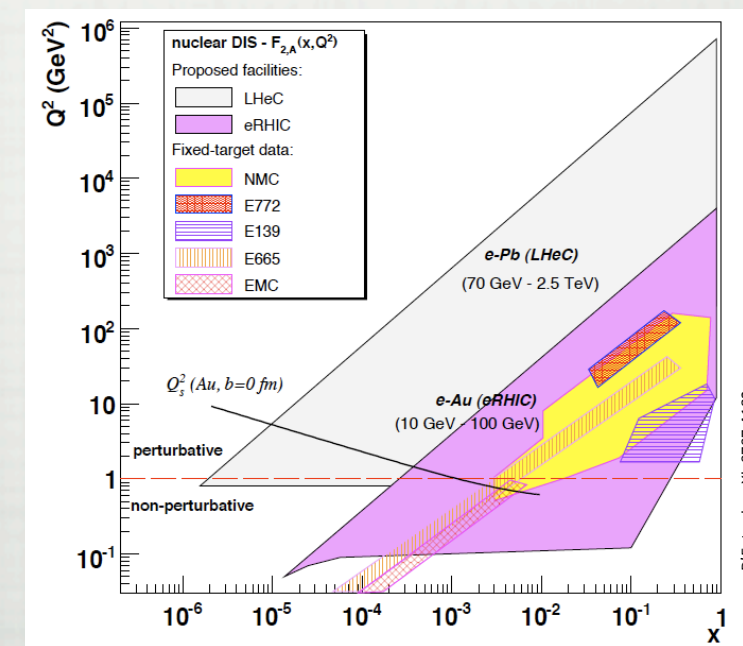


Future experiments

- LHeC
 - 2 different proposals - ring-ring and linac-LHC
 - only Pb (possibly Ca) targets
 - reach to very small-x



- nPDF requirements on LHeC
 - coverage in x - Q^2 plane (small- x)
 - precision (e.g. for gluon PDF)
 - # nuclei



Conclusions & Outlook

- nCTEQ analysis still preliminary - RHIC data being included & analysed at the moment
- nCTEQ has larger uncertainties & larger nuclear suppression for gluon @ low- x
- Some important open questions remain (all can be solved by having more data)
 - uncertainty in nuclear gluon PDF (especially at small- x)
 - important contribution from LHC pA data
 - need HERA-like measurements for many nuclear targets covering also small- x (EIC & LHeC)
 - neutrino DIS on nuclei
 - at the moment NuTeV incompatible with the rest of charged lepton data
 - proton strange quark information from LHC vital
 - new(old) data would solve the problem - NOMAD or NuSonG

THANK YOU