Threshold resummation in SCET vs pQCD: an analytic comparison

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University of Münster, May 6, 2013

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Disclaimer

I'm not a Soft-Collinear Effective Theory expert.

Motivation: threshold resummation for Drell-Yan

MB, Forte, Ridolfi, NPB 847 (2011) 93-159 (arXiv:1006.5918)
Threshold resummation for Drell-Yan pair production
(inclusive invariant-mass and rapidity distributions)
Framework: *perturbative QCD*

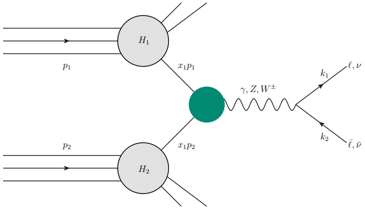
Becher, Neubert, Xu, JHEP 0807 (2008) 030 (arXiv:0710.0680)
Threshold resummation for Drell-Yan pair production
(inclusive invariant-mass and rapidity distributions)
Framework: *SCET*

MB, Forte, Ghezzi, Ridolfi
NPB 861 (2012) 337-360 (arXiv:1201.6364)
arXiv:1301.4502

Outline

Factorization theorem

Production of a system with high invariant mass M
(Higgs, Drell-Yan pair, top pair, ...)
at a collider with center of mass energy $\sqrt{s}$



Inclusive cross-section:

$$\sigma(\tau, M^2) = \int dz \int dx_1 \int dx_2\, f_1(x_1) f_2(x_2)\, C\left(z, \alpha_s(M^2)\right) \delta(x_1 x_2 z - \tau)$$

$$= \int_\tau^1 \frac{dz}{z}\, \mathscr{L}\left(\frac{\tau}{z}\right) C\left(z, \alpha_s(M^2)\right), \qquad \tau = \frac{M^2}{s}$$

parton luminosity [long distance, *universal*]: $\mathscr{L}(x) = \int_x^1 \frac{dy}{y} f_1\left(\frac{x}{y}\right) f_2(y)$

partonic coefficient function [short distance, computable in pQCD]:

$$C(z, \alpha_s) = \delta(1-z) + \alpha_s\, C^{(1)}(z) + \alpha_s^2\, C^{(2)}(z) + \dots$$

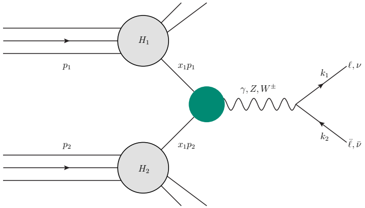
Hadronic vs Partonic Threshold

$$\sigma(\tau, M^2) = \int dz \int dx_1 \int dx_2 \, f_1(x_1) f_2(x_2) \, C\left(z, \alpha_s(M^2)\right) \delta(x_1 x_2 z - \tau)$$
$$= \int_\tau^1 \frac{dz}{z} \, \mathscr{L}\left(\frac{\tau}{z}\right) C\left(z, \alpha_s(M^2)\right)$$

$s = (p_1 + p_2)^2$ hadronic c.m.e.

$\hat{s} = (x_1 p_1 + x_2 p_2)^2$ partonic c.m.e.

$= x_1 x_2 s$

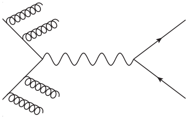


hadronic (physical) threshold: $\tau = \dfrac{M^2}{s} < 1$

partonic threshold: $z = \dfrac{\tau}{x_1 x_2} = \dfrac{M^2}{x_1 x_2 s} = \dfrac{M^2}{\hat{s}} < 1$

Threshold (soft) logarithms

Multiple gluon emissions contribute to the partonic coefficient function



They induce terms

$$C(z,\alpha_s) \ni \alpha_s^n \left[\frac{\ln^k(1-z)}{1-z}\right]_+, \qquad 0 \le k \le 2n-1$$

In the *partonic threshold limit* $\hat{s} \sim M^2$,

$$z = \frac{M^2}{\hat{s}} \to 1$$

the remaining available energy for gluon radiation is low (*soft* gluons).

In this limit, these logs become large, spoiling the perturbativity of the series.

Threshold resummation

In the *partonic* threshold limit $z \to 1$, any finite-order truncation of the *partonic* coefficient function is meaningless.

The threshold logarithms must be resummed

In real life, when is threshold resummation needed?

$$\sigma(\tau) = \int_\tau^1 \frac{dz}{z} \mathscr{L}\left(\frac{\tau}{z}\right) C(z, \alpha_s), \qquad \tau = \frac{M^2}{s}$$

$\tau \sim 1$ (hadronic threshold limit): $z \in [\tau, 1]$ always in the threshold region
⇒ **Resummation is mandatory**

$\tau \ll 1$ (the typical case at LHC!!): $z \sim 1$ always included in the integration region, but is the contribution from that region relevant/dominant?
⇒ **Resummation might be advisable**

[MB, Forte, Ridolfi, NPB 847 (2011) 93-159]
[MB, Forte, Ridolfi, PRL 109 (2012) 102002]

Consider only soft (=threshold) terms

$$C_{\rm soft}(z,\alpha_s) = \delta(1-z) + \sum_{n=1}^{\infty} \alpha_s^n \left(a_n \delta(1-z) + \sum_{k=0}^{2n-1} c_{nk} \left[\frac{\ln^k(1-z)}{1-z} \right]_+ \right)$$

and take the Mellin transform ($z \sim 1 \Rightarrow$ large N)

$$\begin{aligned} C_{\rm soft}(N,\alpha_s) &= \int_0^1 dz\, z^{N-1}\, C_{\rm soft}(z,\alpha_s) \\ &= 1 + \sum_{n=1}^{\infty} \alpha_s^n \sum_{k=0}^{2n} \hat{c}_{nk} \ln^k N \end{aligned}$$

The series can be resummed, up to some finite logarithmic accuracy.

[Catani, Trentadue, NPB 327 (1989) 323] [Sterman, NPB 281 (1987) 310]

Example: leading logarithmic accuracy (LL) and fixed coupling

$$C_{\rm soft}(N,\alpha_s) = 1 + \sum_{n=1}^{\infty} \alpha_s^n \sum_{k=0}^{2n} \hat{c}_{nk} \ln^k N$$

$\hookrightarrow$ **LL, fixed coupling** α_s $\curvearrowright$

$$C^{\rm LL,fc}_{\rm soft}(N,\alpha_s) = 1 + \sum_{n=1}^{\infty} \hat{c}_{n,2n} \left(\alpha_s \ln^2 N\right)^n$$

One can prove that multiple emissions factorize

$$(n\text{-emissions}) \overset{\rm LL,fc}{=} \frac{(\text{single-emission})^n}{n!} \qquad \Rightarrow \qquad \hat{c}_{n,2n} = \frac{(\hat{c}_{1,2})^n}{n!}$$

Therefore we get

$$C^{\rm LL,fc}_{\rm soft}(N,\alpha_s) = \exp\left[\alpha_s\, \hat{c}_{1,2} \ln^2 N\right]$$

Crucial ingredient: **factorization of soft radiation**
Factorization takes place in N space!

Beyond LL and including running-coupling effects:

$$C_{\rm soft}(N,\alpha_s) = g_0(\alpha_s)\,\exp\int_0^1 dz\,\frac{z^{N-1}-1}{1-z} \times\left[\int_{M^2}^{M^2(1-z)^2}\frac{d\mu^2}{\mu^2}2A\left(\alpha_s(\mu^2)\right)+D\left(\alpha_s([1-z]^2M^2)\right)\right]$$

$$= g_0(\alpha_s)\,\exp\left[\underbrace{\frac{1}{\alpha_s}g_1(\alpha_s L)}_{\text{LL}}+\underbrace{g_2(\alpha_s L)}_{\text{NLL}}+\underbrace{\alpha_s g_3(\alpha_s L)}_{\text{NNLL}}+\dots\right]$$

with $L=\ln N$.

Logarithmic counting at the exponent, assuming $\alpha_s L \sim 1$.

Finally, we have to go from N space back to z space.

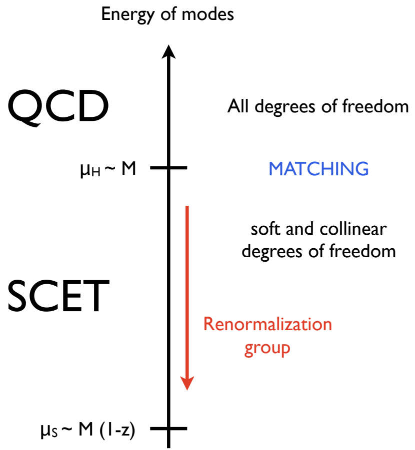
The inverse Mellin transform does not exist, because of the Landau pole of the running coupling α_s.
Alternatively, expand in α_s and invert term by term: divergent series.

A prescription is needed...

Summary:

- Resummation is based on factorization of soft emissions
- Factorization (and hence resummation) takes place in N space
- Due to the Landau pole, going back to z space requires extra work
- The *partonic* logarithms in the *partonic* coefficient function are resummed

General idea of effective theories: at low energy scales, degrees of freedom associated with higher scales are no longer dynamical and can be integrated out.



Factorization in SCET:

$$C_{\rm SCET}(z, M^2, \mu_s^2) = H(M^2)\, U(M^2, \mu_s^2)\, S\left(\mu_s^2, \frac{M^2(1-z)^2}{\mu_s^2}\right)$$

$H(M^2)$:	hard function (matching at the hard scale $\mu_H = M$)
$U(M^2, \mu_s^2)$:	RG evolution from $\mu_H = M$ down to μ_s
$S\left(\mu_s^2, \frac{M^2(1-z)^2}{\mu_s^2}\right)$:	soft function (matching at the soft scale μ_s)
μ_s:	soft scale

The soft scale μ_s should be of the order of $M(1-z)$.

Formally, $C_{\rm SCET}(z, M^2)$ does not depend on μ_s.
However, this is a perturbative statement, so a residual dependence on μ_s remains.

[Becher, Neubert, Xu, JHEP 0807 (2008) 030]

$$U(M^2,\mu_s^2) = \exp\left\{-\int_{M^2}^{\mu_s^2}\frac{d\mu^2}{\mu^2}\left[\Gamma_{\rm cusp}\left(\alpha_s(\mu^2)\right)\ln\frac{\mu^2}{M^2} - \gamma_W\left(\alpha_s(\mu^2)\right)\right]\right\}$$

resums $\ln\frac{\mu_s^2}{M^2}$, and produces single and double logs.

$$S\left(\mu_s^2, \tfrac{M^2(1-z)^2}{\mu_s^2}\right) = (1-z)^{2\eta-1}\ \tilde{s}_{\rm DY}\left(\ln\frac{M^2(1-z)^2}{\mu_s^2} + \partial_\eta,\ \alpha_s(\mu_s)\right)\frac{e^{-2\gamma\eta}}{\Gamma(2\eta)}$$

$$\eta = \int_{M^2}^{\mu_s^2}\frac{d\mu^2}{\mu^2}\Gamma_{\rm cusp}\left(\alpha_s(\mu^2)\right)$$

resums both $\ln\frac{\mu_s^2}{M^2}$ and $\ln(1-z)$,
and produces single logs and *mixed* double logs.

The choice of the soft scale μ_s determines what is being resummed

SCET: choosing the soft scale

$C_{\rm SCET}(z, M^2, \mu_s^2)$ leads to many different results depending on μ_s:

- $\mu_s \sim M$: nothing is resummed, fixed-order result in the soft limit is reproduced (by construction of SCET)
- $\mu_s \sim M(1-z)$: natural partonic choice, resums $\ln(1-z)$
- $\mu_s \sim M/N$: natural partonic choice in N space, resums $\ln N$
- $\mu_s \sim M(1-\tau)$: hadronic choice suggested by Becher, Neubert, Xu

Comparison: $\mu_s \sim M(1-z)$

$C_{\rm QCD}(N, M^2)$ ($C_{\rm soft}$) has *no* inverse Mellin because of the Landau pole.

We could expand in α_s and invert order by order, but the resulting series is divergent.

Conversely, $C_{\rm SCET}(z, M^2, \mu_s^2 = M^2(1-z)^2)$ is formally defined.

Does SCET provide a valid z-space expression?
No.

Order by order in α_s, and away from the endpoint $z = 1$, QCD and SCET expressions coincide.
However, the Landau pole problem is still there: $\alpha_s\left(M^2(1-z)^2\right)$

Moreover, the SCET expression is not defined in $z = 1$

Possible way out: cutoff the convolution integral at $z = \bar{z} < 1$
[Beneke, Falgari, Klein, Schwinn, NPB 855 (2012) 695-741]

Comparison in Mellin space

The QCD resummed expression is in Mellin space, therefore a comparison is appropriate in Mellin space.

Taking the Mellin transform of the SCET result at μ_s fixed:

$$C_{\rm SCET}(N, M^2, \mu_s^2) = C_r(N, M^2, \mu_s^2)\, C_{\rm QCD}(N, M^2)$$

with ($\bar{N} = N e^{\gamma}$) [arXiv:1301.4502]

$$C_r(N, M^2, \mu_s^2) = \frac{E\left(\frac{M^2}{\bar{N}^2}, \mu_s^2\right)}{E(M^2, M^2)} \exp \hat{\mathcal{S}}\left(\mu_s^2, \frac{M^2}{\bar{N}^2}\right)$$

To NNLL, we have

$$\hat{\mathcal{S}}\left(\mu_s^2, \frac{M^2}{\bar{N}^2}\right) = \int_{M^2/\bar{N}^2}^{\mu_s^2} \frac{d\mu^2}{\mu^2}\left[\Gamma_{\rm cusp}\left(\alpha_s(\mu^2)\right) \ln \frac{M^2}{\mu^2 \bar{N}^2} + \hat{\gamma}_W\left(\alpha_s(\mu^2)\right)\right]$$

$$E\left(\frac{M^2}{\bar{N}^2}, \mu_s^2\right) = \tilde{s}_{\rm DY}\left(\ln \frac{M^2}{\mu_s^2 \bar{N}^2}, \alpha_s(\mu_s^2)\right) \exp\left[-\frac{\zeta_2}{2}\frac{C_F}{\pi}\alpha_s(\mu_s^2)\right]$$

Comparison: $\mu_s \sim M/N$

Notice that for $\mu_s = M/\bar{N}$

$$C_r\left(N, M^2, \mu_s^2 = \frac{M^2}{\bar{N}^2}\right) = \frac{E\left(\frac{M^2}{\bar{N}^2}, \frac{M^2}{\bar{N}^2}\right)}{E(M^2, M^2)} = 1 + \mathcal{O}\left(\alpha_s^3 \ln N\right) \qquad \text{(at NNLL)}$$

leading difference = $\alpha_s^3 L \times \alpha_s^n L^{2n} = \alpha_s^{(n+3)} L^{2(n+3)-5} = \alpha_s^m L^{2m-5}$ = NNNLL*

For $\mu_s = M/\bar{N}$, **SCET and QCD coincide**

$$C_{\rm SCET}\left(N, M^2, \mu_s^2 = \frac{M^2}{\bar{N}^2}\right) = C_{\rm QCD}(N, M^2) + \text{higher logarithmic orders}$$

But they also share the same Landau pole problem...

This result has been already proved before to all logarithmic orders:
DIS: [Becher, Neubert, Pecjak, JHEP 0701 (2007) 076]
DY: [Becher, Neubert, Xu, JHEP 0807 (2008) 030]

SCET as an alternative way to obtain the same result as in QCD

Choose μ_s to be related to hadron-level kinematics
[Becher, Neubert, Xu, JHEP 0807 (2008) 030]

$$\mu_s = M(1-\tau)$$

Remarks:

- meaningful at hadron-level only (partonic comparison not possible)

$$\sigma_{\rm SCET}(\tau, M^2) = \int_\tau^1 \frac{dz}{z} \mathscr{L}\left(\frac{\tau}{z}\right) C_{\rm SCET}\left(z, M^2, \mu_s^2 = M^2(1-\tau)^2\right)$$

- resums $\ln\frac{\mu_s}{M} = \ln(1-\tau)$: useful only at large τ
- provided τ is not too close to 1 (so that $\mu_s > \Lambda_{\rm QCD}$), the Landau pole is avoided

Factorization is violated

τ dependence in contrast with the factorization theorem

$$\sigma_{\rm SCET}(\tau, M^2) = \int_\tau^1 \frac{dz}{z} \mathscr{L}\left(\frac{\tau}{z}\right) C_{\rm SCET}\left(z, M^2, M^2(1-\tau)^2\right)$$
$$= \int dx \int dz\, \mathscr{L}(x)\, C_{\rm SCET}\left(z, M^2, M^2(1-\tau)^2\right)\, \delta(xz-\tau)$$

For instance, in N space it does not become a product.

Objection from the SCET community:
in $\mu_s = M(1-\tau)$, τ is just a label, and it has not to be considered as a dynamical variable.

Nevertheless, it's a fact that the *partonic* coefficient function depends on *hadron-level* physics, while it should not.

Hadronic comparison

In this case, we should count powers of $\ln(1-\tau)$ at the level of σ.

$$C_{\rm SCET}(N, M^2, \mu_s^2) = C_r(N, M^2, \mu_s^2)\, C_{\rm QCD}(N, M^2)$$
$$C_{\rm SCET}(z, M^2, \mu_s^2) = \int_z^1 \frac{dz'}{z'}\, C_r(z', M^2, \mu_s^2)\, C_{\rm QCD}\left(\frac{z}{z'}, M^2\right)$$

(formal expression, valid only order by order)

$$\sigma_{\rm SCET}(\tau, M^2, \mu_s^2) = \int_\tau^1 \frac{dz}{z}\, C_r(z, M^2, \mu_s^2)\, \sigma_{\rm QCD}\left(\frac{\tau}{z}, M^2\right)$$

Strategy:
Expanding the NNLL expression of C_r in powers of α_s and plugging it into the previous equation

$$C_r(N, M^2, \mu_s^2) = \left[1 + \mathcal{O}\left(\alpha_s^3 \ln\frac{\mu_s^2}{M^2}\right)\right] \times \left[1 + F_r\left(\alpha_s(\mu_s), \ln\frac{M^2}{\mu_s^2 \bar{N}^2}\right)\right]$$

$$\sigma_{\rm SCET}(\tau, M^2, \mu_s^2 = M^2(1-\tau)^2) = \left[1 + \mathcal{O}\left(\alpha_s^3 \ln(1-\tau)\right)\right] \times \sigma_{\rm QCD}(\tau, M^2)$$

At large τ, the largest logarithmic content of σ is

$$\sigma_{\rm QCD}(\tau, M^2) \sim \sum_n \alpha_s^n \sum_p \ln^{2n+p}(1-\tau)$$

where $\ln^p(1-\tau)$ is a PDF contribution.

Therefore, the largest contribution to the difference QCD-SCET is ($m = n+3$)

$$\sigma_{\rm SCET}(\tau, M^2, M^2(1-\tau)^2) - \sigma_{\rm QCD}(\tau, M^2) \sim \sum_m \alpha_s^m \sum_p \ln^{2m-5+p}(1-\tau)$$

If we neglect p, we can say that the difference is NNNLL*.
(Argument valid to all orders)

However, this conclusion is spoiled by the PDF dependent contribution: the discrepancy can become arbitrarily large depending on p.

What about small τ?

Remember: Higgs at LHC $\tau \sim 10^{-4}$

$M(1-\tau) \simeq M$ is a hard scale!

We are back in a situation in which $\sigma_{\rm SCET}$ reproduces a fixed order result in the soft limit.

What do Becher-Neubert-Xu *exactly* do in the small-τ case?

Becher, Neubert, Xu suggest a soft scale determined by minimization of perturbative contributions of $\tilde{s}_{\rm DY}$ to the cross-section; they propose

$$\mu_s = \frac{M(1-\tau)}{1+7\tau} \sim M \qquad \text{or} \qquad \mu_s = \frac{M(1-\tau)}{\sqrt{6+150\tau}} \sim \frac{M}{\sqrt{6}}$$

In the second case, they resum $\ln\sqrt{6}$ and **single logs** $\ln(1-z)$.
However, this is not threshold resummation (double logs).

Conclusions

- SCET provides an interesting and possibly powerful framework for computations valid in the soft (threshold) limit
- the result of a SCET computation, $C_{\text{SCET}}(z, M^2, \mu_s^2)$, depends in fact on a soft scale μ_s, which is not fixed by the formalism
- different choices of the soft scale lead to different results
- $\mu_s = M$: coincides with fixed-order QCD in the soft limit
- $\mu_s = M(1-z)$: resums $\ln(1-z)$, coincides with QCD order by order, but is not defined in $z = 1$ and has the Landau pole problem
- $\mu_s = M/\bar{N}$: resums $\ln N$, coincides with QCD (same Landau pole problem)
- $\mu_s = M(1-\tau)$:
  - $\tau \sim 1$: resums $\ln(1-\tau)$, no Landau pole, PDF dependence
  - $\tau \ll 1$: nothing is resummed

general comment: this is not threshold resummation in the usual sense.

Backup slides

Logarithmic accuracy

$$C(N,M^2) = 1 + \sum_{n=1}^{\infty} \alpha_s^n \sum_{k=0}^{2n} \hat{c}_{nk} L^k \qquad \ln C(N,M^2) = \sum_{n=1}^{\infty} \alpha_s^n \sum_{k=0}^{n+1} \hat{b}_{nk} L^k$$

QCD:	$A(\alpha_s)$	$D(\alpha_s)$	$g_0(\alpha_s)$	$L = \ln N$	
SCET:	$\Gamma_{\rm cusp}(\alpha_s)$	$\gamma_W(\alpha_s)$	H, $\tilde{s}_{\rm DY}$	$L = \ln(\mu_s/M)$	
				accuracy: $\hat{c}_{nk}$	$\hat{b}_{nk}$
LL	1-loop	—	tree-level	$k = 2n$	$k = n+1$
NLL*	2-loop	1-loop	tree-level	$2n-1 \le k \le 2n$	$n \le k \le n+1$
NLL	2-loop	1-loop	1-loop	$2n-2 \le k \le 2n$	$n \le k \le n+1$
NNLL*	3-loop	2-loop	1-loop	$2n-3 \le k \le 2n$	$n-1 \le k \le n+1$
NNLL	3-loop	2-loop	2-loop	$2n-4 \le k \le 2n$	$n-1 \le k \le n+1$

Starred counting: appropriate for $\ln C(N,M^2)$, assumes $\alpha_s L \sim 1$
Un-starred counting: more appropriate for $C(N,M^2)$, assumes $\alpha_s L^2 \sim 1$

Comparison: $\mu_s \sim M(1-z)$

Example: LL, fixed coupling:

In QCD we have

$$C_{\rm QCD}^{\rm LL,fc}(N, M^2) = \exp\left[\alpha_s\, \hat{c}_{1,2}\, \ln^2 N\right]$$

whose inverse Mellin is

$$C_{\rm QCD}^{\rm LL,fc}(z, M^2) = \delta(1-z) + \left[\frac{1}{1-z}\, \exp\left(\alpha_s\, \hat{c}_{1,2}\, \frac{\partial^2}{\partial\xi^2}\right) \frac{(1-z)^\xi}{\Gamma(\xi)}\bigg|_{\xi=0}\right]_+$$

In SCET we have

$$C_{\rm SCET}^{\rm LL,fc}\left(z, M^2, M^2(1-z)^2\right) = 2\alpha_s \hat{c}_{1,2}\, \frac{\ln(1-z)}{1-z}\, \exp\left[\alpha_s\, \hat{c}_{1,2}\, \ln^2(1-z)\right]$$

The two expression coincide (for $z \neq 1$) at leading $\ln(1-z)$.
However, even the QCD expression with plus-distribution leads to a divergent integral with any test function.
[Catani, Mangano, Nason, Trentadue, NPB 478 (1996) 273]

Comparison: $\mu_s \sim M(1-\tau)$

$$C_r(N, M^2, \mu_s^2) = \frac{E(\mu_s^2, \mu_s^2)}{E(M^2, M^2)} \left[1 + F_r\left(\alpha_s(\mu_s), \ln \frac{M^2}{\mu_s^2 \bar{N}^2} \right) \right]$$

$$C_r(z, M^2, \mu_s^2) = \frac{E(\mu_s^2, \mu_s^2)}{E(M^2, M^2)} \left[\delta(1-z) + F_r\left(\alpha_s(\mu_s), 2\frac{\partial}{\partial \xi} \right) \frac{(1-\tau)^{-\xi} \ln^{\xi-1} \frac{1}{z}}{e^{\gamma \xi} \Gamma(\xi)} \bigg|_{\xi=0} \right]$$

Plugging into $\sigma_{\rm SCET} = C_r \otimes \sigma_{\rm QCD}$ we get

$$\sigma_{\rm SCET}(\tau, M^2, \mu_s^2) = \frac{E(\mu_s^2, \mu_s^2)}{E(M^2, M^2)} \left[\sigma_{\rm QCD}(\tau, M^2) + F_r\left(\alpha_s(\mu_s), 2\frac{\partial}{\partial \xi} \right) \Sigma\left(\tau, M^2, \xi\right)\Big|_{\xi=0} \right]$$

with

$$\Sigma\left(\tau, M^2, \xi\right) = \frac{(1-\tau)^{-\xi}}{e^{\gamma \xi} \Gamma(\xi)} \int_\tau^1 \frac{dz}{z} \left(\ln \frac{1}{z} \right)^{\xi-1} \sigma\left(\frac{\tau}{z}, M^2 \right)$$

$$= \sum_{k=0}^{\infty} c_k(\xi) \frac{d^k \sigma(\tau, M^2)}{d \ln^k (1-\tau)} \left[1 + \mathcal{O}(1-\tau) \right]$$