PHYSICS AT THE TERA SCALE

Helmholtz Alliance

QCD and Monte Carlos

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DESY

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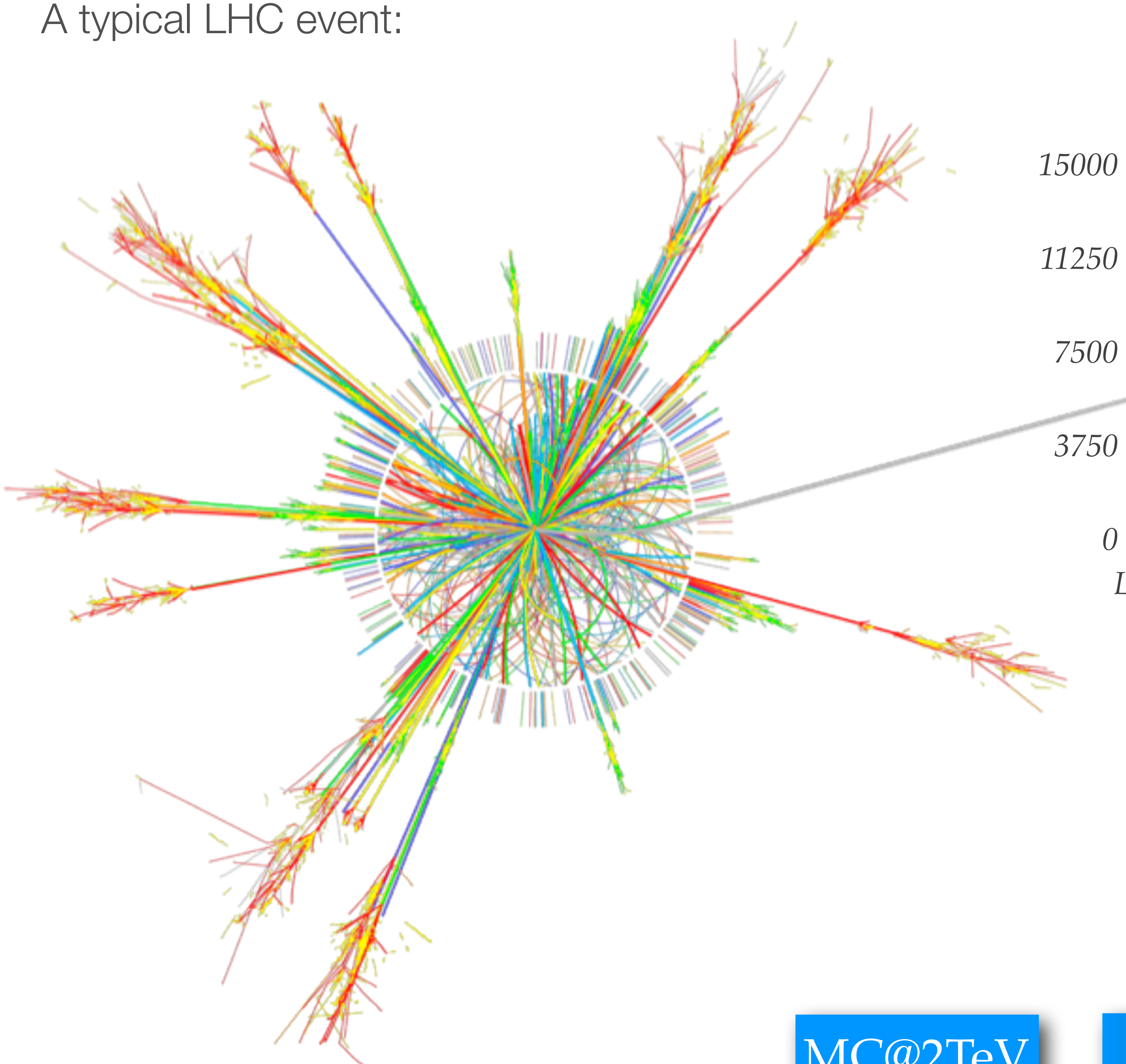
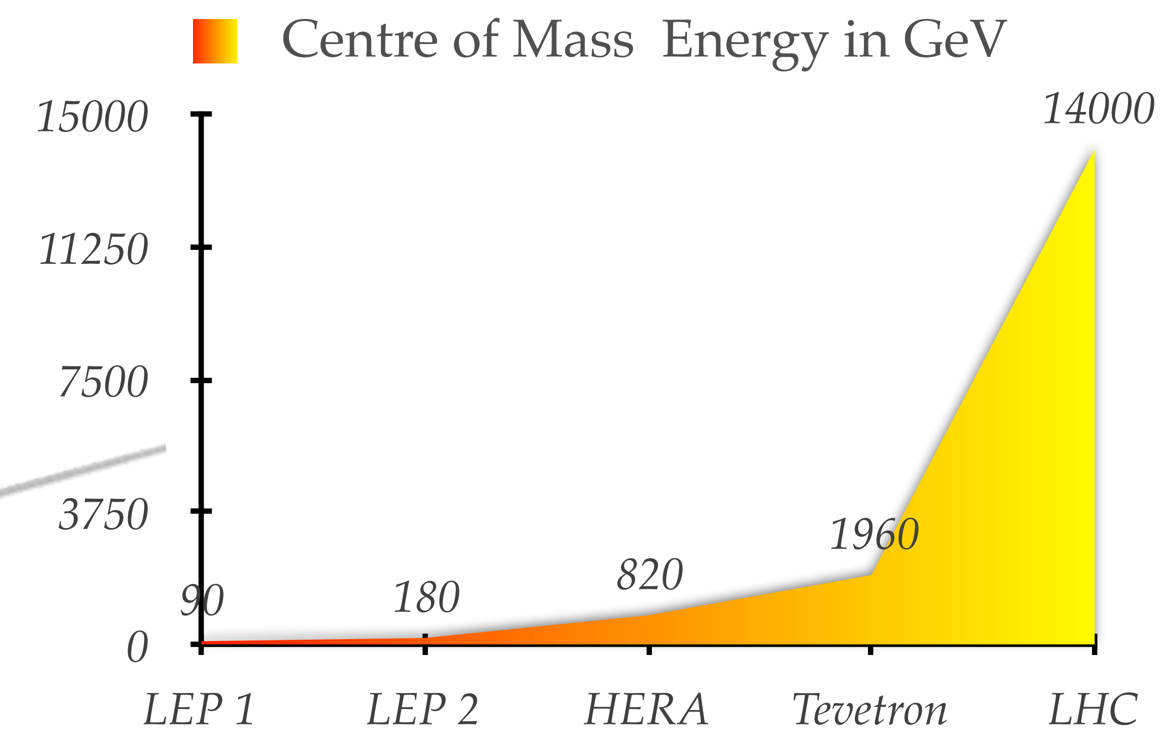
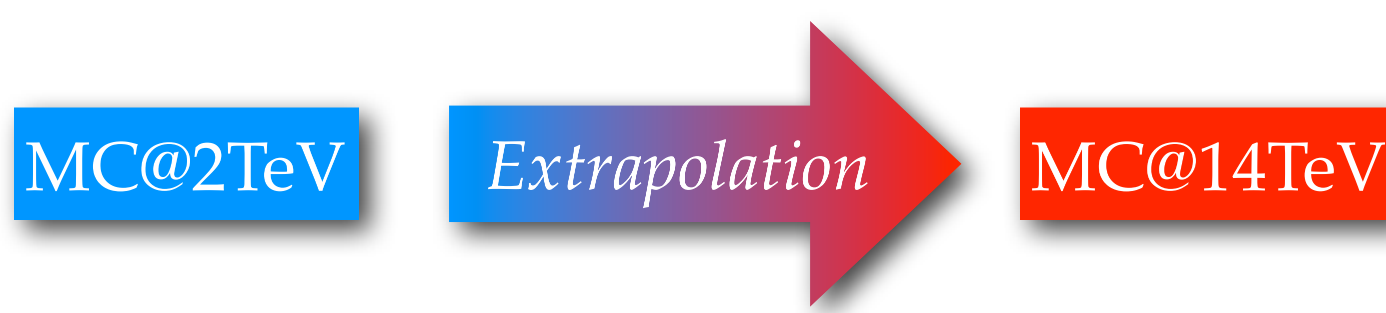
Introduction

A typical LHC event:



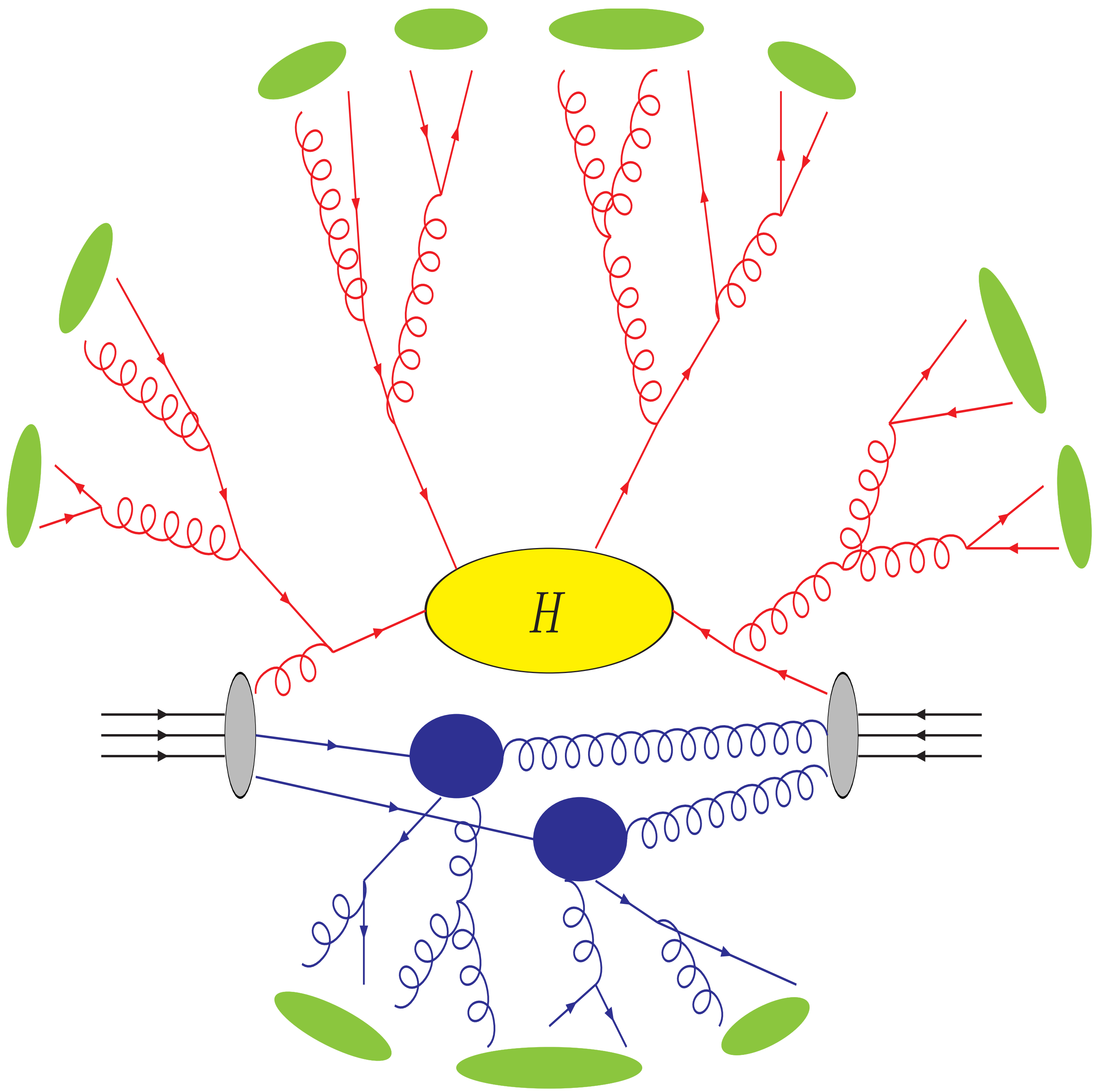




Picture: ATLAS simulation

Introduction

From theory point of view this event looks very complicated



1. Incoming hadron (gray bubbles)
   - Parton distribution function
   - Multi parton distribution functions

2. Hard part of the process (yellow bubble)
   - Matrix element calculation, cross sections at LO, NLO, NNLO level

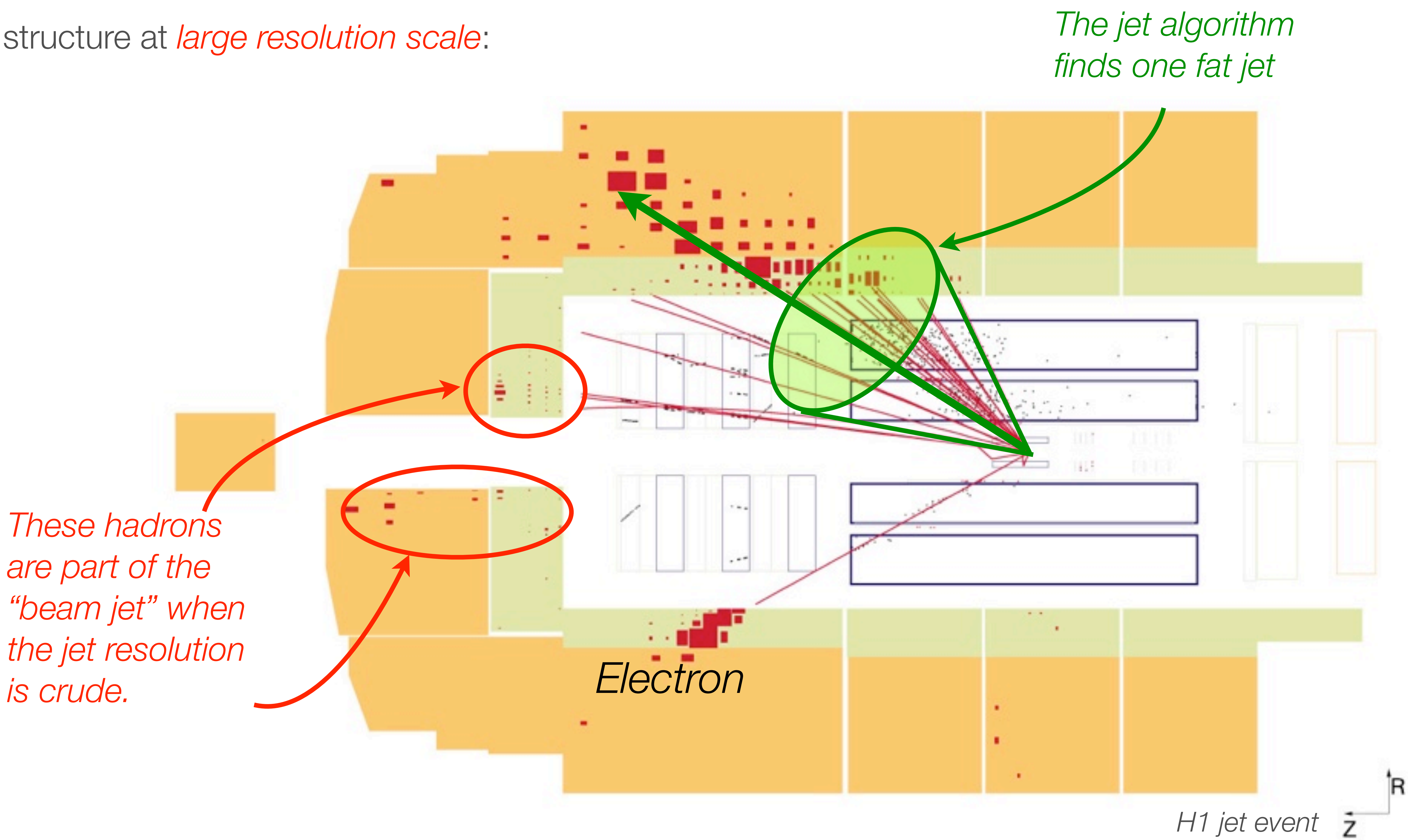
3. Radiations (red graphs)
   - Parton shower calculation
   - Partonic decay
   - Matching to NLO, NNLO

4. Underlying event (blue graphs)
   - Models based on multiple interaction
   - Diffraction

5. Hardonization (green bubbles)
   - Universal models
   - Hadronic decay
   - ....

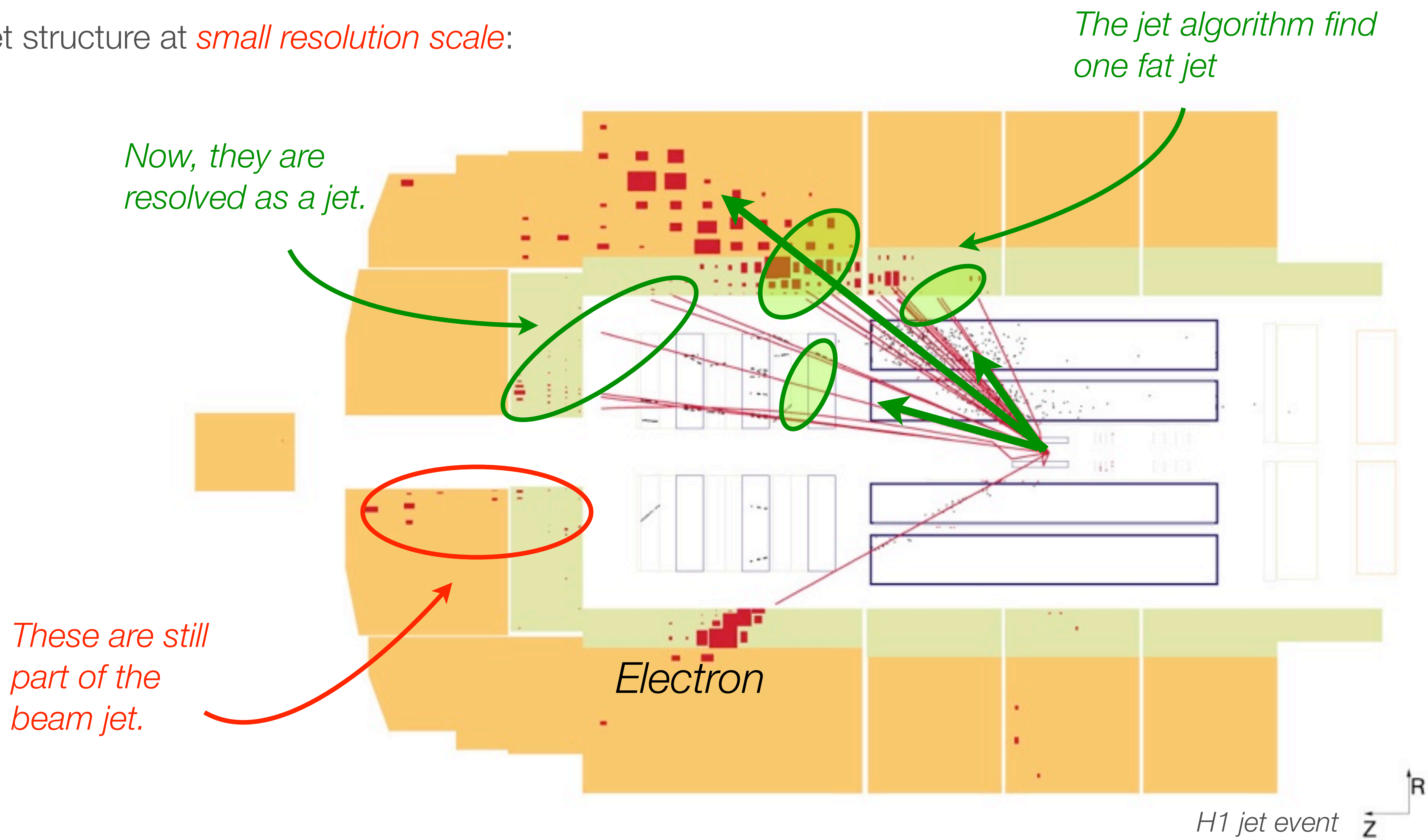
Jet event in DIS process

Jet structure at *large resolution scale*:



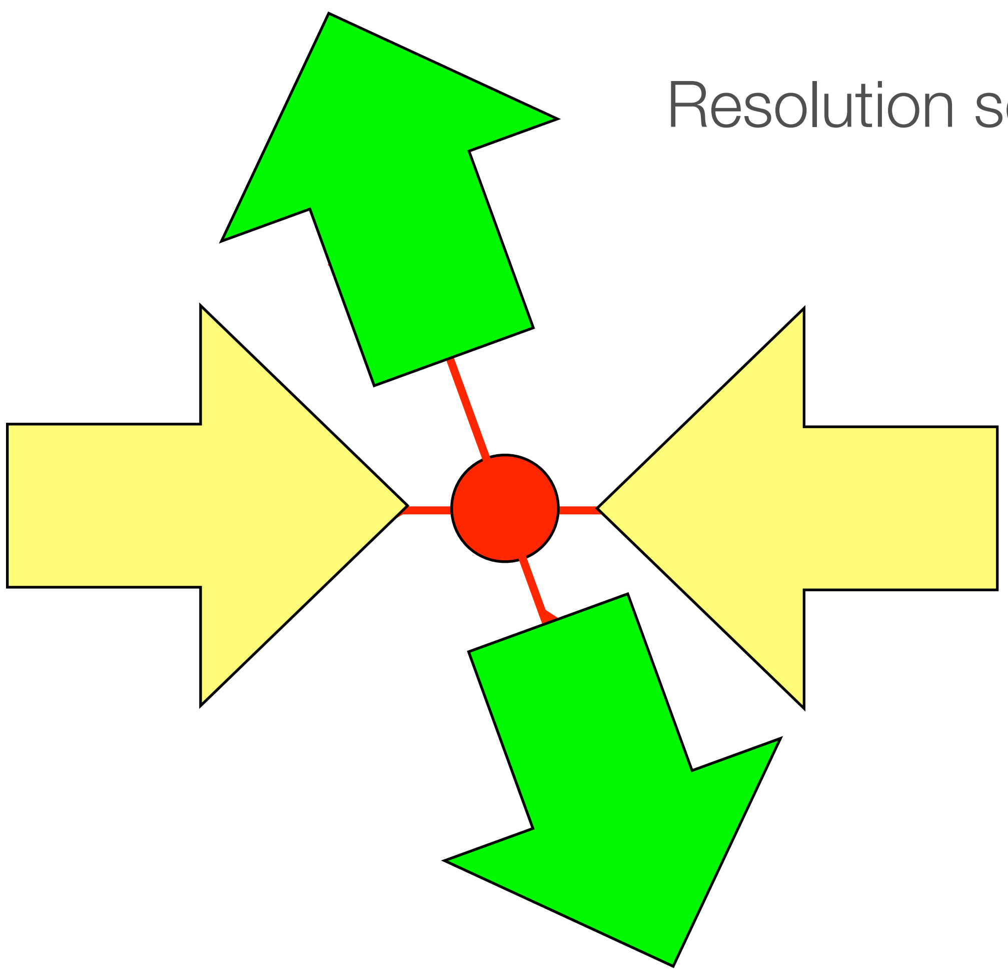
Jet event in DIS process

Jet structure at *small resolution scale*:



Hadron-Hadron Collision

In hadron-hadron collision the picture is more complicated.



Resolution scale: 400 GeV

Decreasing the resolution scale more and more partons are visible and less absorbed by the incoming hadrons and the final state jets.

Important observation: The total cross section *is independent of* the resolution of the measurement (or detector).

We have to also consider the evolution of the final state jets.

Does perturbative QCD support this nice intuitive picture?

Hadron-Hadron Collision

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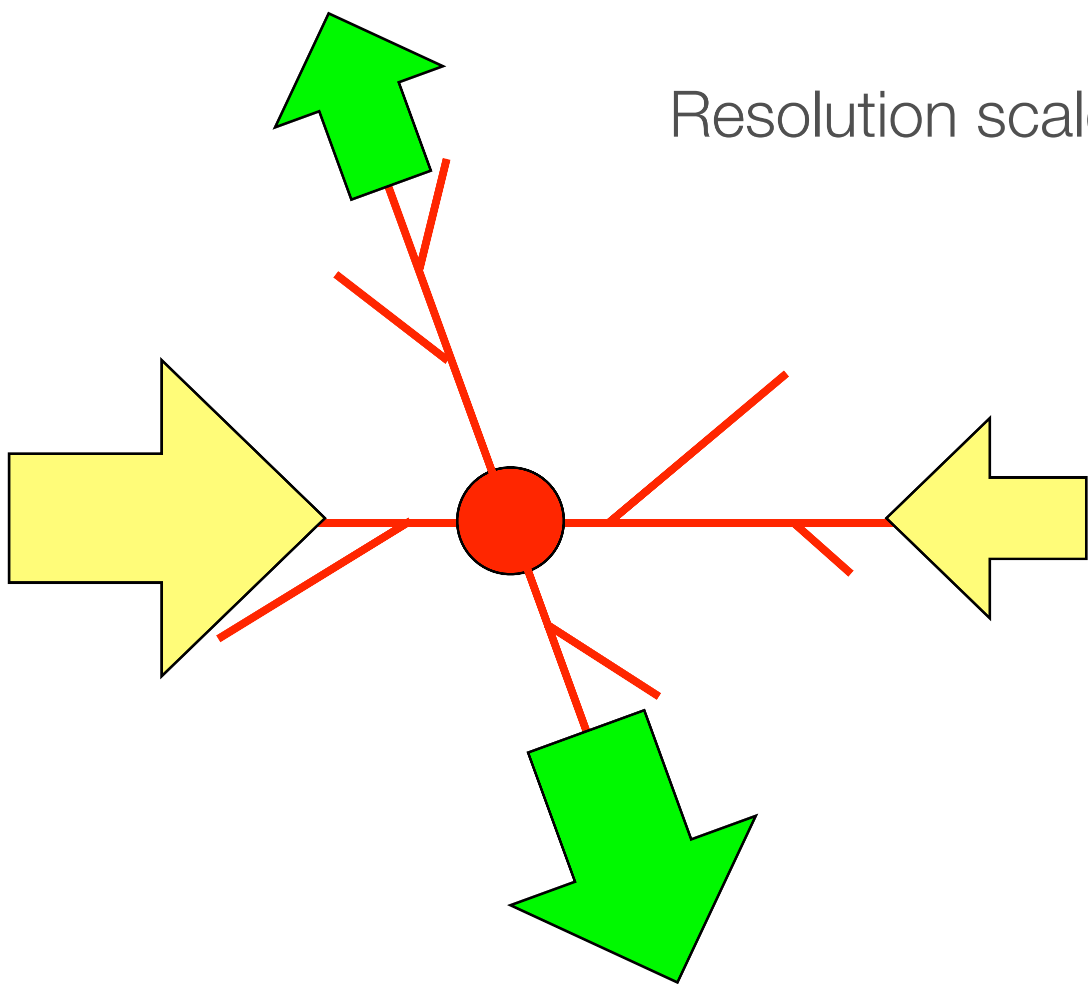

Resolution scale: 100 GeV

Decreasing the resolution scale more and more partons are visible and less absorbed by the incoming hadrons and the final state jets.

<u>Important observation</u>: The total cross section *is independent of* the resolution of the measurement (or detector).

We have to also consider the evolution of the final state jets.

Does perturbative QCD support this nice intuitive picture?

Cross section

The cross section is a phase space integral of all the possible matrix elements and the a convolution to the parton distribution functions.

$$\sigma[F] = \sum_m \int [d\{p,f\}_m] \overbrace{f_{a/A}(\eta_a, \mu_F^2)\, f_{b/B}(\eta_b, \mu_F^2)}^{\textit{parton distributions}} \frac{1}{2\eta_a \eta_b p_A \cdot p_B}$$

$$\times \left\langle \mathcal{M}(\{p,f\}_m) \right| \underbrace{F(\{p,f\}_m)}_{\textit{observable}} \underbrace{\left| \mathcal{M}(\{p,f\}_m) \right\rangle}_{\textit{matrix element}}$$

The fully exclusive final state is described by the QCD density operator, that is the basic object in the Monte Carlos

$$\rho = \sum \rho\big(\{p,f\}_{m+1}\big) \Leftrightarrow \big|\rho\big) = \sum \big|\rho(\{p,f\}_{m+1})\big)$$

- ✗ This is formally an all order expression and it is impossible to calculate out.
- ✗ We can do it at LO, NLO and in some cases NNLO level.
- ✗ Lots of complication with IR singularities.
- ✗ Lots of complication with spin and colors.
- ✓ Try to approximate by using soft and collinear factorization of QCD amplitudes

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Statistical Space

Introducing the density operator, the cross section is

$$\sigma[F] = \sum_m \int [d\{p,f\}_m]\ \mathrm{Tr}\{\underbrace{\rho(\{p,f\}_m)}_{\textit{density operator in color} \otimes \textit{spin space}} F(\{p,f\}_m)\}$$

where the density operator is

$$\begin{aligned}\rho(\{p,f\}_m) &= \big|\mathcal{M}(\{p,f\}_m)\big\rangle \frac{f_{a/A}(\eta_{\mathrm{a}},\mu_F^2) f_{b/B}(\eta_{\mathrm{b}},\mu_F^2)}{2\eta_{\mathrm{a}}\eta_{\mathrm{b}} p_A\cdot p_B} \big\langle \mathcal{M}(\{p,f\}_m)\big| \\ &= \sum_{s,c,s',c'} \big|\{s',c'\}_m\big\rangle \big(\{p,f,s',c',s,c\}_m\big|\rho\big) \big\langle\{s,c\}_m\big|\end{aligned}$$

In the statistical space it is represented by a vector

$$\big|\rho\big) = \sum_m \frac{1}{m!}\int [d\{p,f,s',c',s,c\}_m]\ \underbrace{\big|\{p,f,s',c',s,c\}_m\big)}_{\textit{Basis vector in the statistical space}}\ \big(\{p,f,s',c',s,c\}_m\big|\rho\big)$$

Approx. of the Density Operator

We try to approximate QCD amplitudes by using their *soft collinear factorization properties*.

Real radiations: $\mathcal{H}_I(t) = \mathcal{H}_C(t) + \mathcal{H}_S(t)$

$$\big|\rho(\{\hat{p},\hat{f}\}_{m+1})\big) \approx \int_{t_m}^{\infty} dt\, \Big[\underbrace{\mathcal{H}_C(t)}_{\text{Collinear and soft-collinear contribution}} + \overbrace{\mathcal{H}_S(t)}^{\text{Wide angle soft contributions}}\Big] \big|\rho(\{p,f\}_m)\big)$$

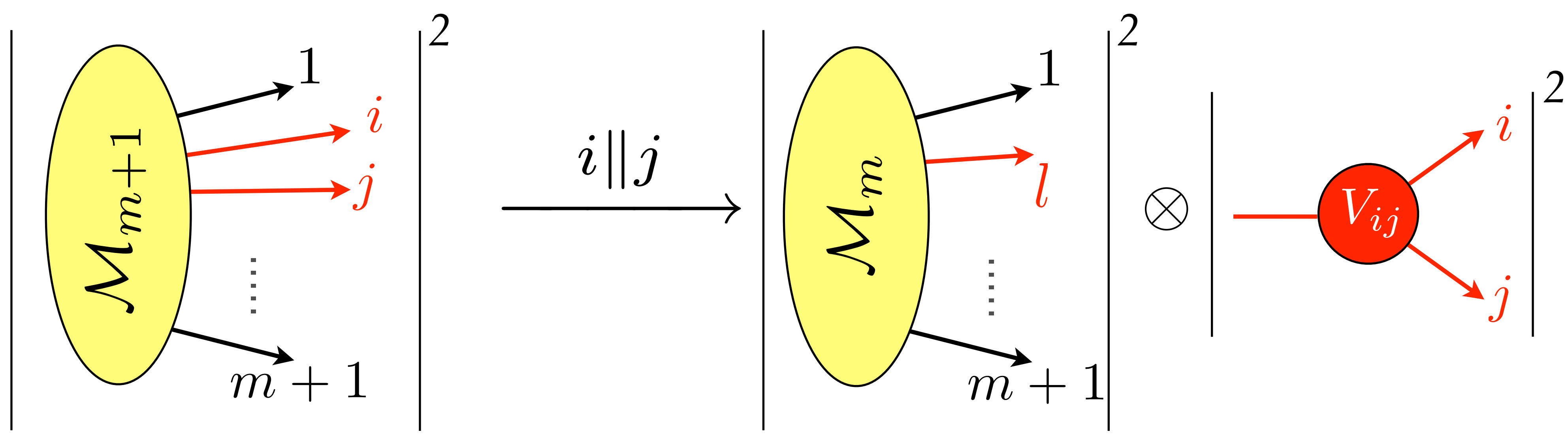
This parameter represents the hardness of the splitting. We will call it shower time.

Now, let us consider a measurement with a resolution scale which correspond to shower time t'

$$\big|\rho^{\mathrm{R}}_{\infty}\big) \approx \underbrace{\int_{t}^{t'} d\tau\, \mathcal{H}_I(\tau)\, \big|\rho(t)\big)}_{\text{Resolved radiations}} + \underbrace{\int_{t'}^{\infty} d\tau\, \mathcal{V}_I^{(\epsilon)}(\tau)\, \big|\rho(t)\big)}_{\substack{\text{Unresolved radiations} \\ \text{This is a singular contribution}}}$$

Collinear Singularities

The QCD matrix elements have universal factorization property when two external partons become collinear

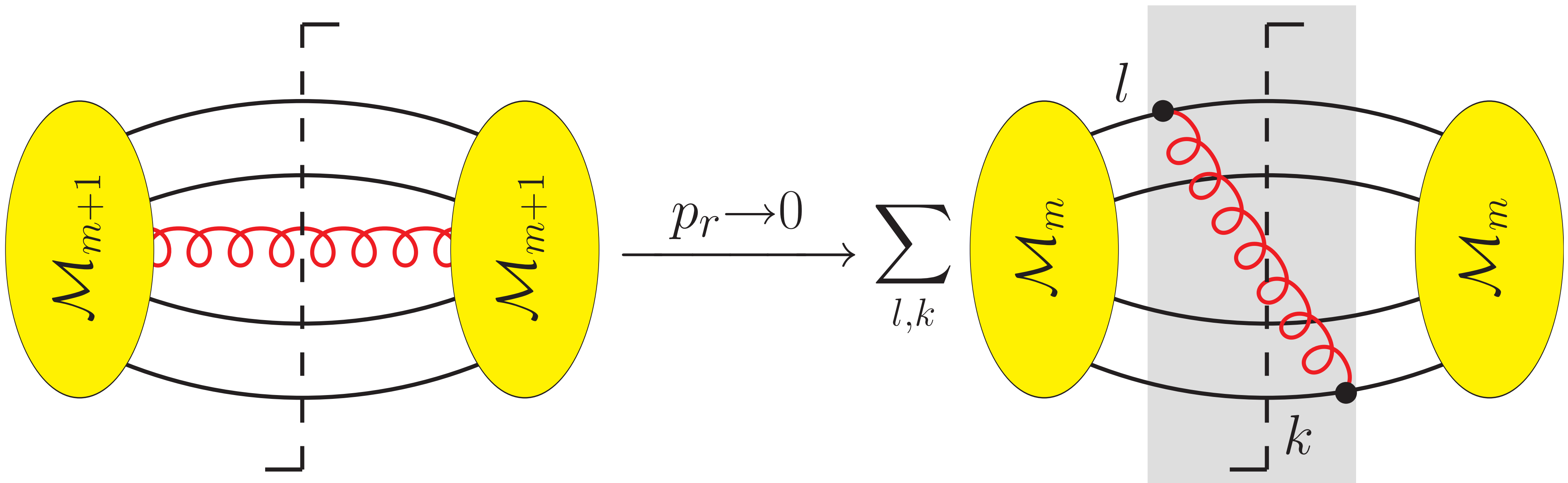


$$\mathcal{H}_C \sim \sum_l t_l \otimes t_l^\dagger \, V_{ij}(s_i, s_j) \otimes V_{ij}^\dagger(s_i', s_j') \Leftrightarrow \frac{\alpha_s}{2\pi} \sum_l \frac{1}{p_i \cdot p_j} P_{f_l, f_i}(z) + \ldots$$

Altarelli-Parisi splitting kernels

Soft Singularities

The QCD matrix elements have universal factorization property when an external gluon becomes soft

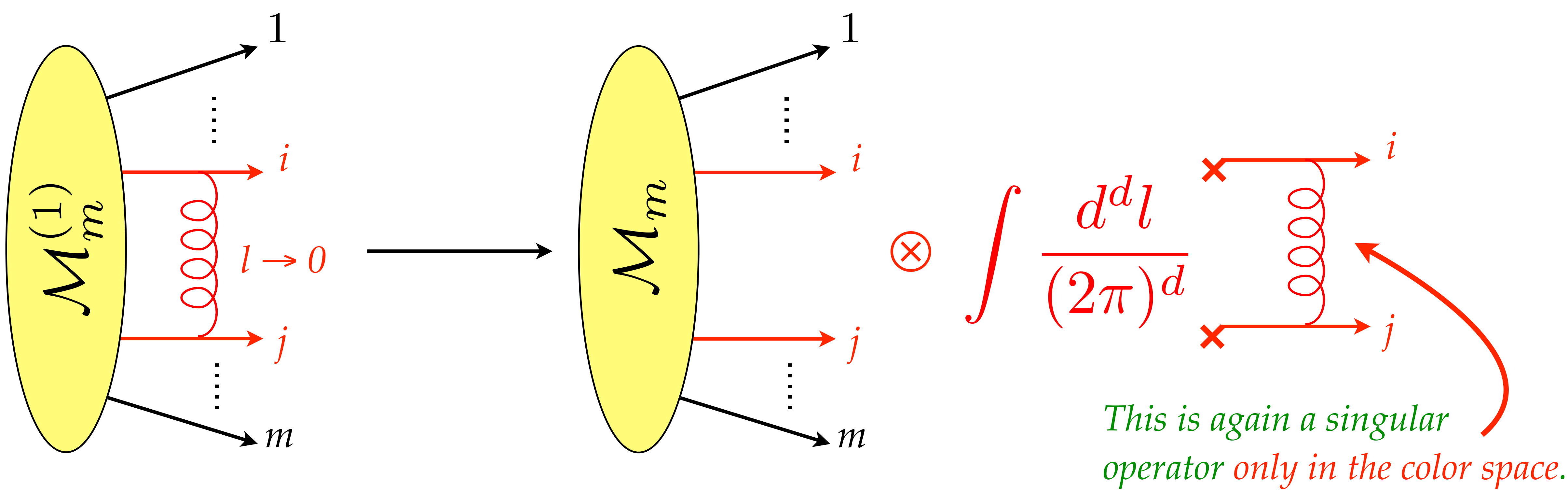


$$\mathcal{H}_S \sim -\sum_{\substack{l,k\\ l\neq k}} \frac{\hat{p}_l \cdot \varepsilon(s)\, \hat{p}_k \cdot \varepsilon(s')}{\hat{p}_l \cdot \hat{p}_{m+1}\, \hat{p}_k \cdot \hat{p}_{m+1}}\, t_l \otimes t_k^\dagger$$

Soft gluon connects everywhere and the color structure is not diagonal; quantum interferences in the color space.

Virtual Contributions

There is another type of the unresolvable radiation, *the virtual (loop graph) contributions.* We have *universal factorization properties* for the loop graphs. E.g.: in the soft limit, when the loop momenta become soft we have



Approx. of the Density Operator

There is another type of the unresolvable radiation, *the virtual (loop graph) contributions.* We have *universal factorization properties* for the loop graphs. E.g.: in the soft limit, when the loop momenta become soft we have

$$\big|\rho_\infty^{\mathrm{V}}\big) \approx - \int_t^\infty d\tau\, \mathcal{V}_I^{(\epsilon)}(\tau)\, \big|\rho(t)\big)$$

Same structure like in the real unresolved case but here with opposite sign.

Combining the real and virtual contribution we have got

$$\big|\rho_\infty^{\mathrm{R}}\big) + \big|\rho_\infty^{\mathrm{V}}\big) = \int_t^{t'} d\tau\, \big[\mathcal{H}_I(\tau) - \mathcal{V}_I(\tau)\big]\big|\rho(t)\big)$$

This operator dresses up the physical state with one real and virtual radiations that *is softer or more collinear than the hard state*. Thus the emissions are ordered.

Physical States

Now we can use this to build up physical states by considering all the possible way to go from t to t'.

$$\begin{aligned}
\big|\rho(t')\big) &= \big|\rho(t)\big) \\
&\quad + \int_t^{t'} d\tau\, [\mathcal{H}_I(\tau) - \mathcal{V}_I(\tau)]\big|\rho(t)\big) \\
&\quad + \int_t^{t'} d\tau_2\, [\mathcal{H}_I(\tau_2) - \mathcal{V}_I(\tau_2)] \int_t^{\tau_2} d\tau_1\, [\mathcal{H}_I(\tau_1) - \mathcal{V}_I(\tau_1)]\big|\rho(t)\big) \\
&\quad + \cdots \\
&= \underbrace{\mathbb{T}\exp\left\{\int_t^{t'} d\tau\, [\mathcal{H}_I(\tau) - \mathcal{V}_I(\tau)]\right\}}_{\mathcal{U}(t',t)\ \textit{shower evolution operator}} \big|\rho(t)\big)
\end{aligned}$$

$$\big|\rho(t')\big) = \mathcal{U}(t',t)\big|\rho(t)\big)$$

$$\frac{d}{dt}\mathcal{U}(t,t') = [\mathcal{H}_I(t) - \mathcal{V}_I((t)]\,\mathcal{U}(t,t')$$

This is an renormalization group equation, the problem is that we don't really know the "theory" that is renormalized by this equation.

Shower Evolution

Unitarity:

Shower evolution operator satisfy the following equation

$$\frac{d}{dt}\mathcal{U}(t,t') = \left[\mathcal{H}_I(t) - \mathcal{V}_I(t)\right]\mathcal{U}(t,t')$$

From $\big(1\big|\mathcal{V}_I(t)\big) = \big(1\big|\mathcal{H}_I(t)\big)$ one can see that the shower preserve the total cross section

$$\big(1\big|\mathcal{U}(t,t') = \big(1\big|$$

Group decomposition property:

$$\mathcal{U}(t,t')\,\mathcal{U}(t',t'') = \mathcal{U}(t,t'')$$

Let us have a physical state evolved to t and consider a measurement F with the typical resolution $t_F < t$. For soft or collinear splittings we have

$$\int_{t_F}^{t} d\tau\,\big(F\big|\mathcal{H}_I(\tau)\big) = \int_{t_F}^{t} d\tau\,\big(F\big|\mathcal{V}_I(t)\big)$$

Now the cross section is

$$\big(F\big|\rho(t)\big) = \big(F\big|\mathcal{U}(t,t_F)\big|\rho(t_F)\big) = \big(F\big|\rho(t_F)\big)$$

The measurement is insensitive for the finer structure, thus they are integrated out to 1.

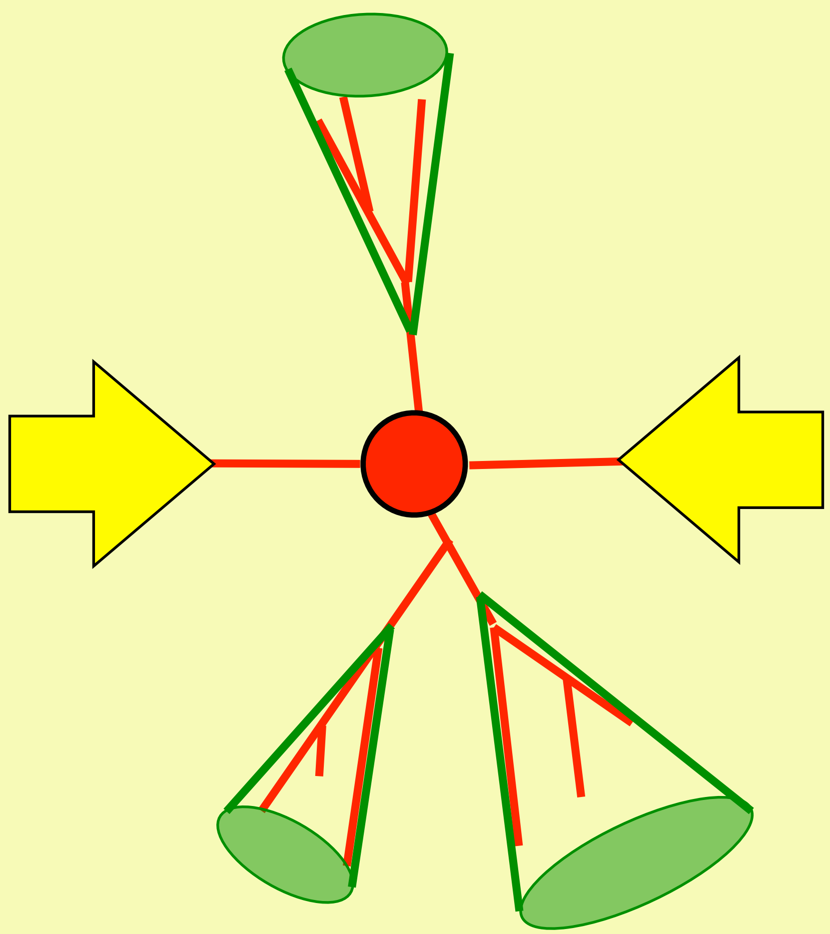
This is depicted in our cartoon!

Shower Evolution

Unitarity:

Shower evolu

$$\big(F\big|\rho(t)\big) = \big(F\big|\mathcal{U}(t,t_F)\big|\rho(t_F)\big) = \big(F\big|\rho(t_F)\big)$$



In this measurement we resolve jets. The resolution is represented by the "green cones"

The measurement is insensitive what is happening inside the "green cones". In the parton shower everything inside the cones are integrated out to 1.

From $\big(1\big|\mathcal{V}_I(t$ … section

Group decompo

Let us have a … typical

resolution $t_F <$

Now the cross

The measurer

structure, thu … *n our cartoon!*

Evolution Equation

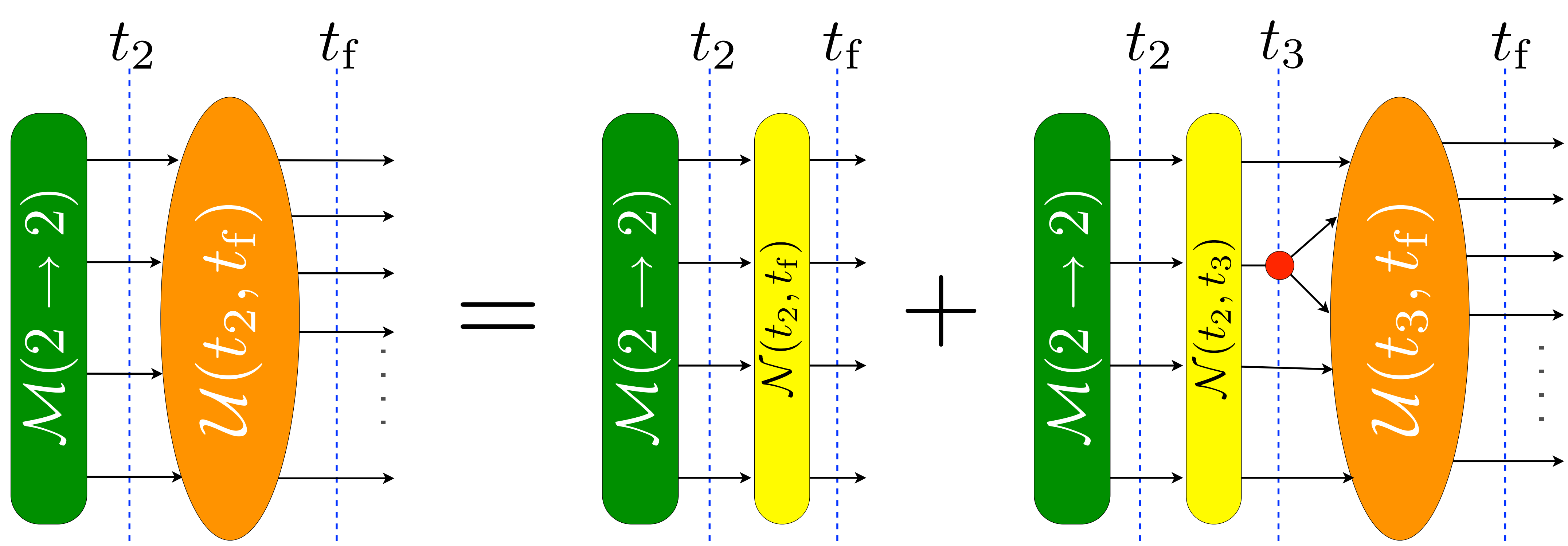
We can write the evolution equation in an integral equation form

$$\mathcal{U}(t_{\mathrm{f}}, t_2) = \underbrace{\mathcal{N}(t_{\mathrm{f}}, t_2)}_{\text{“Nothing happens”}} + \overbrace{\int_{t_2}^{t_{\mathrm{f}}} dt_3\, \mathcal{U}(t_{\mathrm{f}}, t_3) {\color{red}\mathcal{H}_I(t_3)} \mathcal{N}(t_3, t_2)}^{\text{“Something happens”}}$$

where the non-splitting operator is

$$\mathcal{N}(t', t) = \mathbb{T} \exp\left\{ - \int_t^{t'} d\tau\, {\color{red}\mathcal{V}_I(\tau)} \right\}$$

Sudakov operator



Full Splitting Operator

Very general splitting operator (*no spin correlation*) is

Momentum and mapping

$$
\begin{aligned}
&\big(\{\hat p,\hat f,\hat c',\hat c\}_{m+1}\big|\mathcal{H}(t)\big|\{p,f,c',c\}_m\big)\\
&= \sum_{l=\mathrm{a,b},1,\dots,m} \delta\Big(t - T_l\big(\{\hat p,\hat f\}_{m+1}\big)\Big)\,\big(\{\hat p,\hat f\}_{m+1}\big|\mathcal{P}_l\big|\{p,f\}_m\big)\,\frac{m+1}{2}\\
&\quad\times \frac{n_{\mathrm c}(a)n_{\mathrm c}(b)\,\eta_{\mathrm a}\eta_{\mathrm b}}{n_{\mathrm c}(\hat a)n_{\mathrm c}(\hat b)\,\hat\eta_{\mathrm a}\hat\eta_{\mathrm b}}\,\frac{f_{\hat a/A}(\hat\eta_{\mathrm a},\mu_F^2)f_{\hat b/B}(\hat\eta_{\mathrm b},\mu_F^2)}{f_{a/A}(\eta_{\mathrm a},\mu_F^2)f_{b/B}(\eta_{\mathrm b},\mu_F^2)}\sum_k \Psi_{lk}(\{\hat f,\hat p\}_{m+1})\\
&\quad\times \sum_{\beta=L,R}(-1)^{1+\delta_{lk}}\big(\{\hat c',\hat c\}_{m+1}\big|\mathcal{G}_\beta(l,k)\big|\{c',c\}_m\big)
\end{aligned}
$$

Color mapping

Splitting kernel is

It is only LO!

$$
\Psi_{lk} = \frac{\alpha_{\mathrm s}}{2\pi}\,\frac{1}{\hat p_l\cdot\hat p_{m+1}}\left[A_{lk}\,\frac{2\hat p_l\cdot\hat p_k}{\hat p_k\cdot\hat p_{m+1}} + H_{ll}^{\mathrm{coll}}(\{\hat f,\hat p\}_{m+1})\right]
$$

Important:

$$
A_{lk} + A_{kl} = 1
$$

Color operator for gluon emission is

$$
\begin{aligned}
&\big(\{\hat c',\hat c\}_{m+1}\big|\mathcal{G}_R(l,k)\big|\{c',c\}_m\big)\\
&\qquad = {}_D\big\langle\{\hat c\}_{m+1}\big|t_l^\dagger\big|\{c\}_m\big\rangle\,\big\langle\{c'\}_m\big|t_k\big|\{\hat c'\}_{m+1}\big\rangle_D\ .
\end{aligned}
$$

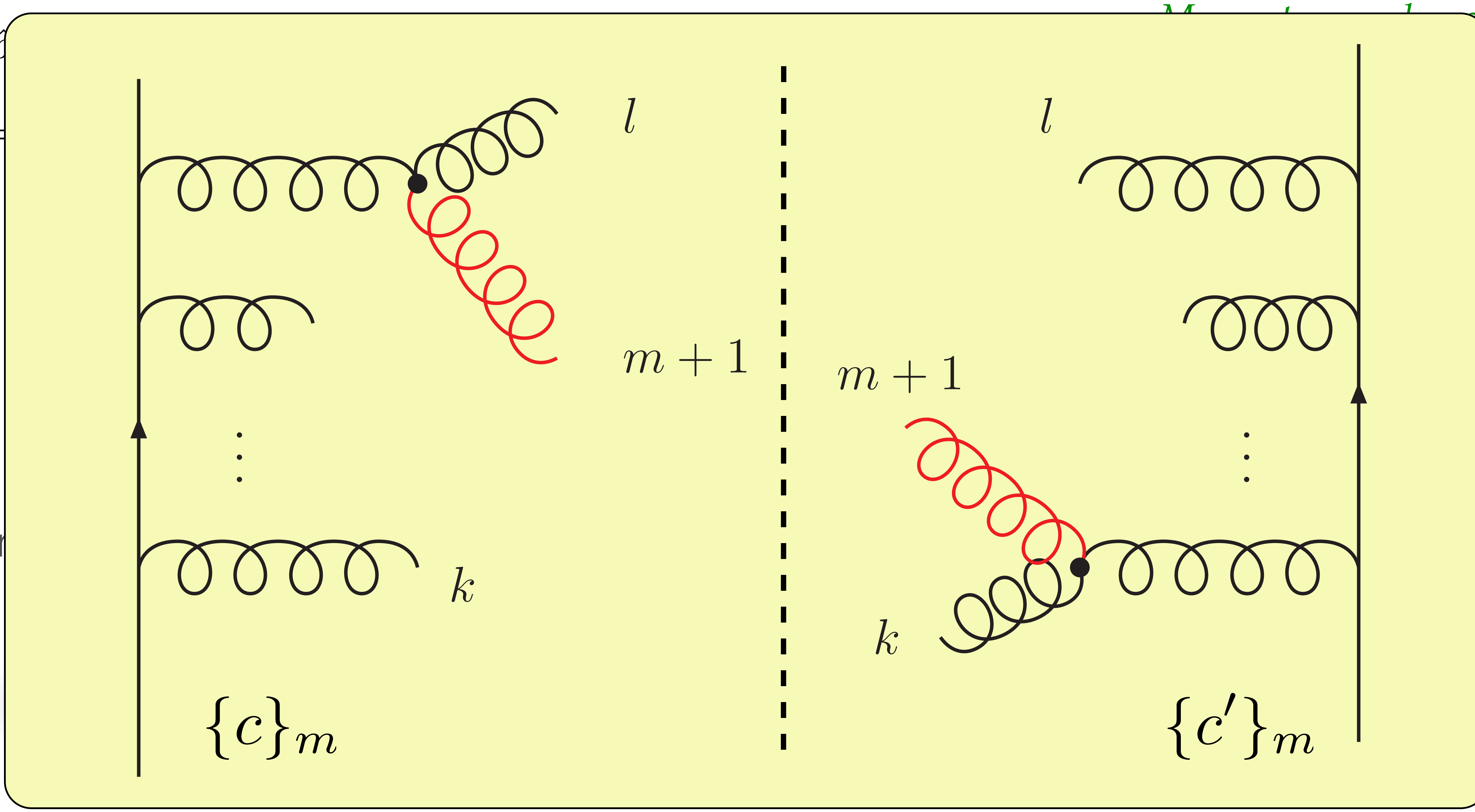
Full Splitting Operator

Very general splitting operator (*no spin correlation*) is

M … pping

$(\{\hat{p}$

$=$



Splitti

Ψ_{lk}

$= 1$

Color

$$\left(\{\hat{c}',\hat{c}\}_{m+1}\middle|\mathcal{G}_R(l,k)\middle|\{c',c\}_m\right)$$

$$= {}_D\!\left\langle\{\hat{c}\}_{m+1}\middle|t_l^\dagger\middle|\{c\}_m\right\rangle\left\langle\{c'\}_m\middle|t_k\middle|\{\hat{c}'\}_{m+1}\right\rangle_D \ .$$

Angular Ordered Shower

Herwig, Herwig++

What would happen if we used angular ordering?

$$t_\angle = T_l\big(\{\hat{p},\hat{f}\}_{m+1}\big) = \log 2 - \log\frac{\hat{p}_l\cdot\hat{p}_{m+1}\,\hat{Q}^2}{\hat{p}_l\cdot\hat{Q}\,\hat{p}_{m+1}\cdot\hat{Q}} = \log\frac{2}{1-\cos\vartheta_{l,m+1}}$$

And let's have a special choice for soft partitioning function:

$$A'_{lk} = \theta(\vartheta_{l,m+1} < \vartheta_{l,k})\,\frac{1-\cos\vartheta_{m+1,k}}{1-\cos\vartheta_{l,k}} \qquad \Rightarrow \qquad A_{lk} + A_{kl} \approx 1$$

$$\Psi_l^{(\mathrm{a.o.})} = \frac{\alpha_\mathrm{s}}{2\pi}\,\frac{2}{\hat{p}_l\cdot\hat{p}_{m+1}}\left[\frac{\hat{p}_l\cdot\hat{Q}}{\hat{p}_{m+1}\cdot\hat{Q}} + H_{ll}^{\mathrm{coll}}\big(\{\hat{f},\hat{p}\}_{m+1}\big)\right]$$

Independent of parton k!!!

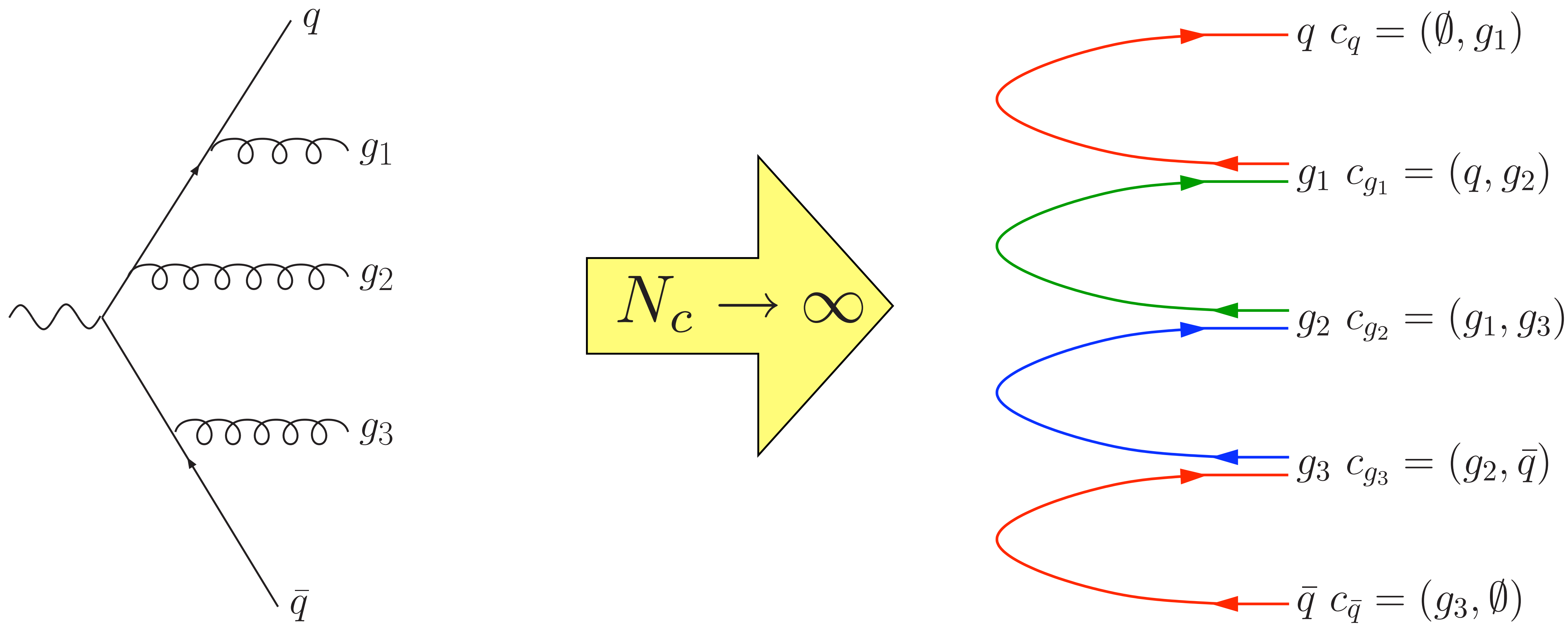
One can perform the sum over the color connected parton analytically

$$-\sum_k \big(\{\hat{c}',\hat{c}\}_{m+1}\big|\mathcal{G}_\beta(l,k)\big|\{c',c\}_m\big) = \big(\{\hat{c}',\hat{c}\}_{m+1}\big|\mathcal{G}_\beta(l,l)\big|\{c',c\}_m\big)$$

No complicated color structure.

Leading Color Approx.

In leading color approximation a the color **8** state is replaced by a color **3**⊗**<u>3</u>** state



Furthermore we have only diagonal color configurations

$$\{c\}_m = \{c'\}_m \quad \Longrightarrow \quad \big|\{p, f, c\}_m\big)$$

Leading Color Approx.

1. Don't have special choice for the evolution variable and the soft partitioning function

Anyway everybody uses transverse momentum
and the simplest soft partitioning function :

PYTHIA, SHERPA

$$t_\perp = T_l\big(\{\hat{p},\hat{f}\}_{m+1}\big) = \log\frac{\hat{Q}^2}{-k_\perp^2}$$

$$A_{lk} = \frac{\hat{p}_k\cdot\hat{p}_{m+1}}{\hat{p}_k\cdot\hat{p}_{m+1} + \hat{p}_l\cdot\hat{p}_{m+1}}$$

2. But do approximation in the color space by considering only the leading color contributions

$$\begin{aligned}
&\big(\{\hat{p},\hat{f},\hat{c}\}_{m+1}\big|\mathcal{H}(t)\big|\{p,f,c\}_m\big) \\
&\quad = \sum_{l=\mathrm{a,b},1,\dots,m} \delta\Big(t - T_l\big(\{\hat{p},\hat{f}\}_{m+1}\big)\Big)\,\big(\{\hat{p},\hat{f}\}_{m+1}\big|\mathcal{P}_l\big|\{p,f\}_m\big) \\
&\qquad\times \frac{n_\mathrm{c}(a)n_\mathrm{c}(b)\,\eta_\mathrm{a}\eta_\mathrm{b}}{n_\mathrm{c}(\hat{a})n_\mathrm{c}(\hat{b})\,\hat{\eta}_\mathrm{a}\hat{\eta}_\mathrm{b}}\,\frac{f_{\hat{a}/A}(\hat{\eta}_\mathrm{a},\mu_F^2)f_{\hat{b}/B}(\hat{\eta}_\mathrm{b},\mu_F^2)}{f_{a/A}(\eta_\mathrm{a},\mu_F^2)f_{b/B}(\eta_\mathrm{b},\mu_F^2)} \\
&\qquad\times (m+1)\sum_k \Psi_{lk}(\{\hat{f},\hat{p}\}_{m+1})\,\big\langle\{\hat{c}\}_{m+1}\big|a_{lk}^\dagger\big|\{c\}_m\big\rangle\ .
\end{aligned}$$

Antenna Dipole Shower

Ariadne, Vincia

The antenna dipole shower is rather a *reorganization of the leading color* partitioned dipole *shower*.

$$\mathcal{H}_{lk}^{\rm part}(t) \propto \left[\mathcal{P}_l \, A_{lk} + \mathcal{P}_k \, A_{kl}\right] \frac{\hat{p}_l \cdot \hat{p}_k}{\hat{p}_{m+1} \cdot \hat{p}_l \;\, \hat{p}_{m+1} \cdot \hat{p}_k}$$

The antenna shower tries to remove the ambiguity of the soft partitioning function A_{lk} by using a new momentum mapping

$$\mathcal{H}_{lk}^{\rm ant}(t) \propto \mathcal{P}_{lk} \, \frac{\hat{p}_l \cdot \hat{p}_k}{\hat{p}_{m+1} \cdot \hat{p}_l \;\, \hat{p}_{m+1} \cdot \hat{p}_k}$$

Now the freedom to choose A_{lk} function resides in the freedom to choose P_{lk}. I think the best mapping for antenna shower would be

$$\mathcal{P}_{lk} = \theta(\vartheta_{l,m+1} < \vartheta_{k,m+1}) \, \mathcal{P}_l + \theta(\vartheta_{k,m+1} < \vartheta_{l,m+1}) \, \mathcal{P}_k$$

Parton Showers

There are basically three points where the essential differences lie, namely the *momentum mapping*, the *evolution parameter* and the choice of the *soft partitioning function*.

Angular ordered shower

- Full color evolution, but no wide angle soft gluon radiation at all.
- Easy to implement
- Loosing the full exclusiveness
- Angle doesn't control the goodness of the underlying approximation (soft and collinear approx.)

Leading color shower

- More flexible
- Systematically improvable
- Easy to implement
- Leading color approximation

Should/Can we do it better?

Do we need subleading color?

Matrix element square is

$$|\mathcal{M}(\{p,f\}_m)|^2 = N_c^n \sum_{\{c\}_m} |A(\{p,f,c\}_m)|^2 + \mathcal{O}\left(\frac{1}{N_c^2}\right)$$

where $A(\{p,f,c\}_m)$ is the color subamplitudes of the color configuration $\{c\}_m$

Cross sections at $\sqrt{s} = 1960$ GeV, with structure functions, in nanobarns, $p_T > 10\,\mathrm{GeV}$ $|\eta| < 2.0$.

Process	σ_0 : Normal	σ_1 : Large Nc component	$\frac{\sigma_1 - \sigma_0}{\sigma_0}$
ud→W+g	0.1029(5)D+01	0.1158(5)D+01	13%
ud→W+gg	0.1018(8)D+00	0.1283(10)D+00	26%
ud→W+ggg	0.1119(17)D-01	0.1564(22)D-01	40%
ud→W+gggg	0.1339(36)D-02	0.2838(71)D-02	120%

Results were calculated by HELAC

Do we need subleading color?

One can think about to go beyond the LO shower or include multi parton interaction (MI). It is clear that there is no way to go higher order with leading color approximation.

$$\alpha_s \approx \frac{1}{N_c^2} \approx 0.1$$

There are two perturbative parameters. The *formal expansion* of the splitting operator is

$$\begin{aligned}\mathcal{H}_{\mathrm{I}} = \frac{\alpha_{\mathrm{s}}}{2\pi}\mathcal{H}_{\mathrm{I}}^{(0,0)} + \frac{\alpha_{\mathrm{s}}}{2\pi}\frac{1}{N_c^2}\mathcal{H}_{\mathrm{I}}^{(0,1)} &+ \left(\frac{\alpha_{\mathrm{s}}}{2\pi}\right)^2 \mathcal{H}_{\mathrm{I}}^{(1,0)} \\ &+ \left(\frac{\alpha_{\mathrm{s}}}{2\pi}\right)^2 \mathcal{H}_{\mathrm{MI}}^{(0,0)} \\ &+ \cdots\end{aligned}$$

Furthermore we *need two color indices* to represent a partonic states (interference terms).

(Note that $\mathcal{H}_I$ is an operator and it is impossible to do this expansion in practice.)

Do we need subleading color?

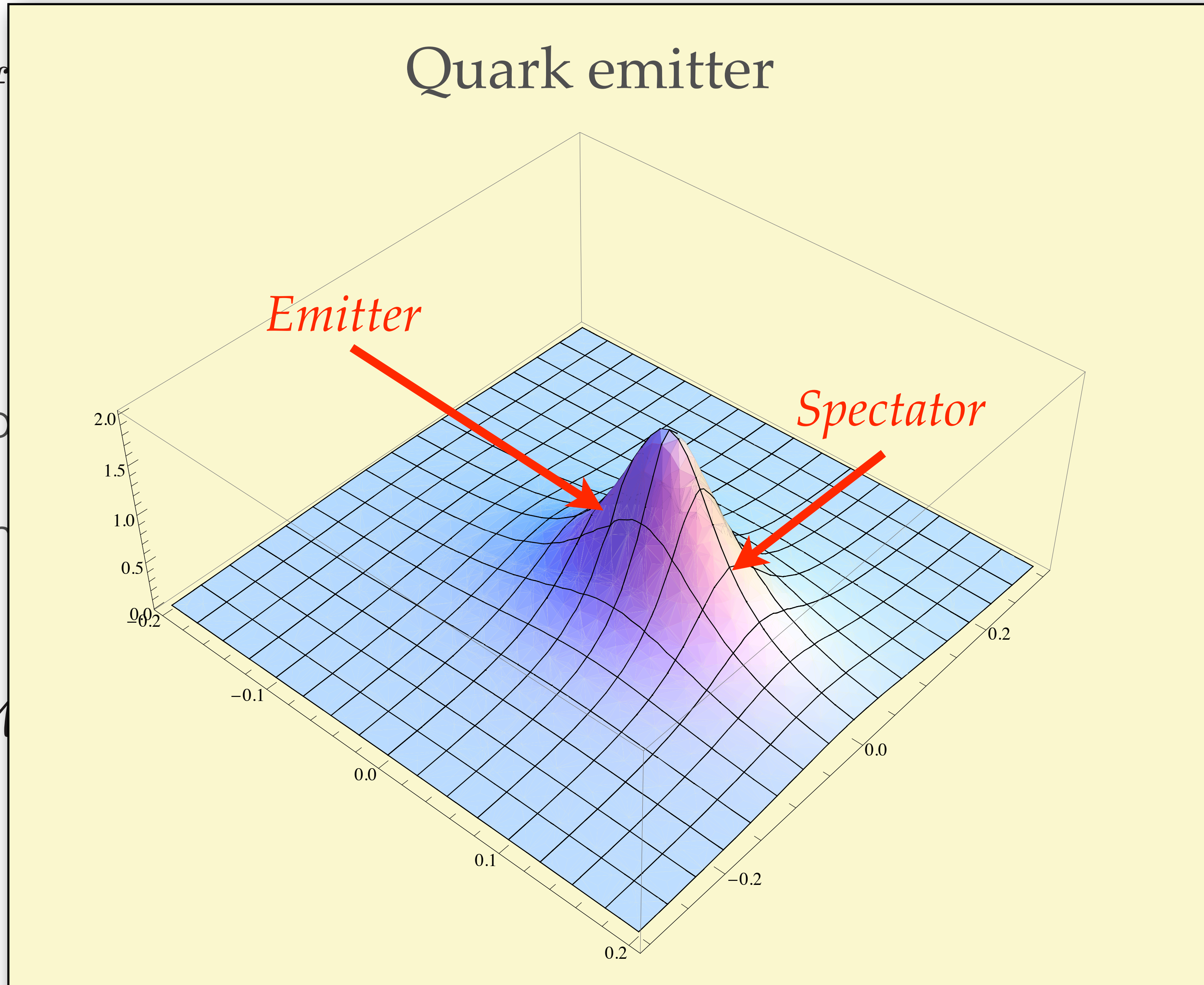
Parton shower starts from the tree level exact matrix elements

$|\mathcal{M}(\{p, f$

$, c'\}_m)^* \}$

How to assign co … ence contributions? *In* … get the normalization righ

$|\mathcal{M}$ … $n)|^2$



Note this is just the standard K-factor trick.

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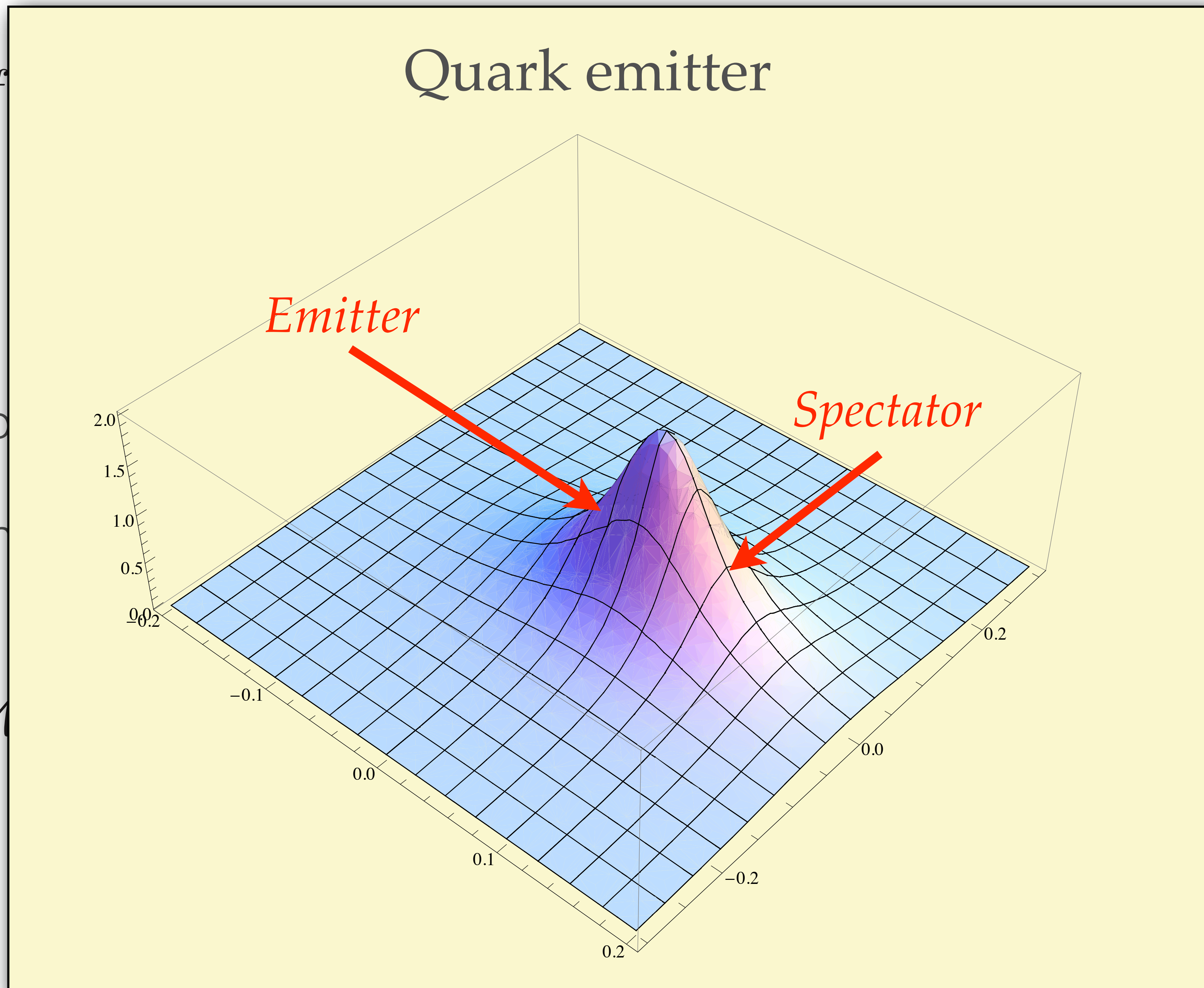

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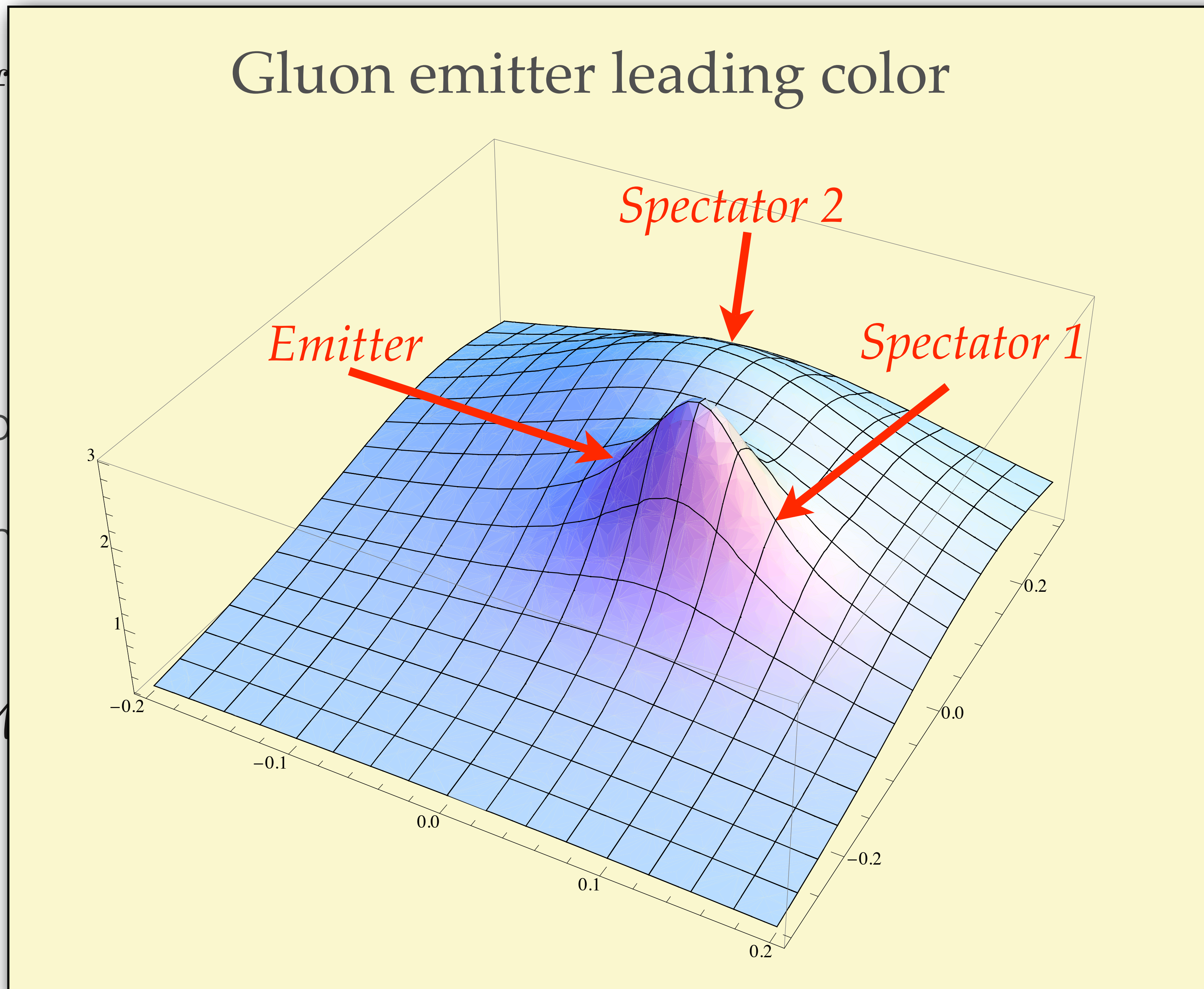

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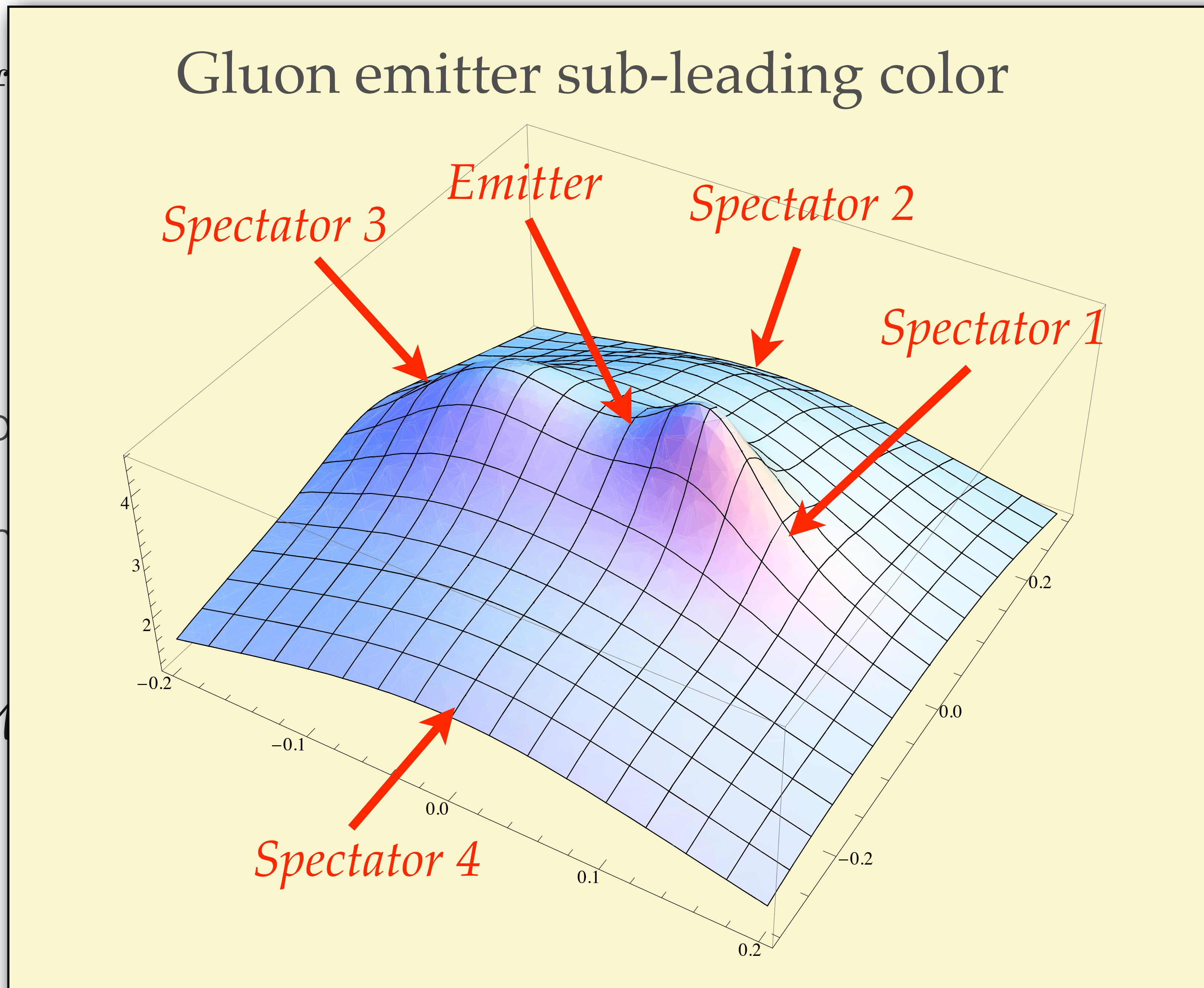
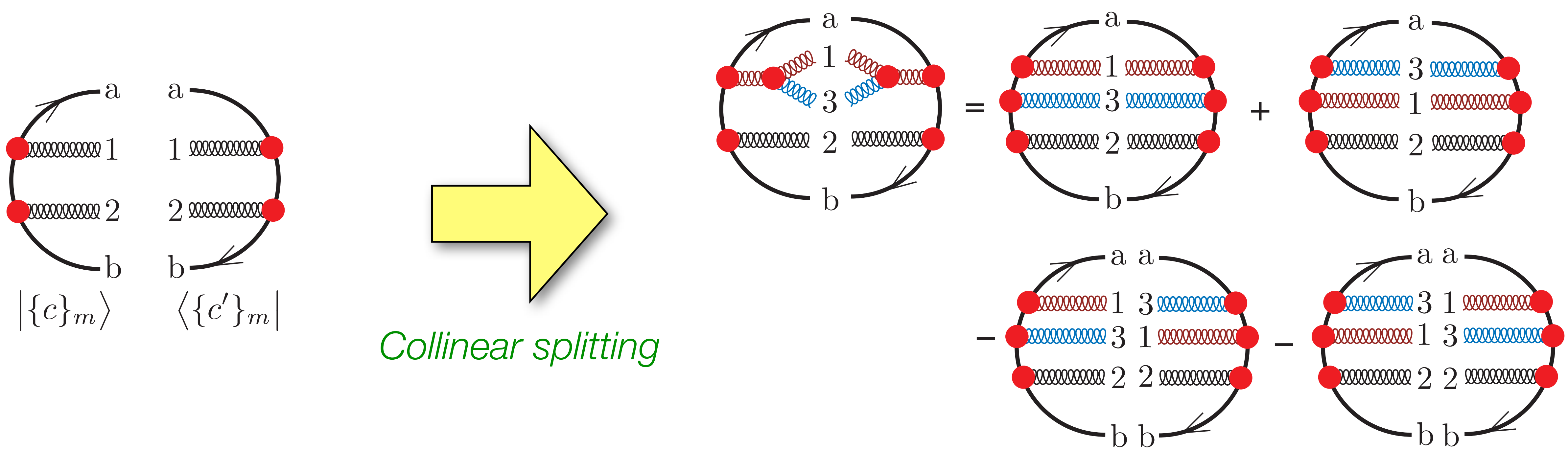


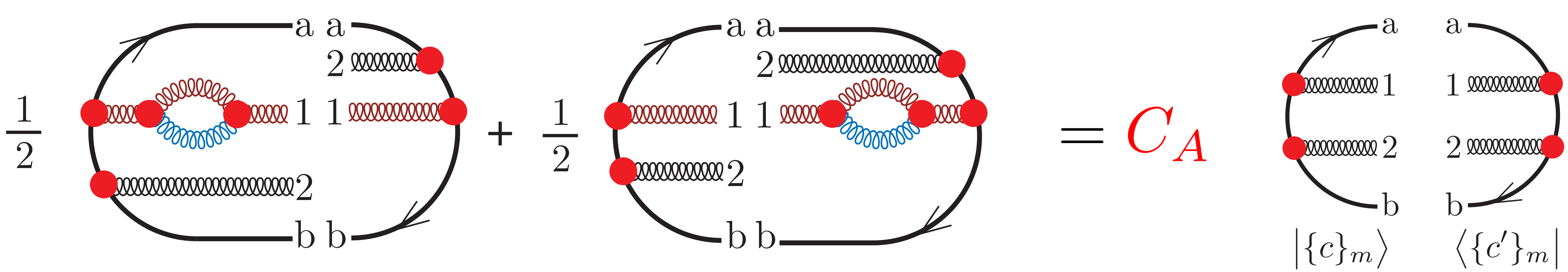
Note this is just the standard K-factor trick.

Why is the color so complicated?

Lets us have a simple color state:



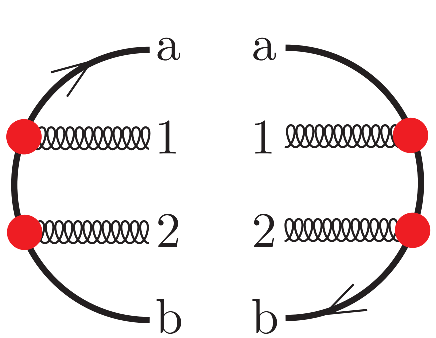
The corresponding virtual graphs (these are exponentiated in the Sudakov operator) are



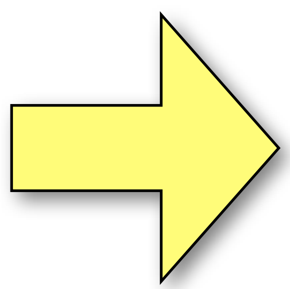
The virtual operator is diagonal for the collinear radiations.

Why is the color so complicated?

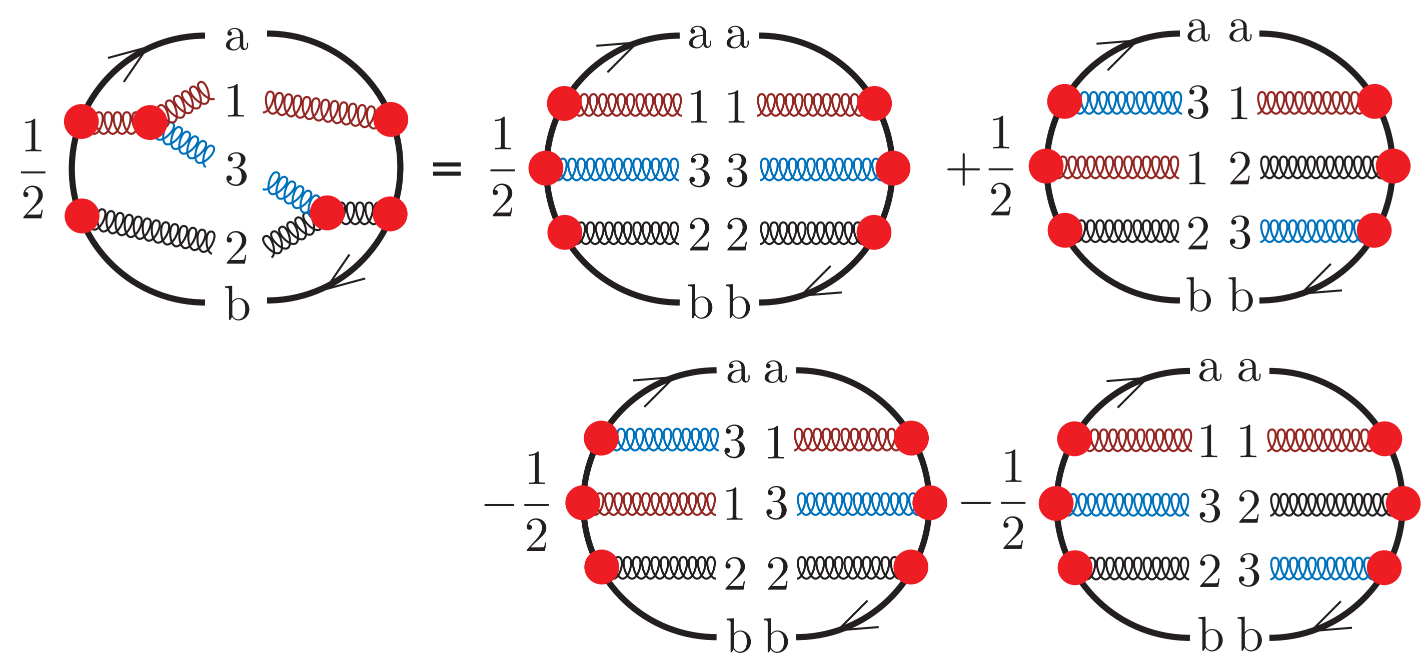
Now, let us consider a soft gluon radiation



$|\{c\}_m\rangle \quad \langle\{c'\}_m|$



Soft splitting



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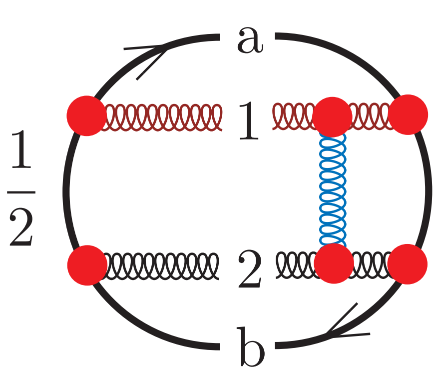
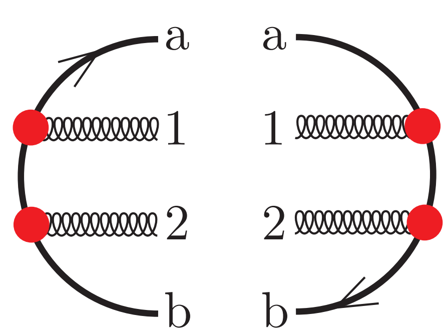


$+c.c. \neq C$

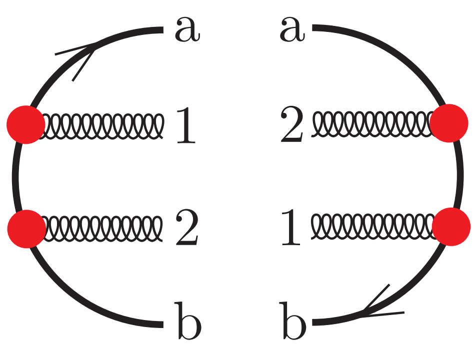


$|\{c\}_m\rangle \quad \langle\{c'\}_m|$

This is not diagonal in the color space. It makes trouble when we try to exponentiate in the Sudakov operator. *At this point it looks really hopeless to go beyond the jolly good leading color approximation.*

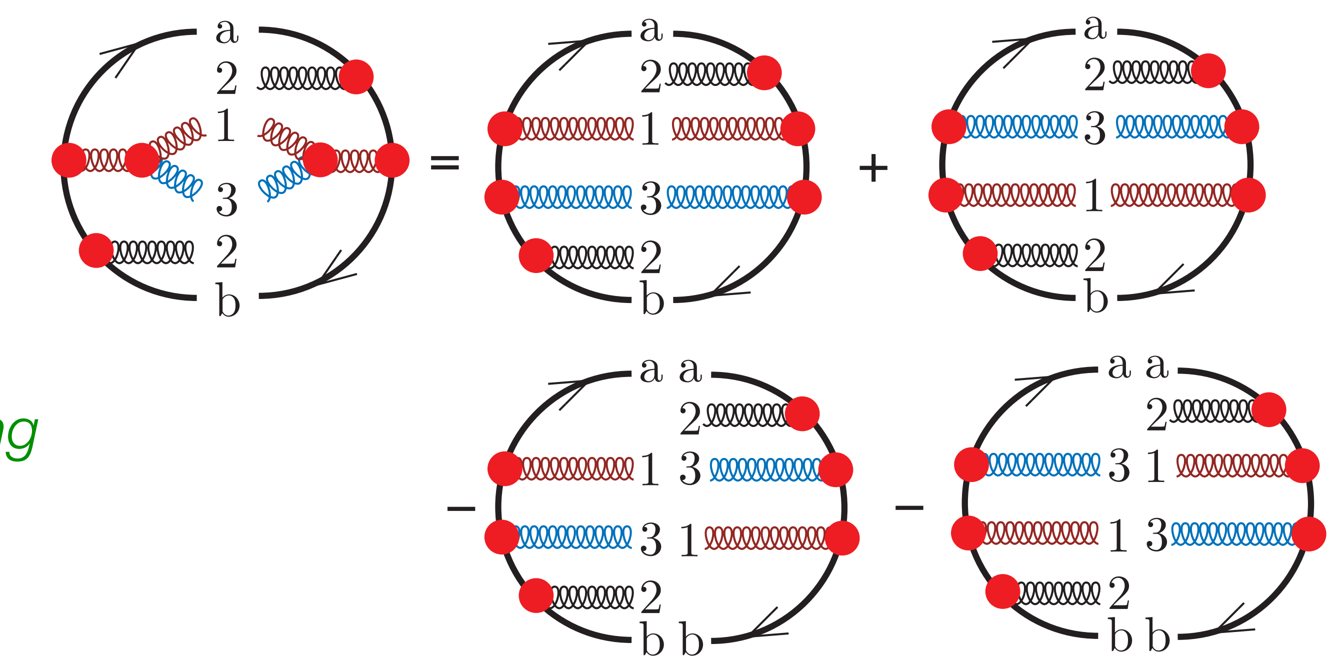
LC+ approximation

ZN, D. Soper, arXiv:1202.4496



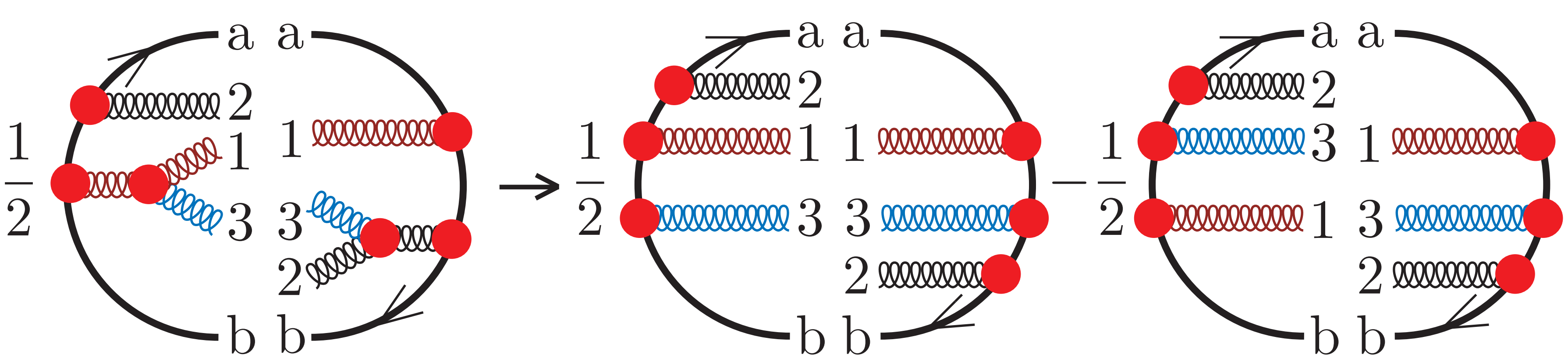
$\left|\{c\}_m\right\rangle \quad \left\langle\{c'\}_m\right|$

Soft-collinear splitting



Pure soft splitting

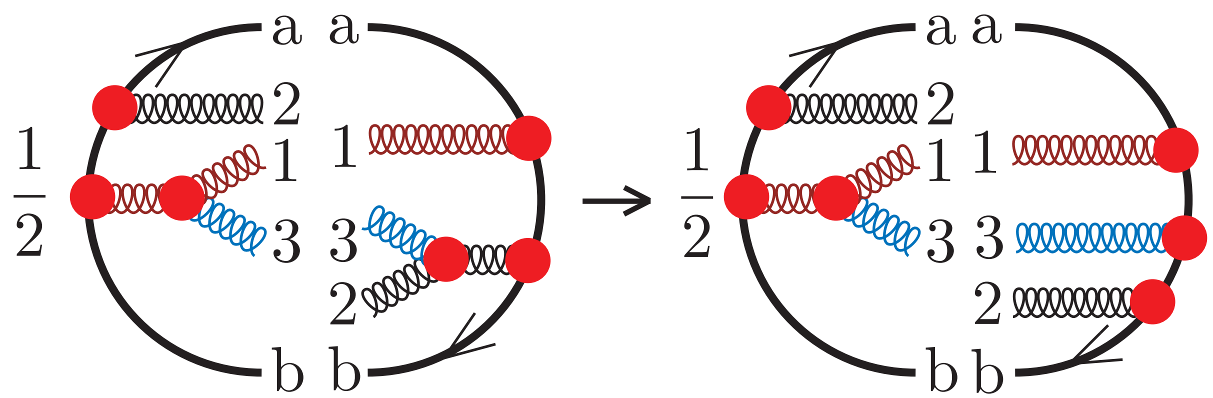
> *LC+ approximation:* *Keep only those terms when the emitted gluon (gluon 3) is color connected with the emitter (gluon 1) in both c and c' states.*



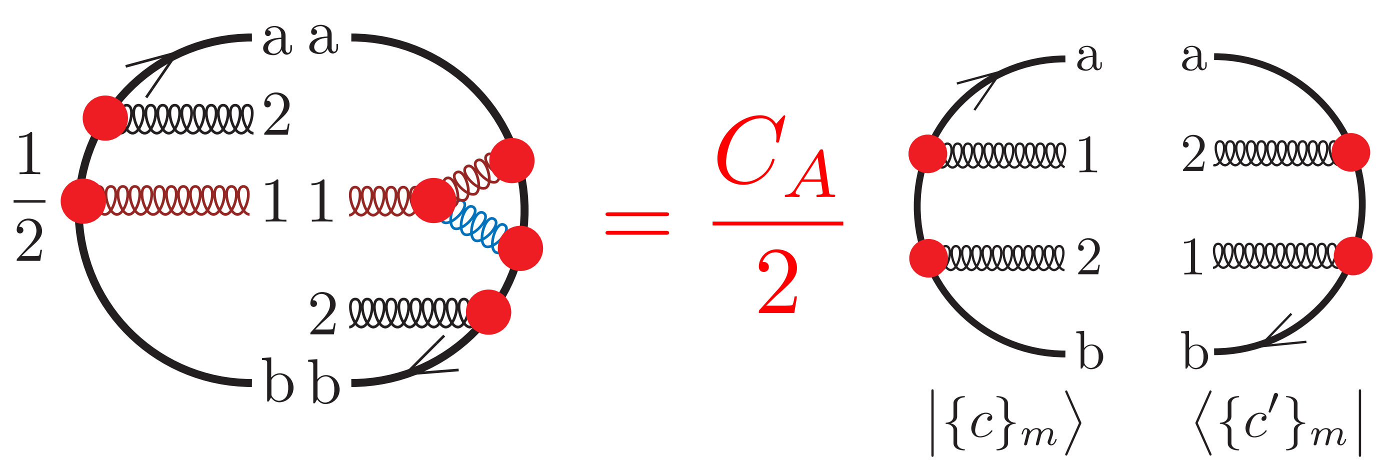
How about the virtual contributions?

LC+ Approximation

The real emission operator is



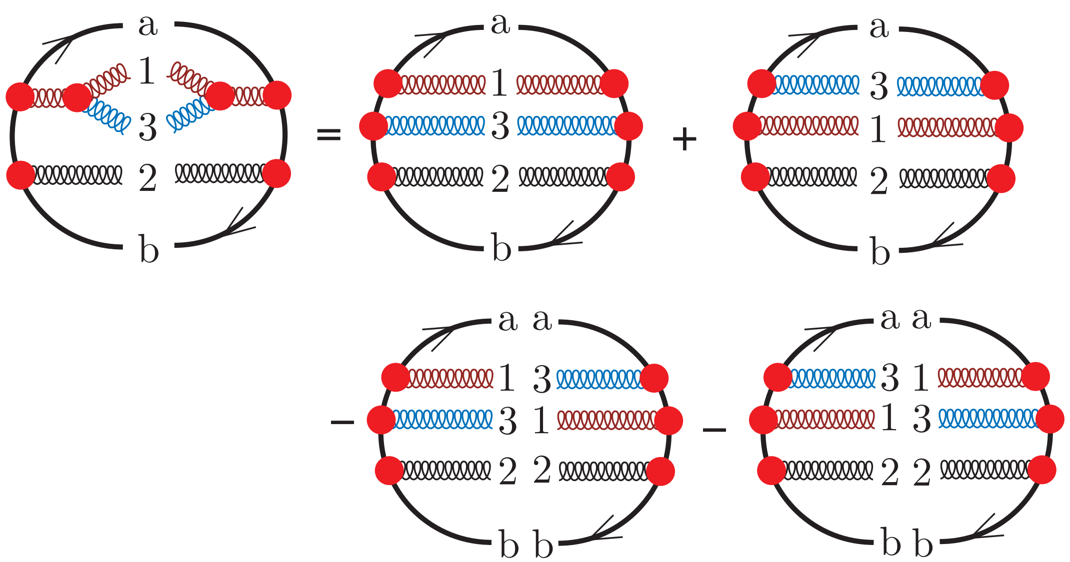
and the corresponding virtual operator is

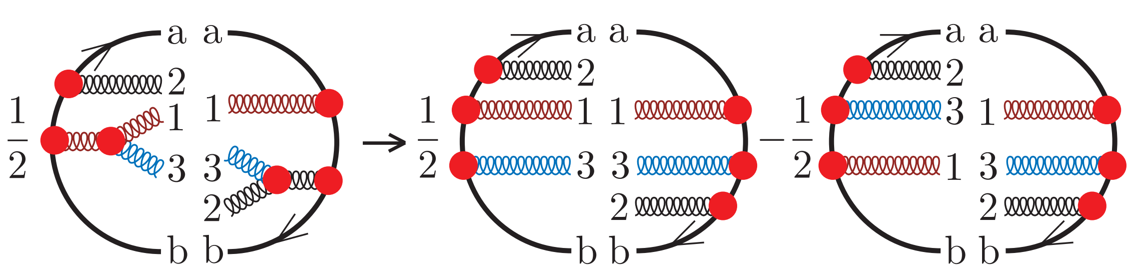


In LC+ approximation the virtual operator is diagonal in color space. Now, it is easy to exponentiate.

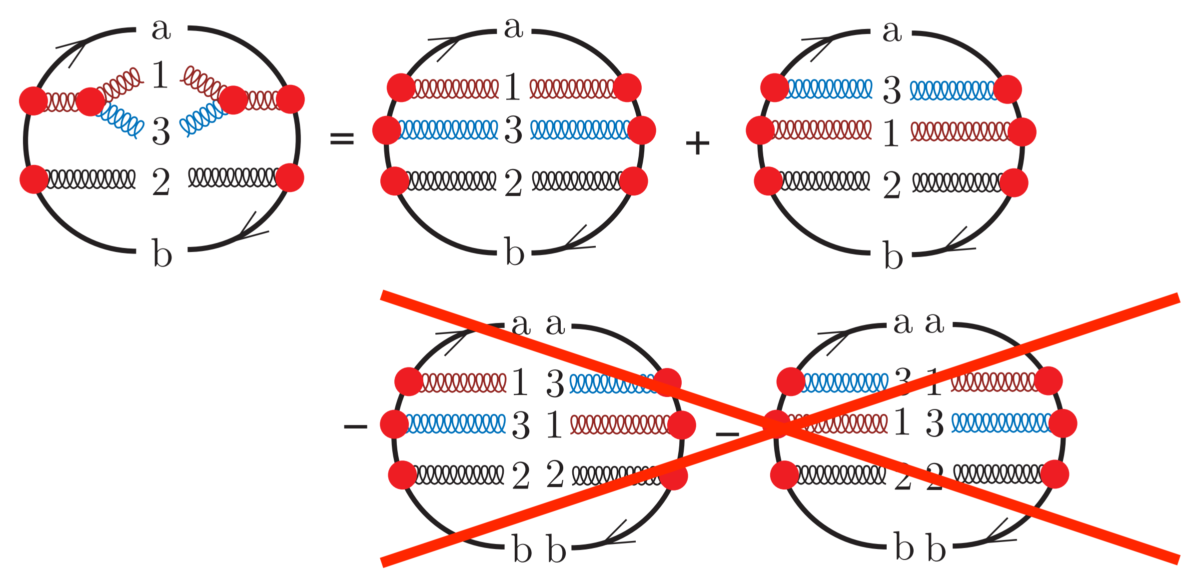
LC+ vs. LC

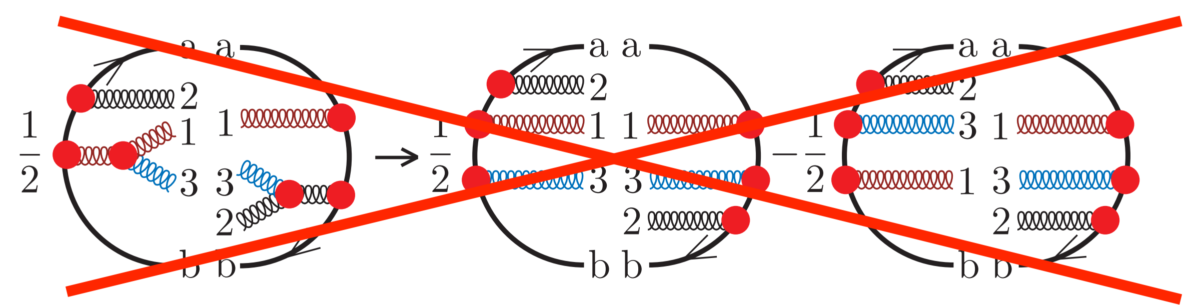
LC+ approximation





LC approximation





LC+ approximation can evolve color interference states. It is very useful in LO (CKKW) and NLO matching.

Full Splitting Operator

Very general splitting operator (no spin correlation)

$$
\begin{aligned}
&\big(\{\hat p,\hat f,\hat c',\hat c\}_{m+1}\big|\mathcal{H}(t)\big|\{p,f,c',c\}_m\big) \\
&= \sum_{l=\mathrm{a,b},1,\dots,m} \delta\Big(t - T_l\big(\{\hat p,\hat f\}_{m+1}\big)\Big)\,\big(\{\hat p,\hat f\}_{m+1}\big|\mathcal{P}_l\big|\{p,f\}_m\big)\,\frac{m+1}{2} \\
&\quad\times \frac{n_\mathrm{c}(a)n_\mathrm{c}(b)\,\eta_\mathrm{a}\eta_\mathrm{b}}{n_\mathrm{c}(\hat a)n_\mathrm{c}(\hat b)\,\hat\eta_\mathrm{a}\hat\eta_\mathrm{b}}\,\frac{f_{\hat a/A}(\hat\eta_\mathrm{a},\mu_F^2)f_{\hat b/B}(\hat\eta_\mathrm{b},\mu_F^2)}{f_{a/A}(\eta_\mathrm{a},\mu_F^2)f_{b/B}(\eta_\mathrm{b},\mu_F^2)}\sum_k \Psi_{lk}(\{\hat f,\hat p\}_{m+1}) \\
&\quad\times \sum_{\beta=L,R}(-1)^{1+\delta_{lk}}\big(\{\hat c',\hat c\}_{m+1}\big|\mathcal{G}_\beta(l,k)\big|\{c',c\}_m\big)
\end{aligned}
$$

Splitting kernel is

$$
\Psi_{lk} = \frac{\alpha_\mathrm{s}}{2\pi}\,\frac{1}{\hat p_l\cdot\hat p_{m+1}}\left[A_{lk}\,\frac{2\hat p_l\cdot\hat p_k}{\hat p_k\cdot\hat p_{m+1}} + H_{ll}^{\mathrm{coll}}(\{\hat f,\hat p\}_{m+1})\right]
$$

Important:

$$
A_{lk} + A_{kl} = 1
$$

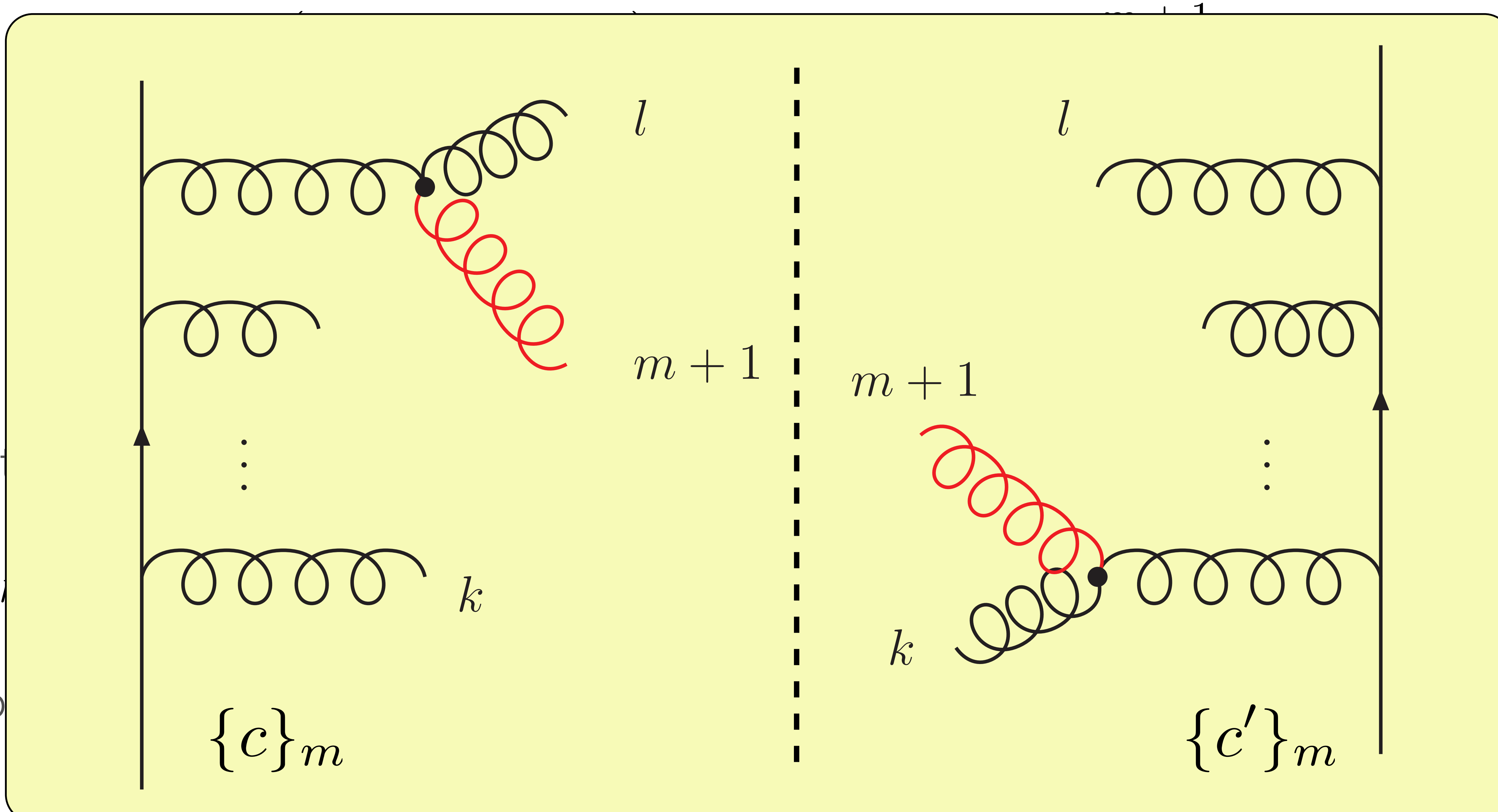
Color operator for gluon emission is

$$
\begin{aligned}
&\big(\{\hat c',\hat c\}_{m+1}\big|\mathcal{G}_R(l,k)\big|\{c',c\}_m\big) \\
&\qquad = {}_D\big\langle\{\hat c\}_{m+1}\big|t_l^\dagger\big|\{c\}_m\big\rangle\,\big\langle\{c'\}_m\big|t_k\big|\{\hat c'\}_{m+1}\big\rangle_D\ .
\end{aligned}
$$

Full Splitting Operator

Very general splitting operator (no spin correlation)

$$\big(\{\hat{p},\hat{f},\hat{c}',\hat{c}\}_{m+1}\big|\mathcal{H}(t)\big|\{p,f,c',c\}_m\big)$$



Split

Ψ_{l} $= 1$

Colo

$$\big(\{\hat{c}',\hat{c}\}_{m+1}\big|\mathcal{G}_R(l,k)\big|\{c',c\}_m\big)$$

$$= {}_D\big\langle\{\hat{c}\}_{m+1}\big|t_l^\dagger\big|\{c\}_m\big\rangle\big\langle\{c'\}_m\big|t_k\big|\{\hat{c}'\}_{m+1}\big\rangle_D \ .$$

LC+ Projection Operator

We approximate the color operator using a projection

$$\big(\{\hat{c}', \hat{c}\}_{m+1}\big|\mathcal{G}(k, l; \{\hat{f}\}_{m+1})\big|\{c', c\}_m\big) = \big(\{\hat{c}', \hat{c}\}_{m+1}\big|t_k^\dagger \otimes t_l\big|\{c', c\}_m\big)$$

$$\big(\{\hat{c}', \hat{c}\}_{m+1}\big|\mathcal{C}(l, m+1)\,\mathcal{G}(k, l; \{\hat{f}\}_{m+1})\big|\{c', c\}_m\big)$$

The projection keep the color connected part

$$\mathcal{C}(l, m+1)\big|\{c', c\}_{m+1}\big) = \begin{cases} \big|\{c', c\}_{m+1}\big) & l \text{ and } m+1 \text{ color connected} \\ & \text{in } \{c'\}_{m+1} \text{ and in } \{c\}_{m+1} \\ 0 & \text{otherwise} \end{cases}$$

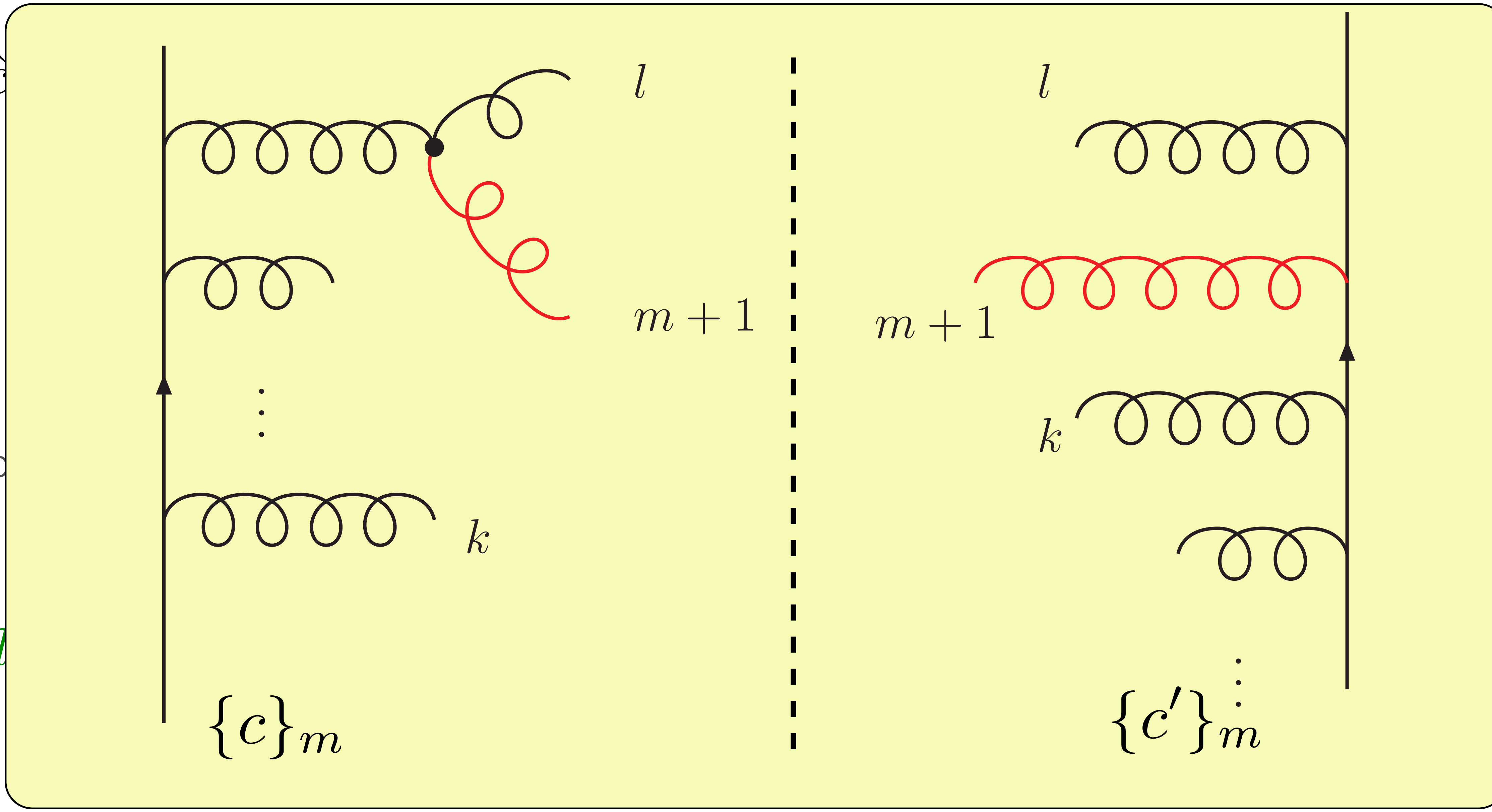
The corresponding quantum level operator is

$$\mathcal{C}(l, m+1) = C(l, m+1)^\dagger \otimes C(l, m+1)$$

LC+ Projection Operator

We approximate the color operator using a projection

$(\{\hat{c}$ … $_m)$



The p … ted

$\mathcal{C}(l$ …

The corresponding quantum level operator is

$$\color{green}{\mathcal{C}(l,m+1)} = \color{red}{C(l,m+1)^\dagger} \otimes \color{blue}{C(l,m+1)}$$

Solution of the Evolution Equation

The idea is split the splitting operator "*good*" and "*bad*" part and expand the evolution operator in the "bad" splitting operator.

$$\mathcal{H}_I(t) = \mathcal{H}_I^{(J)}(t) + \mathcal{H}_I^{(S)}(t)$$

Fully exponentiated *Subtracted*

The inclusive splitting operators are

$$\big(1\big|\mathcal{V}^{(J)}(t) = \big(1\big|\mathcal{H}_I^{(J)}(t) \qquad \text{and} \qquad \big(1\big|\mathcal{V}^{(S)}(t) = \big(1\big|\mathcal{H}_I^{(S)}(t)$$

Now the *good part* of the evolution operator is

$$\mathcal{U}^{(J)}(t,t') = \mathcal{N}^{(J)}(t,t') + \int_{t'}^{t} d\tau\, \mathcal{U}^{(J)}(t,\tau)\mathcal{H}_I^{(J)}(\tau)\mathcal{N}^{(J)}(\tau,t')$$

The full evolution operator is given by

$$\mathcal{U}(t,t') = \mathcal{U}^{(J)}(t,t') + \int_{t'}^{t} d\tau\, \mathcal{U}(t,\tau)\left[\mathcal{H}_I^{(S)}(\tau) - \mathcal{V}^{(S)}(\tau)\right]\mathcal{U}^{(J)}(\tau,t')$$

Solution of the Evolution Equation

The idea is split the splitting operator "*good*" and "*bad*" part and expand the evolution operator in the "bad" splitting operator.

$$\mathcal{H}_I(t) = \underbrace{\mathcal{H}_I^{(J)}(t)}_{\text{Fully exponentiated}} + \underbrace{\mathcal{H}_I^{(S)}(t)}_{\text{Subtracted}}$$

The i

$$\begin{aligned}\mathcal{U}(t,t') = {} & \mathcal{U}^{(J)}(t,t') + \int_{t'}^{t} d\tau\, \mathcal{U}^{(J)}(t,\tau)\left[\mathcal{H}_I^{(S)}(\tau) - \mathcal{V}^{(S)}(\tau)\right]\mathcal{U}^{(J)}(\tau,t') \\ & + \int_{t'}^{t} d\tau_2 \int_{t'}^{\tau_2} d\tau_1\, \mathcal{U}^{(J)}(t,\tau_2)\left[\mathcal{H}_I^{(S)}(\tau_2) - \mathcal{V}^{(S)}(\tau_2)\right] \\ & \qquad \times \mathcal{U}^{(J)}(\tau_2,\tau_1)[\mathcal{H}_I^{(S)}(\tau_2) - \mathcal{V}^{(S)}(\tau_1)]\,\mathcal{U}^{(J)}(\tau_1,t') \\ & + \cdots\end{aligned}$$

Now

The full evolution operator is given by

$$\mathcal{U}(t,t') = \mathcal{U}^{(J)}(t,t') + \int_{t'}^{t} d\tau\, \mathcal{U}(t,\tau)\left[\mathcal{H}_I^{(S)}(\tau) - \mathcal{V}^{(S)}(\tau)\right]\mathcal{U}^{(J)}(\tau,t')$$

Solution of the Evolution Equation

The idea is sp ... the
evolution oper

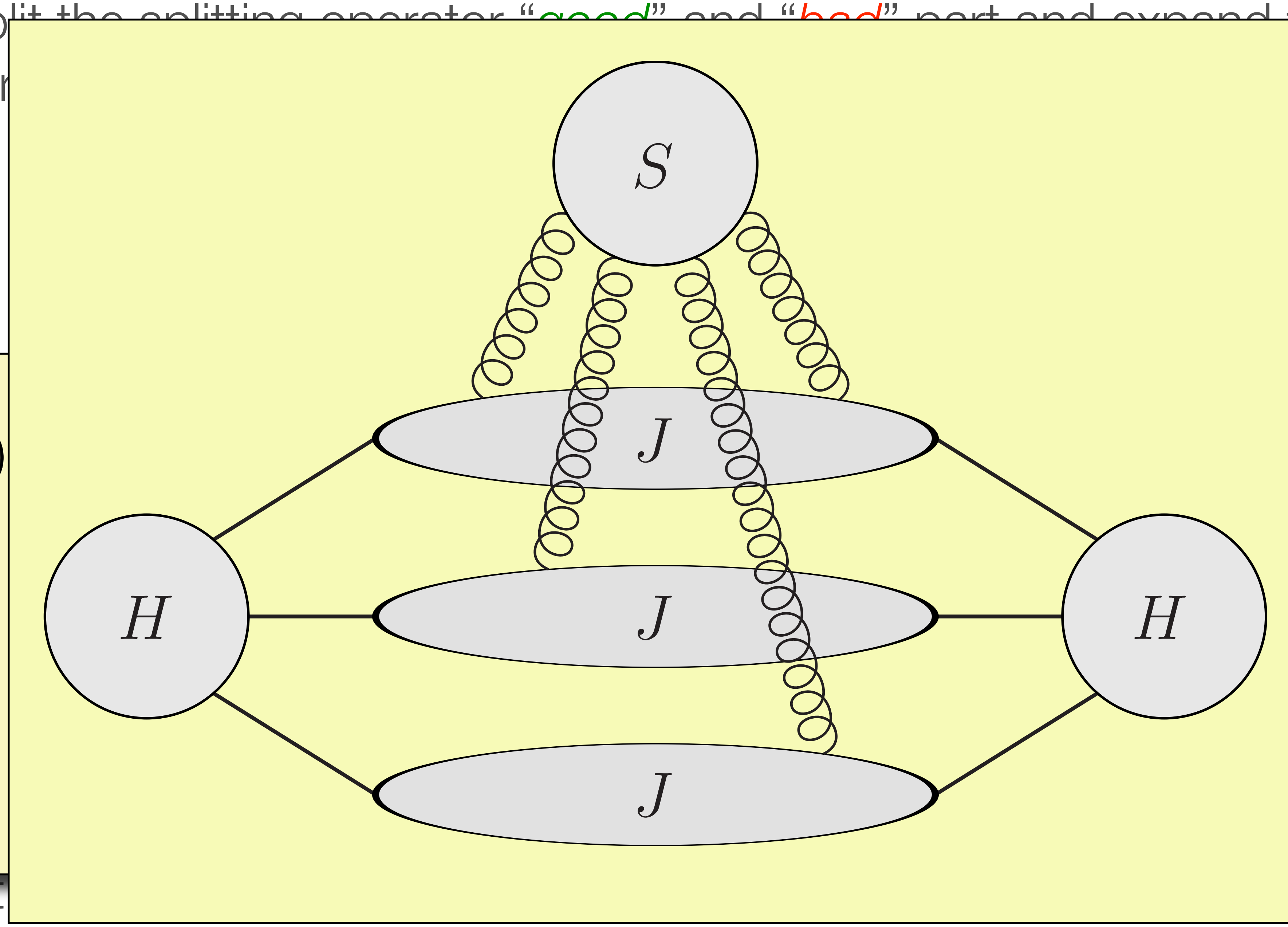
The i

$\mathcal{U}(t,t')$ ... $^{J)}(\tau,t')$

Now

$t')$

The full evolut



$$\mathcal{U}(t,t') = \mathcal{U}^{(J)}(t,t') + \int_{t'}^{t} d\tau\, \mathcal{U}(t,\tau)\left[\mathcal{H}_I^{(S)}(\tau) - \mathcal{V}^{(S)}(\tau)\right]\mathcal{U}^{(J)}(\tau,t')$$

Numerical Result

We implemented this scheme in a general purpose MC program. It is still in very early stage and requires more tests.

$$\sigma_{\rm tot} = \sigma_{\rm tot}\left[\sigma^{(0)} + \frac{1}{N_c^2}\sigma^{(1)} + \frac{1}{N_c^4}\sigma^{(2)} + \frac{1}{N_c^6}\sigma^{(3)} + \cdots\right]$$

For e+e- at 2TeV energy the total cross section is

$$\sigma_{\rm tot} = \sigma_{\rm tot}\left[0.28706 + \frac{1}{N_c^2}2.46618 + \frac{1}{N_c^4}17.229996 + \frac{1}{N_c^6}164.902716 + \cdots\right]$$

Here we generated events up to NNNLC.

For Drell-Yan at the LHC the total cross section is

$$\sigma_{\rm tot} = \sigma_{\rm tot}\left[0.344608 + \frac{1}{N_c^2}1.9949175 + \frac{1}{N_c^4}12.1350555 + \frac{1}{N_c^6}206.9773155 + \cdots\right]$$

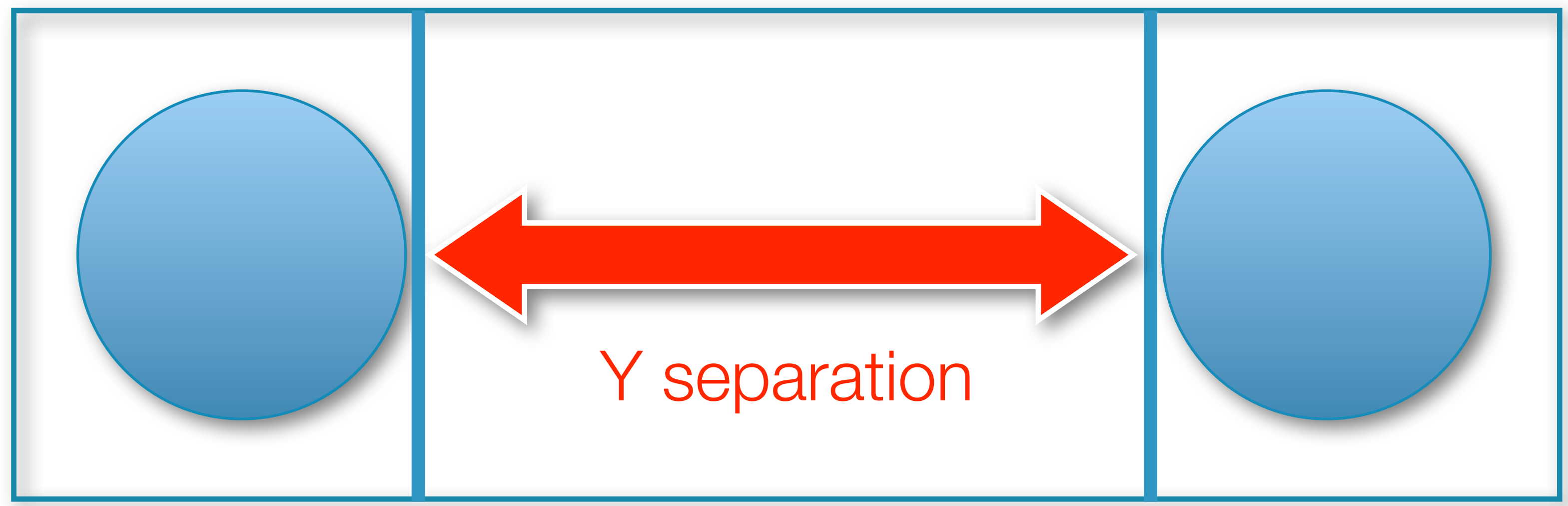
LC+ Approximation

- This operator can evolve color interference contribution.
- LC+ approximation is very useful in LO and NLO matiching.
- Full collinear and soft+collinear contributions are included.
- Wide angle pure soft contributions are not fully included. Omitted part is suppressed by $1/N_c^2$. It is treated perturbative.
- If the observable is double log variable then we can sum up the large logarithms at next-to-leading log accuracy.
- The corresponding inclusive splitting operator can be exponentiated easily.
- Leads to a Markovian process but we have to deal with positive and negative weights.
- Actually the LC+ approximation can do even more...

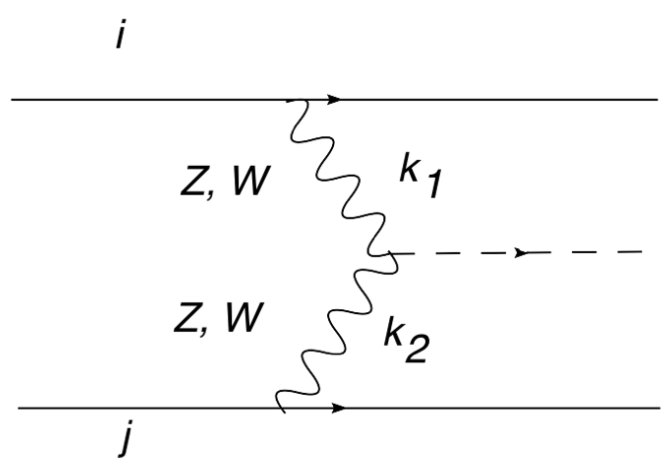
Non-global Observables

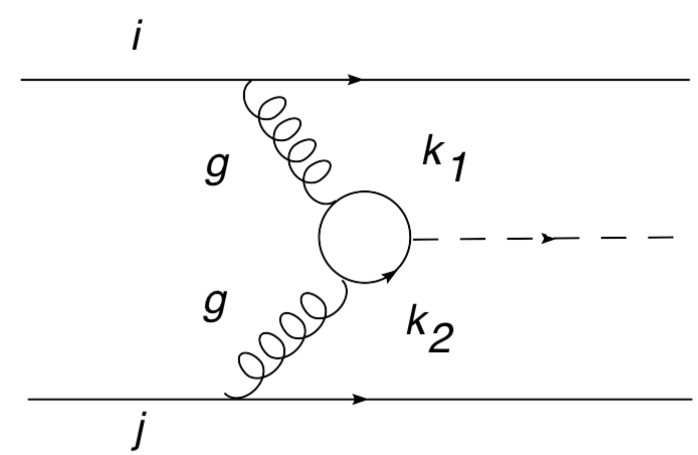
Production of two jets:

- with transverse momentum Q
- with rapidity separation Y
- emissions with $k_T > Q_0$



Motivation:





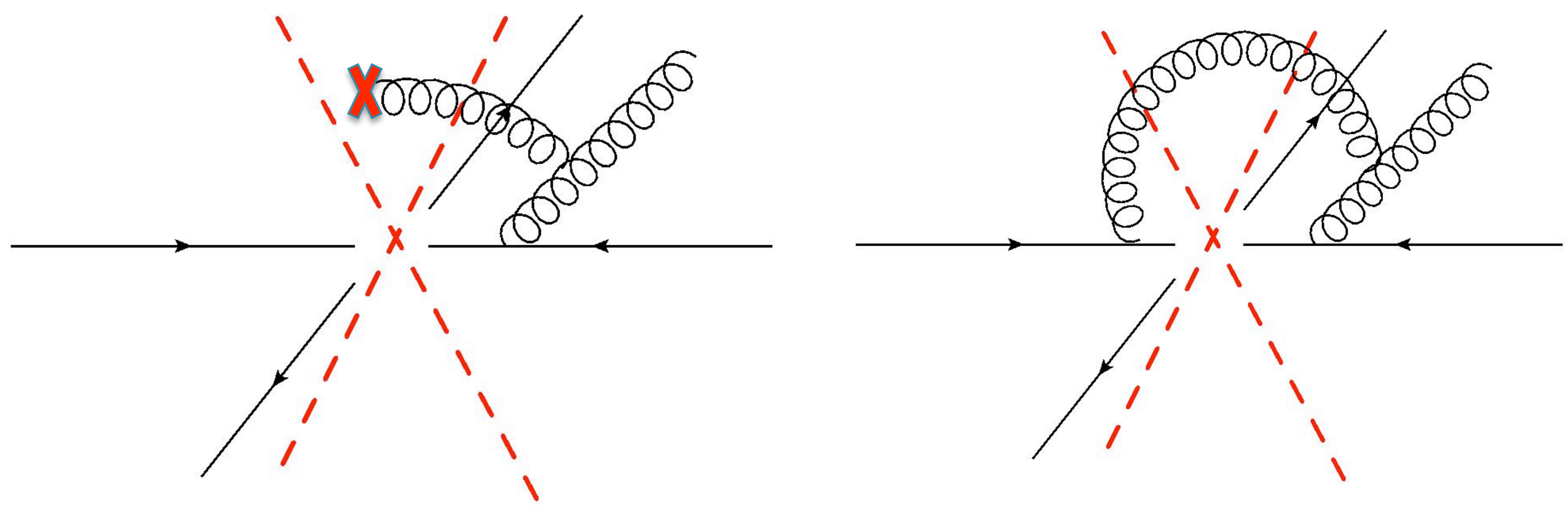
- Possible Higgs discovery channel
- Important to extract the VVH coupling
- Different QCD radiation in the interjet region

What happens if we dress the hard scattering with soft gluons?

Non-global Effects

Virtual contributions are not the whole story because real emissions out of the gap are forbidden to remit back into the gap

Dasgupta and Salam: hep-ph/0104277



This configurations lead to the so-called *Super-Leading Logs (SLL)*

$$\sigma^{(1)} \sim -\alpha_s^4 L^5 \pi^2 + \cdots$$

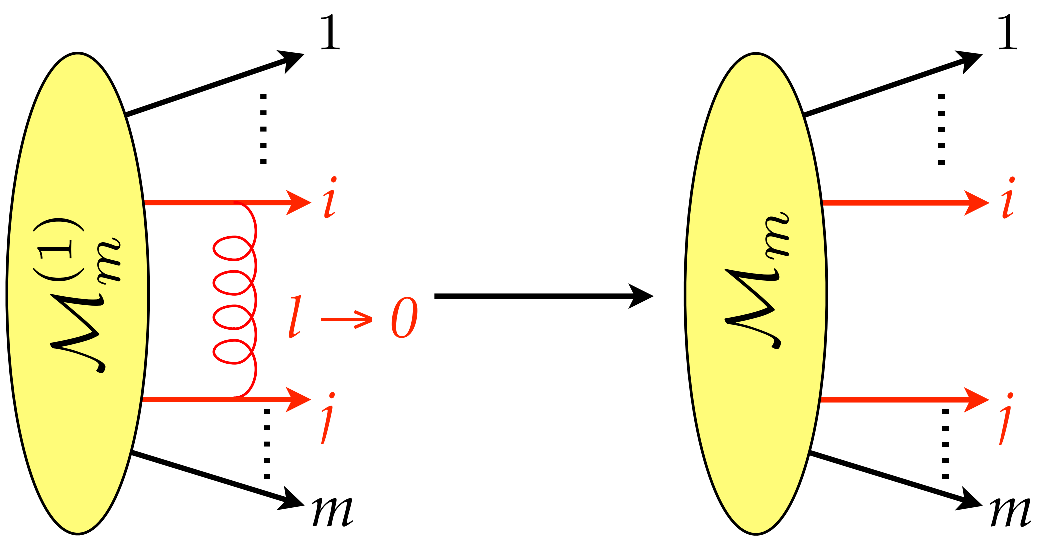
Forshaw, Kyrieleis, Seymour hep-ph/0604094

This logarithms are entirely due to the emission of the Coulomb gluons:

$$\mathbf{\Gamma} = \mathrm{i}\pi \, \boldsymbol{T}_1 \cdot \boldsymbol{T}_2 + \cdots$$

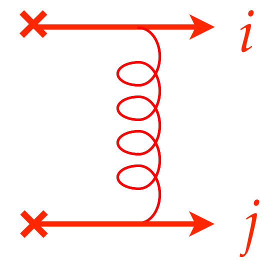
Does any other shower algorithm know about these logarithms?

Virtual Contributions



This is a singular operator in the color space.

$$\otimes \int \frac{d^d l}{(2\pi)^d}$$



$$\mathcal{V}_I(t)$$

In standard parton shower this operator is obtained from the unitarity condition

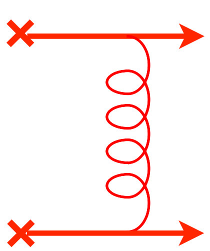
$$\big(1\big|\mathcal{V}_I(t) = \big(1\big|\mathcal{H}_I(t)$$

Always real

NO leading color contribution!!!

But it turns out that we have imaginary contribution from the virtual graphs

$$\int \frac{d^d l}{(2\pi)^d}$$



$$\propto \mathcal{V}_I(t) + \mathrm{i}\pi\widetilde{\mathcal{V}}_I(t) = \mathcal{V}_I(t) \underbrace{- \mathrm{i}\pi \sum_{l\neq k} \frac{\theta_{kl}}{v_{kl}} \frac{\alpha_{\mathrm{s}}(t)}{2\pi} \boldsymbol{T}_k \cdot \boldsymbol{T}_l}_{\text{Coulomb phase}} + c.c.$$

and we still have unitary

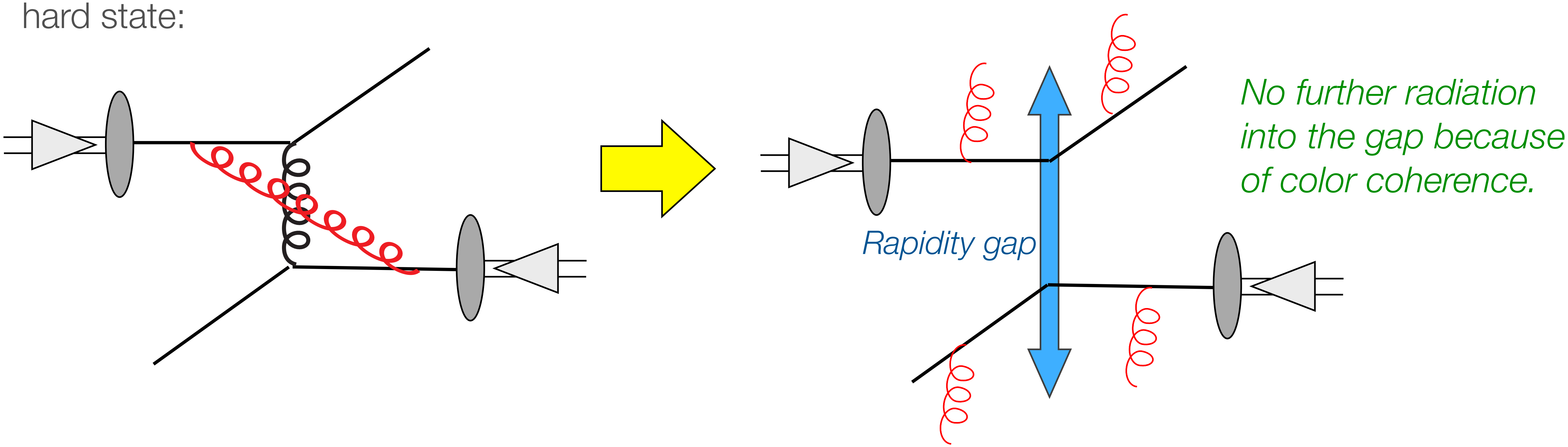
$$\big(1\big|\widetilde{\mathcal{V}}(t) = 0$$

What can Coulomb gluon do?

Coulomb Gluon

1. Coulomb gluon changes the color configuration and the color flow. It is pure virtual contribution, thus it is unresolvable. *It does the same thing what color reconnection does.*

2. It always make color correlation between the two incoming partons. Let's consider a color octet hard state:



This is a contribution to the diffractive events.

3. Leads to *"Super Leading Logs"* in the case of some non-global observables.

4. In LC approximation there is no contribution, but in LC+ approximation some of these logs can be summed up.

Workshop

Event Generators & Resummation

29 May - 1 June 2012

DESY, Hamburg

Topics:

- Validating Monte Carlo event generators against known analytical results
- Developing analytical ideas and using those developments to better understand the accuracy provided by event generators
- Accuracy of the different shower schemes with respect to resummation of large logarithms and the limitations of these schemes
- Using event generators for resumming large logarithms at least at NLL level and going beyond the leading colour approximation
- Categorizing observables according to their analytical properties and their relations to event generators
- Parton distribution functions in event generators
- Continued development of better Monte Carlo event generators

Organizing committee: Mrinal Dasgupta (Univ. of Manchester), Jeff Forshaw (Univ. of Manchester), Bernd Kniehl (Univ. Hamburg), Anna Kulesza (RWTH Aachen), Zoltan Nagy (DESY), Mike Seymour (Univ. of Manchester), Frank Tackmann (DESY)

http://psr12.desy.de