Computation of the Static Quark-Antiquark Potential

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Content

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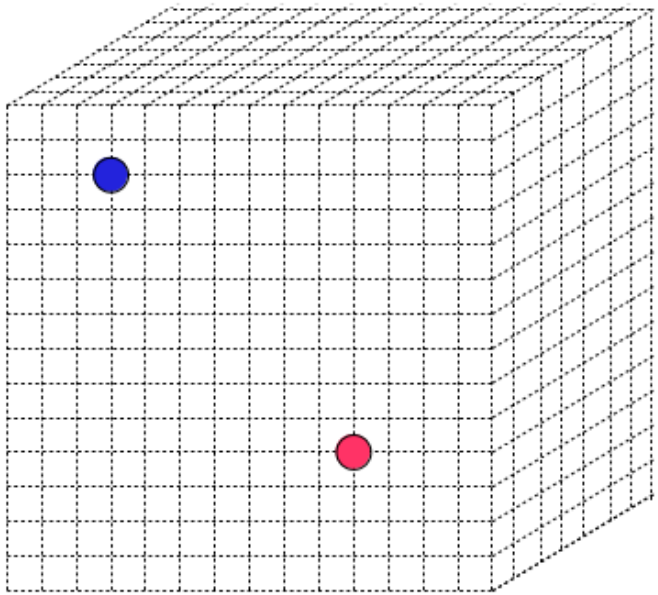
Fields on the Lattice



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- Ansatz: Use the path integral formalism

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$$\begin{aligned}\langle x', t'|x, t\rangle = \int dx_1 \dots dx_{n-1} \left\langle x'|e^{-iH\Delta t}|x_{n-1}\right\rangle \cdot \\ \cdot \left\langle x_{n-1}|e^{-iH\Delta t}|x_{n-2}\right\rangle \cdots \left\langle x_1|e^{-iH\Delta t}|x\right\rangle\end{aligned} \tag{3}$$

- Approximation for the single transition amplitudes:

$$\left\langle x_{k+1}\right| e^{-iH\Delta t} \left|x_k\right\rangle \approx \sqrt{\frac{m}{2\pi i \Delta t}} \exp\left\{ i\Delta t \left[\frac{m}{2} \left(\frac{x_{k+1} - x_k}{\Delta t} \right)^2 - V(x_k) \right] \right\} \tag{4}$$

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- Using this approximation for every transition amplitude gives

$$\left\langle x' \right| \left. x \right\rangle = \int \frac{dx_1 \dots dx_{n-1}}{(2\pi i \Delta t/m)^{n/2}} \exp\left\{ i \sum_{k=0}^{n-1} \Delta t \left[\frac{m}{2} \left(\frac{x_{k+1}-x_k}{\Delta t} \right)^2 - V(x_k) \right] \right\} \tag{5}$$

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- In the limit $n \to \infty$ (the path is getting finer and finer) the argument is

$$\sum_{k=0}^{n-1} \Delta t \left[\frac{m}{2} \left(\frac{x_{k+1}-x_k}{\Delta t} \right)^2 - V(x_k) \right]$$

$$\to \int_0^T dt \left[\frac{m}{2} \left(\frac{dx}{dt} \right)^2 - V(x) \right] = \int_0^T dt\ L(x,\dot{x}) = S \tag{6}$$

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$\rightarrow$ strong fluctuating paths are suppressed

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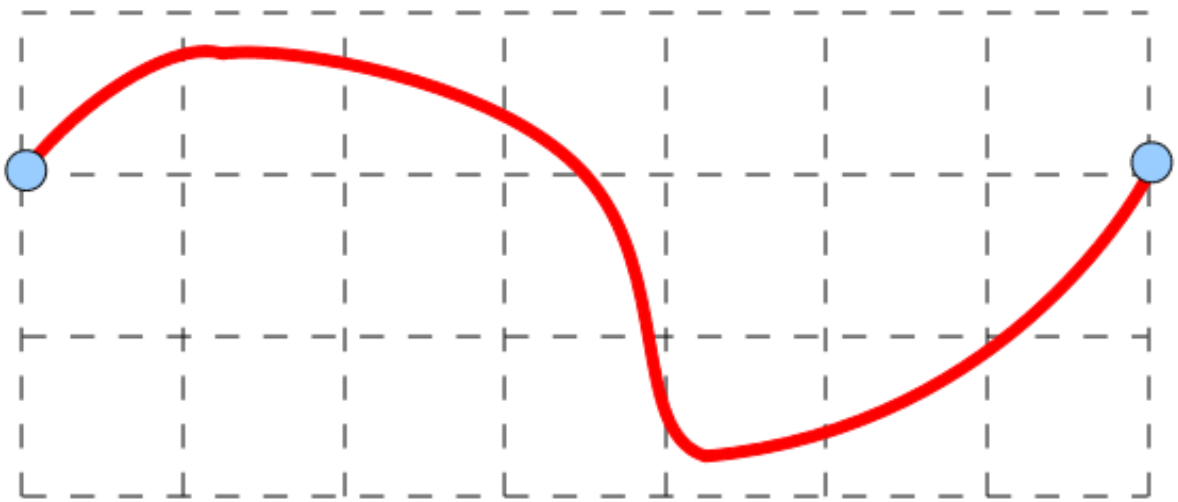
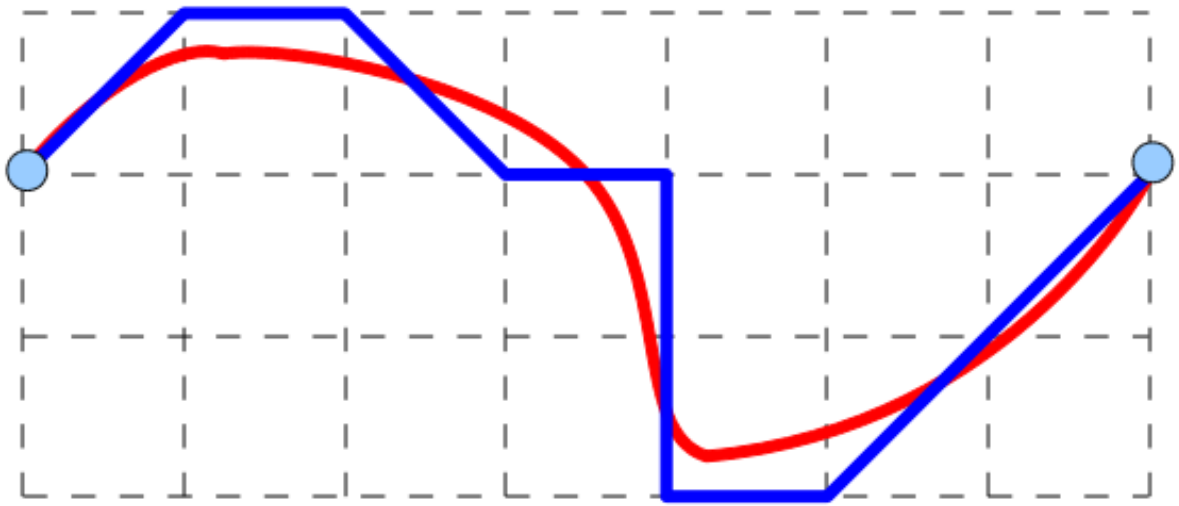
- *Infinite* dimensional integrals $\rightarrow$ *finite* dimensional integrals
  $\rightarrow$ Set of allowed curves is finite on the lattice





- Discretization can be introduced by the translation rules

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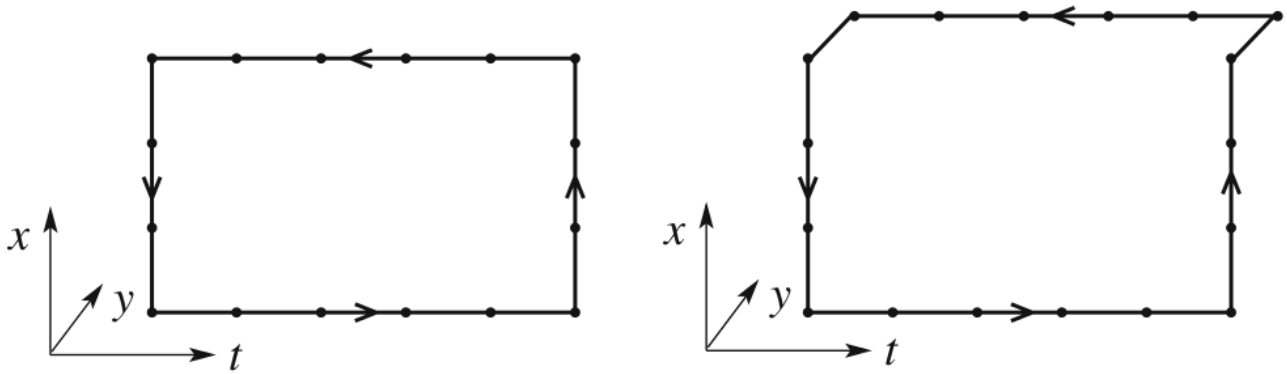
$\to$ In principle, numerical computation is possible now

Basic Remarks about Lattice QCD



- In QCD the free fermionic action (without flavors) is

$$S_F^0[\psi, \overline{\psi}] = \int d^4x \; \overline{\psi}(x)(\gamma_\mu \partial_\mu + m)\psi(x) \qquad (11)$$

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- Like in QCD, this is *not* gauge invariant

- Physics should be invariant under the following transformation:

$$\psi(n) \to \psi'(n) = \Omega(n)\psi(n), \tag{13}$$

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  $\to$ choose gauge fields that obey

$$U_\mu(n) \to U'_\mu(n) = \Omega(n)U_\mu(n)\Omega(n+\hat{\mu})^\dagger \tag{16}$$

- In the end

$$S_F[\psi,\overline{\psi},U] = a^4 \sum_{n\in\mathrm{lat}} \overline{\psi}(n) \left\{ \sum_{\mu=1}^{4} \gamma_\mu \frac{U_\mu(n)\psi(n+\hat{\mu}) - U_{-\mu}(n)\psi(n-\hat{\mu})}{2a} + m\psi(n) \right\} \tag{17}$$

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- Note: the gauge fields are elements of $SU(3)$ (in contrast to lie algebra elements)!

Wilson's Gauge Action

- pure gauge theory: kinematics and dynamics of the gluons

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- This is one possibility, there exist other - more complex - choices

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- Wilson loop:

$$W_{\mathcal{L}}[U] = tr[S(m,n,n_t)\,T(n,n_t)^{\dagger}\,S(m,n,0)^{\dagger}\,T(m,n_t)] \quad (19)$$

with

$$\text{Wilson line:} S(m,n,n_t) = \prod_{k,j \in \mathcal{C}_{m,n}} U_j(k,n_t)$$

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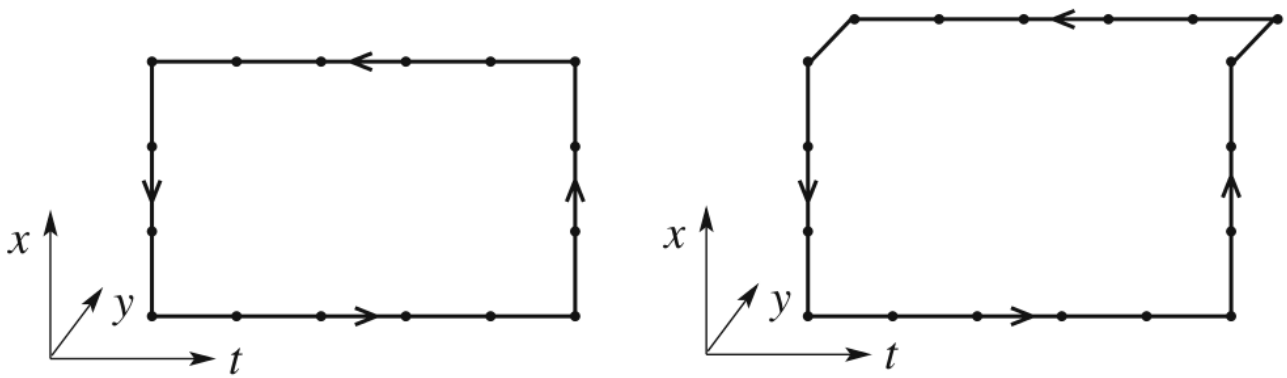

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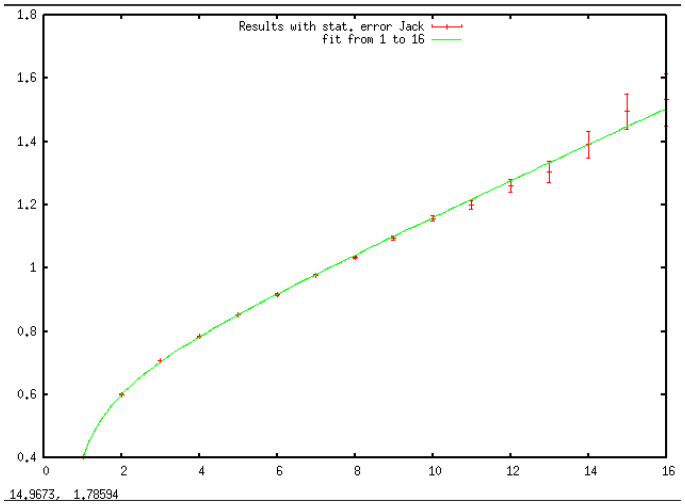
  where $r = a \cdot |m - n|$ and $t = n_t \cdot a$

  $\rightarrow$ computation of the expectation values allows the fitting of the potential $V(r)$
- Higher orders (excited states) are exponentially suppressed in time

Computation with Wilson Loops



Starting point for computation: data file with $\langle W_{\mathcal{L}} \rangle = \langle W(R,T) \rangle$

Öffnen | Speichern | Rückgängig

Measurements-big1.seq

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1
(MM) : 200 : Smeared Wilson loop                    : 1,1 : 6.79158690577333357119016454817028716206550598144531250e-01
(MM) : 200 : Smeared Wilson loop                    : 2,1 : 4.52317431338058684175962298468220978975296020507812500e-01
(MM) : 200 : Smeared Wilson loop                    : 3,1 : 3.01331717115361108483284624526277184486389160156250000e-01
(MM) : 200 : Smeared Wilson loop                    : 4,1 : 2.00796858850142534436145069776102900505065917968750000e-01
(MM) : 200 : Smeared Wilson loop                    : 5,1 : 1.34007792223782756790839698624040465801954269409179688e-01
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(MM) : 200 : Smeared Wilson loop                    : 8,1 : 4.00475963402988702211082738813274772837758064270019531e-02
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(MM) : 200 : Smeared Wilson loop                    : 16,1 : 1.93272145723536077632775942447551642544567584991455078e-03
(MM) : 200 : Smeared Wilson loop                    : 1,2 : 5.47282621137037295078187071339925751090049743652343750e-01
(MM) : 200 : Smeared Wilson loop                    : 2,2 : 2.95997230607195627172245622205082327127456665039062500e-01
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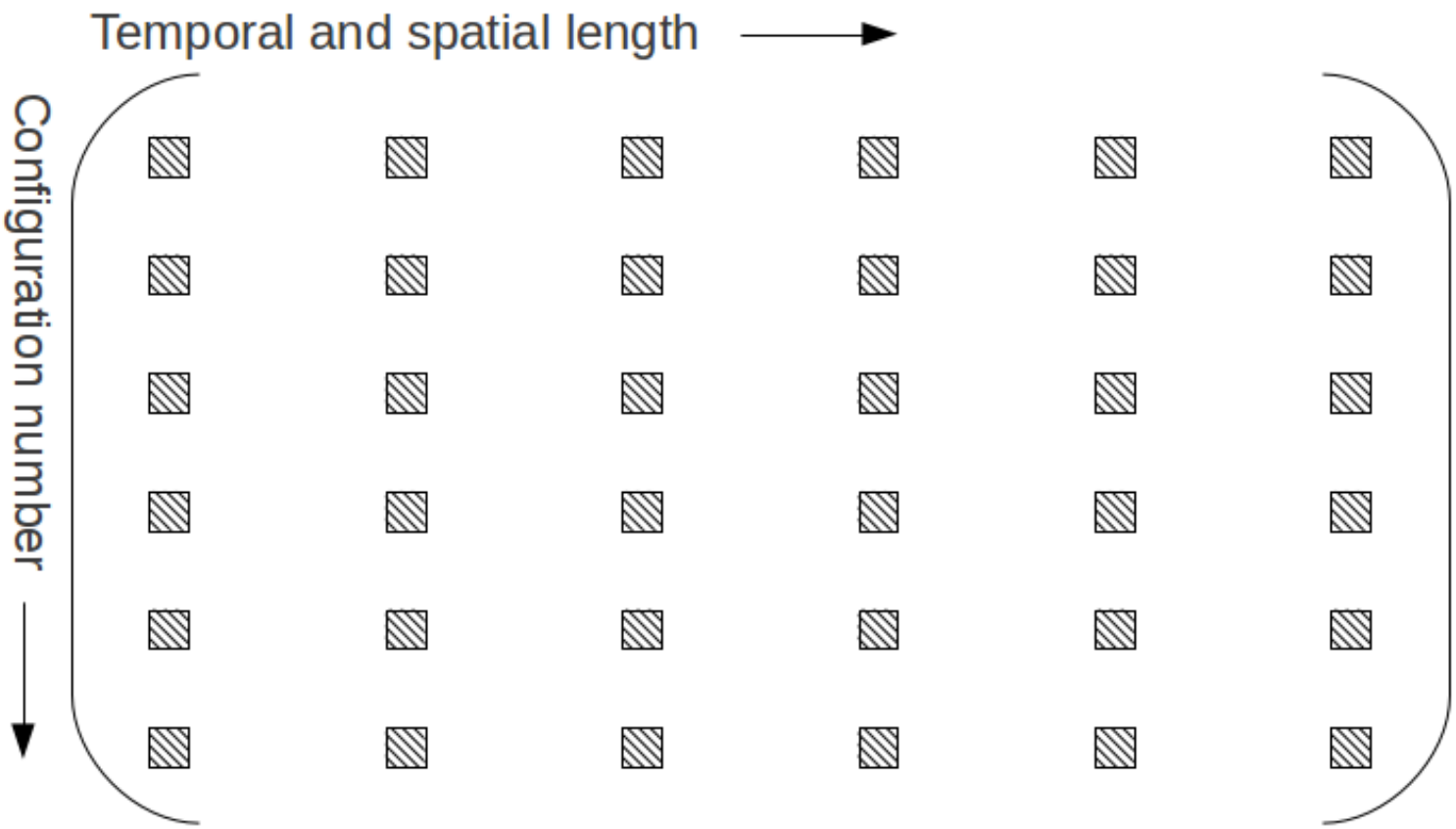
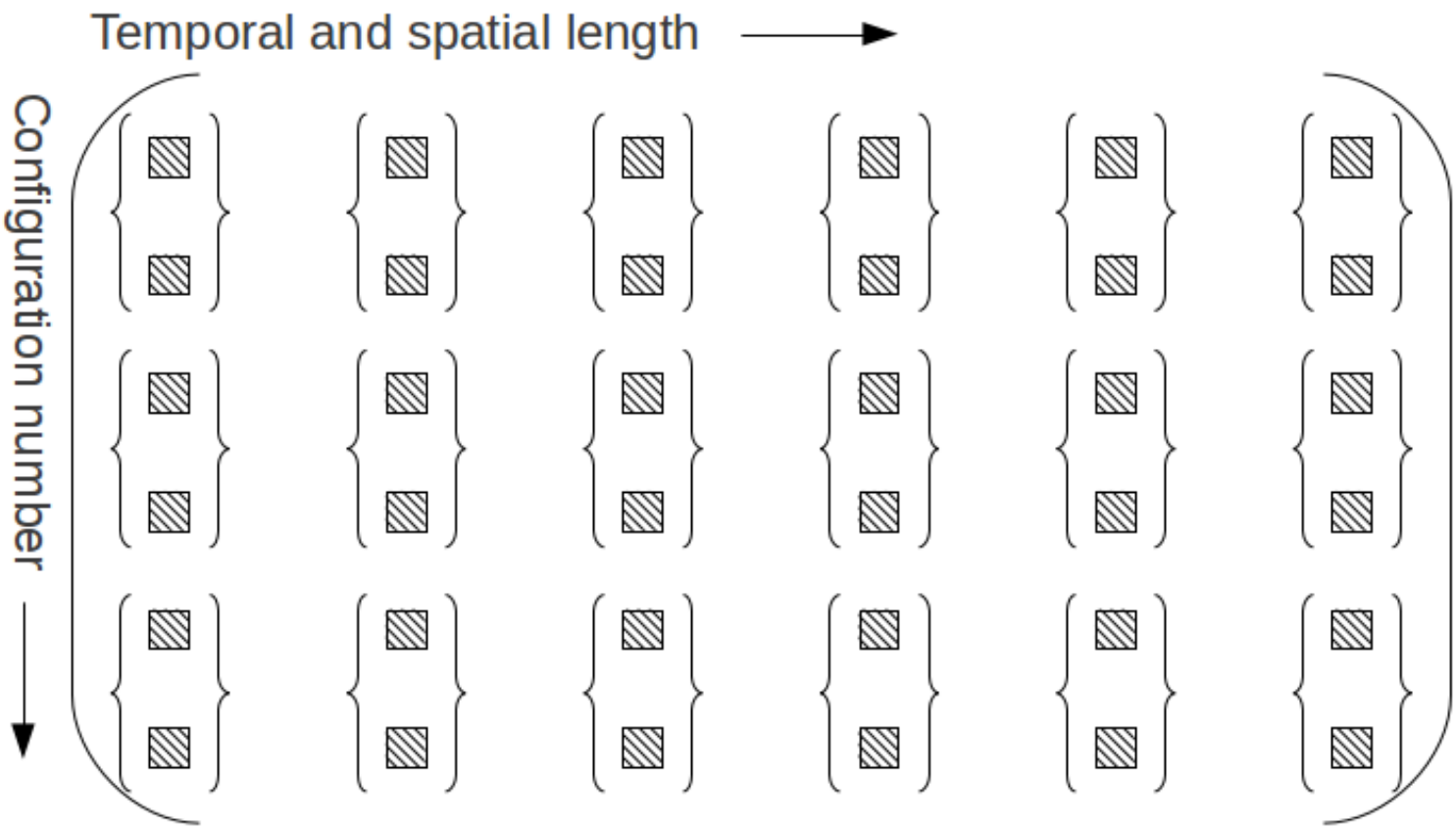
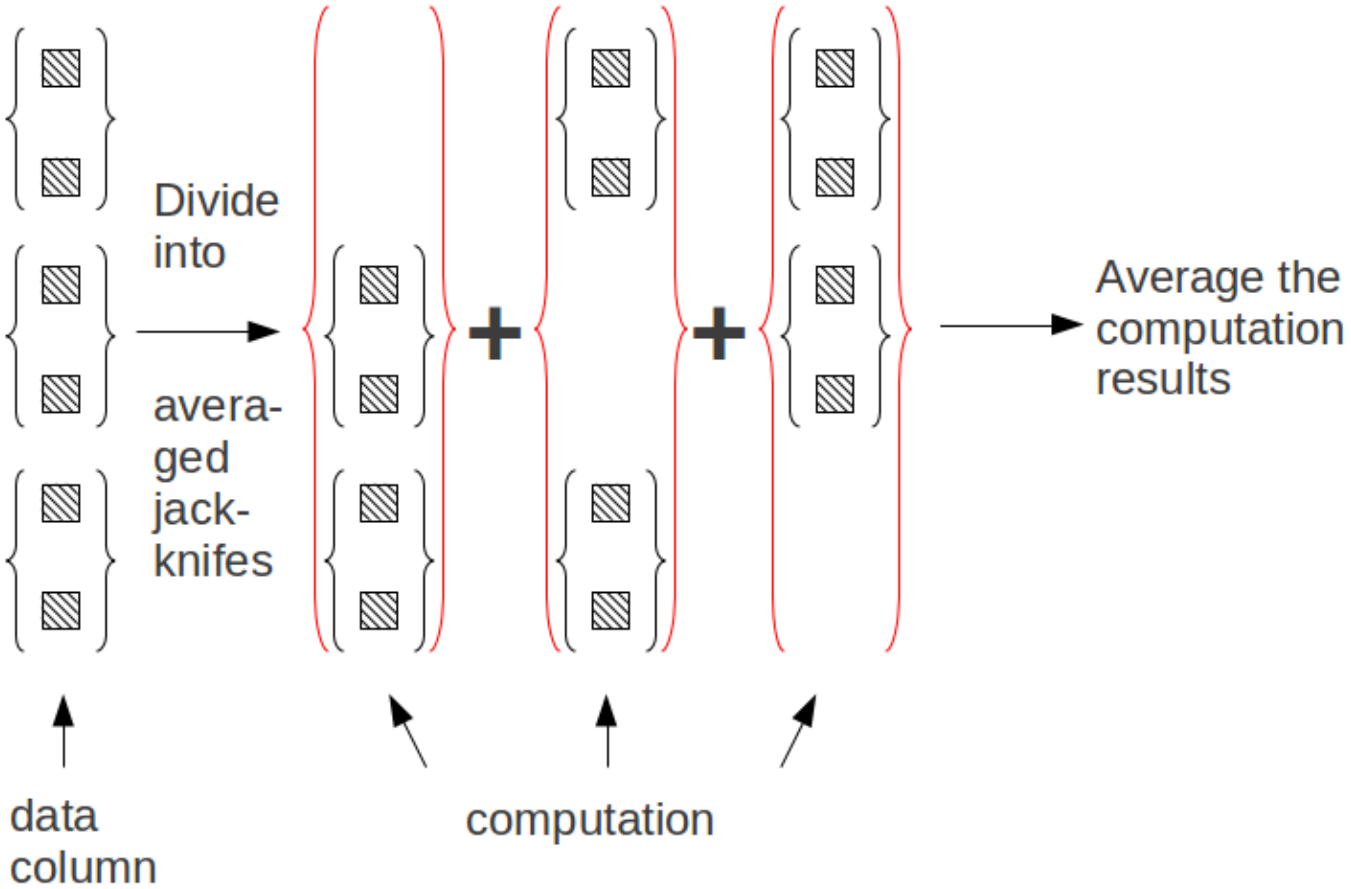
Reading:



Binning: Create bins by averaging over N_{bin} configurations for specific R and T (correlation)



Jackknifing: Create *jackknifes* by averaging over all bins for (R, T) where one bin is left out for every jackknife; compute results on every jackknife (stat. method to get more realistic errors)



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  - Fit $V(R) = A + \frac{B}{R} + \sigma R$ to obtain potential parameters (Coloumb-type and linear rising potential expected, improvements possible)

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- As stated above: the final results are obtained by averaging over all jackknifes

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- For small lattices this is problematic because the exponential decay fit is performed for only a few datapoints.

References

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