The QCD Field Strength Correlator on the Lattice

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Outline

Quantum ChromoDynamics

Lagrangian

$$\mathscr{L}(\overline{\psi}, \psi, A_\mu) = \sum_{f,c} \overline{\psi}_{f,c}(\gamma_\mu D_\mu + m_f)\psi_{f,c} + \frac{1}{4} F^A_{\mu\nu} F^A_{\mu\nu}$$

- covariant derivative: $D_\mu = \partial_\mu + \mathrm{i}gA_\mu$
- field strength tensor: $[D_\mu, D_\nu] = \mathrm{i}gF^A_{\mu\nu} T^A$

$$\Rightarrow \quad F^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - gf^{ABC} A^B_\mu A^C_\nu$$

Adjoint Representation

fundamental representation ⇒ adjoint representation
physical quark-triplet ⇒ non-physical quark-octet

- Lie Algebra: $[T^A, T^B] = \mathrm{i} f^{ABC} T^C, \quad A, B, C = 1, \ldots, 8$
- Generators: $T^a_{\mathrm{adj}} = (T^a)_{bc} = -\mathrm{i} f^{abc}$
- $f^{acd} f^{bcd} = N\delta^{ab}$,
  $\delta^{aa} = N^2 - 1, \qquad a, b, c, d = 1, \ldots, N^2 - 1$

Memento Gluelumps

A system containing a valence gluon connected by an adjoint string to an adjoint source is called *gluelump*.

In Remembrance of Gluelumps *http://www.hanisauland.de/zoom/hihu1*

Perturbation Theory I

Expectation Value (general)

$$\langle \mathcal{O} \rangle_{\mathscr{L}} \equiv \frac{1}{Z} \int \mathcal{D}[\overline{\psi}, \psi, A_\mu] \, \mathcal{O}(\overline{\psi}, \psi, A_\mu) \, \mathrm{e}^{-\int \mathrm{d}^4 x \, \mathscr{L}(x)}$$

- Normalisation $Z = Z[J = 0]$
- Rewrite $\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_\mathrm{I}$

Expectation Value (Interaction)

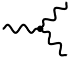
$$\langle \mathcal{O} \rangle_{\mathrm{int}} \equiv \left\langle \mathcal{O} \, \mathrm{e}^{-\int \mathrm{d}^4 x \, \mathscr{L}_\mathrm{I}(x)} \right\rangle$$

Perturbation Theory II

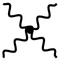
What about $\mathscr{L}$?

$$\mathscr{L}_0 = \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{1}{2\xi}(\partial_\mu A_\mu^a)^2 - \overline{c}^a \partial^2 c^a$$

$$\mathscr{L}_\mathrm{I} = -\frac{g}{2} f^{abc}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) A_\mu^b A_\nu^c$$

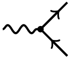


$$+\frac{g^2}{4} f^{abc} f^{cde} A_\mu^a A_\nu^b A_\mu^d A_\nu^e$$



$$-g f^{abc} \overline{c}^a (\partial_\mu A_\mu^c) c^b$$

$$+\mathrm{i} g \overline{\psi} \gamma_\mu A_\mu^a T^a \psi$$



1. expand exponential function
2. rearrange in powers of g

Feynman Rules - Propagators

- Feynman rules vertices ✓
- Feynman rules propagators?
  $\rightarrow$ 2-point-functions $G_2(x_1, x_2) = \langle 0 | T\{\phi(x_1)\phi(x_2)\}|0\rangle$

Example:

Gluon propagator

$$D^{ab}_{\mu\nu}(x-y) = \delta^{ab} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \left[\delta_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right] \frac{\mathrm{e}^{ik(x-y)}}{k^2}$$

$$\stackrel{\xi=1}{=} \delta^{ab}\, \delta_{\mu\nu} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\mathrm{e}^{ik(x-y)}}{k^2}$$

- Feynman-'t Hooft gauge $\xi = 1$
- Need result in position space in n dimensions (dimensional regularisation) $\rightarrow$ compute integrals

Gluon Propagator in Position Space

Schwinger-Parametrisation

$$\frac{1}{(m^2-k^2)^\alpha} = \frac{1}{\Gamma(\alpha)} \int_0^\infty \mathrm{d}\beta\ \beta^{\alpha-1} \mathrm{e}^{-\beta(m^2-k^2)}$$

- Result gluon propagator:

$$\begin{aligned} D(z) &= \int \frac{\mathrm{d}^n k}{(2\pi)^n} \frac{\mathrm{e}^{\mathrm{i}kz}}{k^2} \\ &= \int_0^\infty \mathrm{d}\beta \int \frac{\mathrm{d}^n k}{(2\pi)^n}\ \mathrm{e}^{-\beta k^2 + \mathrm{i}kz} \\ &= \frac{\Gamma(\frac{n}{2}-1)}{4\pi^{\frac{n}{2}} z^{n-2}} \end{aligned}$$

- Useful relation:

$$\int \mathrm{d}^n q\ \mathrm{e}^{-q^2} = \pi^{\frac{n}{2}}$$

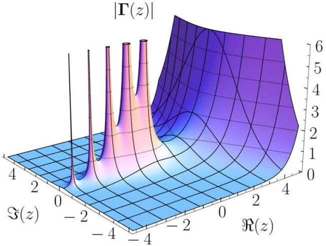
Consider $n = 4$

- Gluon propagator:

$$D(z) = \frac{\Gamma(\frac{n}{2}-1)}{4\pi^{\frac{n}{2}} z^{n-2}} \overset{n=4}{=} \frac{1}{4\pi^2 z^2} \qquad \checkmark$$

- One loop standard integral:

$$I = \int \frac{\mathrm{d}^n k}{(2\pi)^n} \frac{1}{(k^2+m^2)^2}$$
$$= \frac{\Gamma(2-\frac{n}{2})}{(4\pi)^{\frac{n}{2}}\Gamma(2)} \frac{1}{m^{4-n}}$$
$$\overset{n=4}{=} \text{UV-divergent} \quad ↯$$



$\Rightarrow$ Renormalisation

Renormalisation

1. Regularisation
   - Dimensional Regularisation
   - Lattice Regularisation
   - Pauli-Villars-Regularisation
2. Renormalisation
   - Minimal Subtraction Scheme (MS-Scheme) (mass independent)
     $\longrightarrow$ modified MS-Scheme ($\overline{\mathrm{MS}}$-Scheme)
   - Momentum Subtraction Scheme (mass dependent)
     $\longrightarrow$ intermediate renormalisation
     $\longrightarrow$ on-shell renormalisation

Dimensional Regularisation I *'t Hooft, Veltman*

Example: One loop standard integral

$$I = \int \frac{\mathrm{d}^n k}{(2\pi)^n} \frac{1}{(k^2+m^2)^2} = \frac{\Gamma(2-\frac{n}{2})}{(4\pi)^{\frac{n}{2}}\Gamma(2)} \frac{1}{m^{4-n}}$$

1. define $n = 4 - 2\varepsilon$
2. expand around poles:

$$\Gamma(-m+\epsilon) = \frac{(-1)^m}{m!}\left\{\frac{1}{\epsilon} + \psi(m+1) + \mathcal{O}(\epsilon)\right\}$$

$$\psi(m+1) = 1 + \frac{1}{2} + \ldots + \frac{1}{m} - \gamma_{\mathrm{E}}, \quad \psi(1) = -\gamma_{\mathrm{E}} = -0,5772\ldots$$

3. use features of gamma function: $\Gamma(x+1) = x\,\Gamma(x)$
4. use approximation: $x^\varepsilon = \mathrm{e}^{\varepsilon \ln x} = 1 + \varepsilon \ln x + \mathcal{O}(\varepsilon^2)$

Dimensional Regularisation II *'t Hooft, Veltman*

Example: One loop standard integral

$$I = \frac{\Gamma(2-\frac{n}{2})}{(4\pi)^{\frac{n}{2}}\Gamma(A)} \frac{1}{(m^2-p^2)^{2-\frac{n}{2}}}$$
$$= \frac{1}{(4\pi)^2}\left(\frac{1}{\varepsilon} - \gamma_{\mathrm{E}} + \ln 4\pi - \ln m^2 + \mathcal{O}(\varepsilon)\right)$$

- finite value for fixed ε
- UV-divergence $\rightarrow$ logarithmic divergence in ε for $\varepsilon \rightarrow 0$

$\Downarrow$

Subtract pole for fixed $\varepsilon \Rightarrow$ Minimal Subtraction

MS-Scheme *'t Hooft*

- subtract counterterm:

$$\mathscr{L} = \mathscr{L}_0 - \Delta\mathscr{L}$$

  - $\mathscr{L}_0$: original Lagrangian
  - $\Delta\mathscr{L}$: counterterm Lagrangian, same structure as $\mathscr{L}_0$, with renormalised quantities

  $\Rightarrow$ finite $\mathscr{L}$

- coupling constant renormalisation:

$$g_0 \longrightarrow \nu^{2-\frac{n}{2}} g$$

  (introduce mass scale ν)

- symmetries conserved (except chiral symmetries)

Renormalisability

Definition

A theory is called renormalisable if the counterterm of this theory is of the same structure as the original Lagrangian. No new counterterms must be introduced at higher orders in perturbation theory.

Reasonable for computations using dimensional regularisation:
$\Rightarrow$ subtract prefactor $\ln 4\pi - \gamma_{\mathrm{E}}$:

modified MS-Scheme

$\overline{\mathrm{MS}}$-Scheme

Redefine mass scale parameter:

$$\nu^2 = \mu^2 \frac{\mathrm{e}^{\gamma_{\mathrm{E}}}}{4\pi}$$

Field Strength Correlator

Definition

$$\mathcal{D}_{\mu\nu\lambda\omega}(z) = \langle 0 | \mathrm{T}\{ F^a_{\mu\nu}(y) \mathrm{P}\, \mathrm{e}^{-gf^{abc} z^\tau \int_0^1 \mathrm{d}\sigma A^c_\tau(x+\sigma z)} F^b_{\lambda\omega}(x) \} | 0 \rangle$$

- time ordering:

$$\mathrm{T}\ \exp\left\{ \int_0^t \mathrm{d}t' \mathcal{O}(t') \right\} = 1 + \int_0^t \mathrm{d}t'\, \mathcal{O}(t') + \int_0^t \mathrm{d}t' \int_0^{t'} \mathrm{d}t''\, \mathcal{O}(t')\, \mathcal{O}(t'') + ..$$

- Schwinger-line:

$$S^{ab}_C(y,x) = \mathrm{P} \exp\left\{ -g\, f^{abc} z^\tau \int_{0,C}^1 \mathrm{d}\sigma\, A^c_\tau(x+\sigma z) \right\}$$

Used for

- QCD sum rules *Shifman, Vainshtein, Zakharov*
- gluon condensates *Campostrini, Di Giacomo, Olejnik*
- stochastic vacuum model *Dosch, Simonov*
- high energy hadron-hadron scattering *Nachtmann, Dosch et al.*

Lorentz-Structure

Field strength correlator parametrisable:

$$\mathcal{D}_{\mu\nu\lambda\omega}(z) = (\delta_{\mu\lambda}\delta_{\nu\omega} - \delta_{\mu\omega}\delta_{\nu\lambda})\left(\mathcal{D}(z^2) + \mathcal{D}_1(z^2)\right) + (\delta_{\mu\lambda}z_\nu z_\omega - \delta_{\mu\omega}z_\nu z_\lambda - \delta_{\nu\lambda}z_\mu z_\omega + \delta_{\nu\omega}z_\mu z_\lambda)\frac{\partial\mathcal{D}_1(z^2)}{\partial z^2}$$

- $\mathcal{D}(z^2)$, $\mathcal{D}_1(z^2)$: scalar functions
- use antisymmetry $\mu \leftrightarrow \nu$ and $\lambda \leftrightarrow \omega$:

$$\mathcal{D}_{\mu\nu\lambda\omega}(z) \equiv \mathcal{D}_{[\mu\nu][\lambda\omega]}(z)$$

- so that: $\mathscr{D}_{\mu\nu\lambda\omega}(z) = \delta_{\mu\lambda}\delta_{\nu\omega}\,\mathcal{A}(z^2) + \delta_{\mu\lambda}\frac{z_\nu z_\omega}{z^2}\,\mathcal{B}(z^2)$

$$\Rightarrow \quad 2\,\mathcal{A}(z^2) = \mathcal{D}(z^2) + \mathcal{D}_1(z^2), \quad \frac{1}{z^2}\mathcal{B}(z^2) = \frac{\partial\mathcal{D}_1(z^2)}{\partial z^2}$$

Perturbation Theory

Expectation Value (general)

$$\langle \mathcal{O} \rangle_{\mathscr{L}} \equiv \frac{1}{Z} \int \mathcal{D}[\overline{\psi}, \psi, A_\mu]\, \mathcal{O}(\overline{\psi}, \psi, A_\mu)\, \mathrm{e}^{-\int \mathrm{d}^4 x\, \mathscr{L}(x)}$$

- Normalisation $Z = Z[J = 0]$
- Rewrite $\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_\mathrm{I}$

Expectation Value (Interaction)

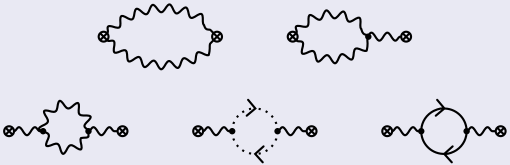
$$\langle \mathcal{O} \rangle_{\mathrm{int}} \equiv \left\langle \mathcal{O}\, \mathrm{e}^{-\int \mathrm{d}^4 x\, \mathscr{L}_\mathrm{I}(x)} \right\rangle$$

Contributions Continuum

$$\mathcal{D}_{\mu\nu\lambda\omega}(z) = \left\langle 0 \middle| \mathrm{T}\left\{ F^a_{\mu\nu}(y) \mathrm{P}\ \mathrm{e}^{-g f^{abc} z^\tau \int_0^1 \mathrm{d}\sigma A^c_\tau(x+\sigma z)} F^b_{\lambda\omega}(x) \right\} \middle| 0 \right\rangle$$

$=$

$+$



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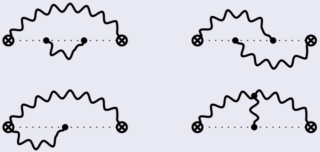

Useful Parametrisations

Schwinger-Parametrisation

$$\frac{1}{(m^2-k^2)^\alpha} = \frac{1}{\Gamma(\alpha)} \int_0^\infty \mathrm{d}\beta \; \beta^{\alpha-1} \mathrm{e}^{-\beta(m^2-k^2)}$$

Feynman-Parametrisations

$$\frac{1}{a^\alpha b^\beta} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 \mathrm{d}x \, \frac{x^{\alpha-1}(1-x)^{\beta-1}}{(xa+(1-x)b)^{\alpha+\beta}}$$

$$\frac{1}{abc} = \int_0^1 \mathrm{d}y \, 2y \int_0^1 \mathrm{d}x \, \frac{1}{[ayx+by(1-x)+c(1-y)]^3}$$

$$\frac{1}{a_1^\alpha a_2^\beta \ldots a_n^\omega} = \frac{\Gamma(\alpha+\beta+\ldots+\omega)}{\Gamma(\alpha)\Gamma(\beta)\ldots\Gamma(\omega)} \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \ldots \mathrm{d}x_n \, \delta(1-\sum_{i=1}^n x_i) \times \frac{x_1^{\alpha-1} x_1^{\beta-1} \ldots x_n^{\omega-1}}{(a_1x_1+a_2x_2+\ldots+a_nx_n)^{\alpha+\beta+\ldots+\omega}}$$

Standard Integral

$$
\begin{aligned}
I_{\alpha\beta} &= \int \mathrm{d}^d z' \frac{z'_\alpha z'_\beta}{(z'-z_1)^n (z'-z_2)^m} \\
&= \pi^{\frac{d}{2}} \frac{\Gamma(\frac{n+m-d}{2})\Gamma(\frac{d-n}{2})\Gamma(\frac{d-m}{2})}{\Gamma(\frac{n}{2})\Gamma(\frac{m}{2})\Gamma(d-\frac{m+n}{2}+2)} \frac{1}{(z_1-z_2)^{n+m-d}} \\
&\quad \times \Big\{ (\tfrac{d-n}{2}+1)(\tfrac{d-n}{2}) z_{2\alpha} z_{2\beta} \\
&\quad + (\tfrac{d-m}{2})(\tfrac{d-n}{2})(z_{2\alpha} z_{1\beta} + z_{1\alpha} z_{2\beta}) \\
&\quad + (\tfrac{d-m}{2}-1)(\tfrac{d-m}{2}) z_{1\alpha} z_{1\beta} \\
&\quad + \frac{\delta_{\alpha\beta}}{2} \frac{(\frac{d-m}{2})(\frac{d-n}{2})}{(\frac{n+m-d}{2}-1)} (z_1-z_2)^2 \Big\}
\end{aligned}
$$

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&\quad + \frac{\delta_{\alpha\beta}}{2} \frac{(\frac{d-m}{2})(\frac{d-n}{2})}{(\frac{n+m-d}{2}-1)} (z_1-z_2)^2 \Big\}
\end{aligned}
$$

Results Continuum

Leading Order

$$\mathcal{D}_1^{(0)}(z^2) = \frac{N^2-1}{\pi^4 z^4}$$

Field Strength Correlator in $\mathcal{O}(g^2)$

$$\mathcal{D}^{(2)}(z^2) = N(N^2-1)\frac{1}{\pi^2 z^4}\frac{\alpha_s}{\pi}\left[-\frac{1}{4}L + \frac{3}{8}\right]$$

$$\mathcal{D}_1^{(2)}(z^2) = N(N^2-1)\frac{1}{\pi^2 z^4}\frac{\alpha_s}{\pi}\left[\left(\frac{\beta_1}{2N} - \frac{1}{4}\right)L + \frac{\beta_1}{3N} + \frac{29}{24} + \frac{\pi^2}{3}\right]$$

- results coincide with work of *Eidemüller, Jamin*
- results on the lattice *Di Giacomo, Meggiolaro, Panagopoulos et al.*
- comparison difficult ⇒ need calculation in lattice perturbation theory

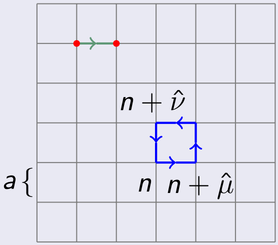
Lattice Discretisation

- hypercubic lattice
- replacements:

  $x_\mu \longrightarrow n_\mu a$ (site)

  $\int \mathrm{d}^4 x \longrightarrow a^4 \sum_n$
- restriction to first Brillouin zone:

  $\mathrm{BZ} = \left\{ k \,\middle|\, -\frac{\pi}{a} < k_\mu \leq \frac{\pi}{a} \right\}$

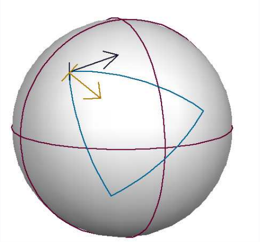


- parallel transporter $U_\mu(n)$

  (link variable)
- covariant derivative:

  $D_\mu \phi(x) = \frac{1}{a}\left(U_\mu^{-1}(n)\phi(n+\hat{\mu}) - \phi(n) \right)$
- plaquette:

  $P_{\mu\nu}(n) = U_\mu(n) U_\nu(n+\hat{\mu}) U_\mu^\dagger(n+\hat{\nu}) U_\nu^\dagger(n)$



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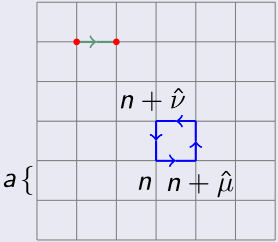

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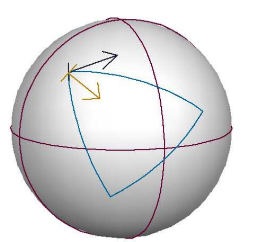


Lattice Gauge Theory

- connection lattice $\leftrightarrow$ continuum via gauge fields

$$U_\mu(n) \equiv \mathrm{e}^{\mathrm{i}agT^B A_\mu^B(n)} \equiv \mathrm{e}^{\mathrm{i}aA_\mu(n)}$$

- use expansion $A_\mu(n+\hat{\nu}) = A_\mu(n) + a\partial_\nu A_\mu(n) + \dots$
  and Baker-Campbell-Hausdorff formula

$$P_{\mu\nu}(n) = \mathrm{e}^{\mathrm{i}ga^2 G_{\mu\nu}(n)}, \qquad G_{\mu\nu}(n) = F_{\mu\nu}(n) + \mathcal{O}(a)$$

- $F_{\mu\nu}$ traceless $\Rightarrow$ construct gauge invariant action

$$\Downarrow$$

Wilson Action

$$S_\mathrm{G} = \beta \sum_x \sum_{\mu\neq\nu} (N_\mathrm{C} - \mathrm{Re\,Tr}\, P_{\mu\nu}), \quad \beta = \frac{2N_\mathrm{C}}{g^2}$$

Haar Measure

Generating Functional

$$Z = \int \prod_{n,\mu} \mathcal{D}U_\mu(n)\, \mathrm{e}^{-S_{\mathrm{total}}(n)}$$

- g small $\Rightarrow$ saddle-point-expansion in $A_\mu(n)$
- problem: infinite number of terms $\Rightarrow$ infinite number of new vertices
- rewrite *Haar Measure* $\mathcal{D}U_\mu$ in $\mathrm{d}A_\mu(n)$

$$\prod_{n,\mu} \mathcal{D}U_\mu(n) = \mathrm{e}^{-S_{\mathrm{M}}} \prod_{n,\mu,a} \mathrm{d}A^a_\mu(n)$$

$$\text{where} \qquad S_{\mathrm{M}} = \frac{g^2}{8a^2} \sum_x (A^a_\mu(x))^2$$

Feynman Rules

Total Action

$$S_{\mathrm{total}} = S_{\mathrm{G}} + S_{\mathrm{M}} + S_{\mathrm{FP}} + S_{\mathrm{GF}}$$

1. Feynman rules analogue to continuum, but use:

$$\partial_\mu^{\mathrm{R}} f(x) = \frac{1}{a}(f(x+a\hat{\mu}) - f(x)),$$
$$\partial_\mu^{\mathrm{L}} f(x) = \frac{1}{a}(f(x) - f(x-a\hat{\mu})),$$

$$\Box = \partial_\mu^{\mathrm{R}} \partial_\mu^{\mathrm{L}}$$

2. momenta:

$$k_\mu \longrightarrow \widetilde{k}_\mu = \frac{2}{a} \sin\left(\frac{ak_\mu}{2}\right)$$

Propagators and New Vertices

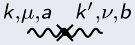
Propagators

$$D^{ab}_{\mu\nu}(x) = \delta^{ab}\delta_{\mu\nu}\int_{\frac{\pi}{a}}^{\frac{\pi}{a}} \frac{\mathrm{d}^4 k}{(2\pi)^4}\frac{\mathrm{e}^{\mathrm{i}k(x+a\frac{\hat{\mu}-\hat{\nu}}{2})}}{\widetilde{k}^2}$$

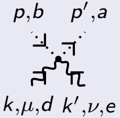
$$\text{where} \quad \widetilde{k}^2 = \sum_{\kappa=1}^{4}(\widetilde{k}_\kappa)^2 = \frac{4}{a^2}\sum_\kappa \sin^2\frac{ak_\kappa}{2}$$

New Vertices

$$-\frac{g^2}{4a^2}\delta_{\mu\nu}\delta_{ab}$$



$$\frac{a^2g^2}{12}\delta_{\mu\nu}(f^{aec}f^{bdc}+f^{adc}f^{bec})\widetilde{p}\,'_\mu\widetilde{p}_\mu$$



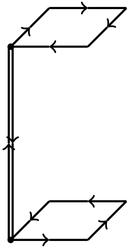
Field Strength Correlator

Definition

$$\mathcal{D}_{\mu\nu\lambda\omega}(x) = \left\langle 0 \middle| \mathrm{T}\left\{ G^a_{\mu\nu}(x) S^{ab}_{\mathrm{adj}}(x,0) G^b_{\lambda\omega}(0) \right\} \middle| 0 \right\rangle$$

where

$$S_{\mathrm{adj}}(x,0) = \mathrm{P}\exp\left\{ g\, f^{abc} \int_0^x \mathrm{d}y^\mu\, A^c_\mu(y) \right\}$$



- two sources, connected by an adjoint string
  ⇒ gluelump *Jorysz, Michael*
- measure gluelump mass *Laine, Philipsen*
- in case of SUSY: glueballino *Jorysz, Michael*
  (static adjoint source + dynamical gluon)

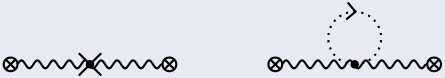
In Remembrance of Gluelumps

Contributions on the Lattice

Field Strength Correlator

$$\mathcal{D}_{\mu\nu\lambda\omega}(x) = \left\langle 0 \left| \mathrm{T}\left\{ G^a_{\mu\nu}(x) S^{ab}_{\mathrm{adj}}(x,0) G^b_{\lambda\omega}(0) \right\} \right| 0 \right\rangle$$

$=$ continuum-like contributions

$+$ 

1. convergent integrals on the lattice
   $\Rightarrow$ power counting theorem of Reisz
2. divergent integrals
   $\Rightarrow$ asymptotic expansion *Becher, Melnikov*

Power Counting Theorem of Reisz *Capitani, Rothe*

General Feynman Integral on the Lattice

$$F(q; M, a) = \int_{\mathrm{BZ}} \prod_{i=1}^{L} \frac{\mathrm{d}^4 k_i}{(2\pi)^4} \frac{N(k, q; M, a)}{D(k, q; M, a)}$$

L: number of loops

q_i: external momenta

M: mass ($\{M_i\} \equiv M$)

N: contributions of vertices/ numerators of propagators

D: product of propagators

- leading order fulfils theorem ✓
- coincides with continuum result ✓
- What about divergent integrals?

Divergent Integrals on the Lattice

1. general bosonic integral:

$$\mathcal{F}^{(4)}(p; n_1, n_2, n_3, n_4) = \int_{-\pi}^{\pi} \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\widetilde{k}_1^{2n_1} \widetilde{k}_2^{2n_2} \widetilde{k}_3^{2n_3} \widetilde{k}_4^{2n_4}}{D_{\mathrm{B}}(k, m)^p}$$

   - express in terms of Z_0, Z_1 and F_0 *Capitani*
   - need result in coordinate space ↯

2. use asymptotic expansion *Becher, Melnikov*

$$\mathcal{F}^1(m') = \frac{1}{4\pi^d} \int_{-\infty}^{\infty} \prod_{\kappa=1}^{d} \frac{\mathrm{d}\eta_\kappa}{(1+\eta_\kappa^2)} \left(m'^2 + D_{\mathrm{B}}(\eta)\right)^{-1}, \quad \left(\eta_\mu = \tan \frac{k_\mu}{2}\right)$$

   - split: *soft* and *hard scale* $\mathcal{F}^1(m') = \mathcal{F}^1_{\mathrm{soft}}(m') + \mathcal{F}^1_{\mathrm{hard}}(m')$
   - soft part: Taylor-expansion in η_κ, hard part: expansion in m'
   - lattice perturbation theory reduces to continuum perturbation theory ✓
   - but still need result in coordinate space: $\int_{-\pi}^{\pi} \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{k_\mu^{n_1} \ldots k_\omega^{n_n}}{(\widetilde{k}^2+m^2)^\alpha}$ ↯

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Numerical Approach

Example

$$I_{\mu\lambda}(n) = \partial_\mu^n \partial_\lambda^n \int_{-\pi}^{\pi} \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\mathrm{e}^{\mathrm{i}kn}}{(\widetilde{k}^2)^2}$$

1. CUBA-Library
   - suitable for computations of Z_0, Z_1 and F_0
   - inappropriate for needed integrals (even LO result does not agree with analytic result)
2. Brute Force
   - rewrite lattice integral as Riemann sum
   - compute for different values of n
   - fit $I(n)$ for comparison with analytical continuum results

Coordinate-Space-Method *Lüscher, Weisz, (Vohwinkel)*

Large x Behaviour

$$\mathcal{D}_{\mu\nu\lambda\omega} \overset{|x|\to\infty}{\simeq} \frac{\beta(g)}{g} \langle G^a_{\mu\nu}(0) G^a_{\mu\nu}(0) \rangle \, \mathrm{e}^{-x/\lambda_\mathrm{G}}$$

gluon condensate: $\beta(g)/g \langle G^a_{\mu\nu}(0) G^a_{\mu\nu}(0) \rangle$

correlation length: $\lambda_\mathrm{G} = 1/M_\mathrm{G}$ *D′Elia et al., Bali et al.*

- coordinate-space result ✓
- made for numerical approach
  $\Rightarrow$ not sensible for perturbation theory (great distances) ↯

Gluelumps - Rest in Peace

$\ddot{\smile}$

When things are going well,
something will go wrong.

When things just can't get any worse,
they will.

Anytime things appear to be going better,
you have overlooked something.