Simulation of Supersymmetric theories on the lattice

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This is NOT an apple

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Hint: To prove it is an apple .. Try to eat it !!

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This is how an apple looks like .. a simulation
[for more details see online Oxford dictionnary]

Outline

Two motivations for LHC

- Discover Higgs boson (complete SM)
- Discover physics beyond SM (SUSY, Little Higgs model, extra-dimension, ..)

Motivations for SUSY

Standard Model (SM):

- solution to the *naturalness*[1] problem
- Gauge couplings unification

Quantum Gravity:

- SUSY softens UV divergences of quantum gravity and string theory
- AdS/CFT: conjecture relating quantum gravity to SYM theory

Cosmology: Lightest Supersymmetric Particles (LSP) are candidate for dark matter: neutralino, gravitino,..

[1] the parameters are proteced by symmetry from huge quantum correction. This symmetry is recovered in the limit of vanishing parameters i.e. setting the electron mass in QED, the chiral symmetry is exact.

The Higgs sector instabilities

- the Higgs (scalar particle) sector of SM contains instabilities.
- the lowest dimensional relevant term in SM is the Higgs mass term
- the scalar gains also mass from virtual fermion loops[2] contribution (quadratically divergent) allowed at each energy scale even very large scale.
- there is no symmetry in SM which protect the Higgs mass
- particles (not from SM) from *new* theory could contribute to these quantum corrections to stabilize the scalar mass.

[2] also self-coupling and gauge boson coupling

more details

- in the scalar self-energy, the correction to the mass square m_H^2 at one loop order in perturbation theory is represented by

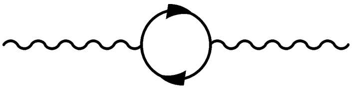


Figure: Fermionic loop contribution to Higgs-Boson two point function

$$\Pi^f_{hh}(0) = -\int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left(-i\frac{\lambda_f}{\sqrt{2}}\right)\frac{i}{\not{k}-m_f}\left(-i\frac{\lambda_f}{\sqrt{2}}\right)\frac{i}{\not{k}-m_f} \quad (1)$$

$$= -2\lambda_f^2 \int \frac{d^4k}{(2\pi)^4}\left[\frac{1}{k^2-m_f^2}+\frac{2m_f^2}{(k^2-m_f^2)^2}\right]. \quad (2)$$

- observe the first term is quadratically divergent and independent of the scalar mass m_H
- Renormalizability:

$$m_H^2(\nu) \sim m_H^2(\Lambda) - (\Lambda^2 - \nu^2) \, , \nu = \langle H \rangle \sim 250\text{GeV} \tag{3}$$

- Λ scale can be set to the Planck mass $M_P \sim 10^{19}$. keeping $m_H \leq \mathcal{O}(1\text{Tev})$ requires fine tuning among the bare parameters at a level of 10^{-34}: the *gauge hierarchy problem*

$$\frac{m_H^2(\Lambda) - \Lambda^2}{\Lambda^2} \sim \frac{\nu^2}{\Lambda^2} = \mathcal{O}(10^{-34}) \tag{4}$$

Possible solutions:

- SUSY symmetry: fermion $\longleftrightarrow$ boson
- Extra dimension
- Litte Higgs theory

SUSY as solution ..

SUSY is an elegant candidate:

- stabilize the Higgs mass
- unify the (running) gauge couplings
- it has a soft breaking

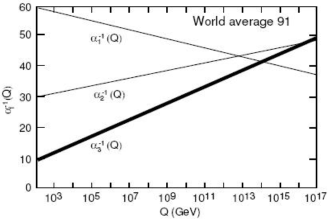
SUSY is a symmetry of the nature ?! actually not *exact*

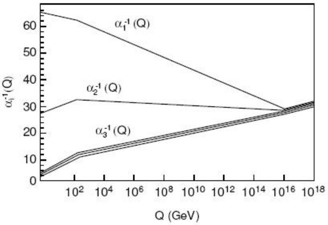
- particle with the same mass as the electron has never been observed $\rightarrow$ SUSY must be broken
- the soft SUSY breaking M_s induces mass splitting of order $\mathcal{O}(M_s)$. Using naluralness arguments to weak scale has suggested that $M_s \leq \mathcal{O}(TeV)$. The sparticle should be discovered in forthcoming high energy acceleartor.

Unification of gauge couplings

- The SM couplings depend on the energy scale (running couplings).
- Matching of the running couplings occurs at GUT scale in MSSM.

[Particle Data Group]





(left)**SM** , (right)**MSSM**

SUSY, back to naturalness problem

sfermion loop (scalars) contributing to the Higgs self energy is considered

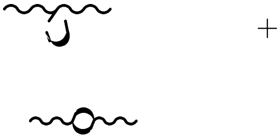


Figure: Sfermion loop contribution to Higgs self energy

- now, the first term cancels quadratic divergences in the loop fermion
- only logarithmic divergences remains $\longrightarrow$ cancelled by renormalization procedure

Relevance and role of symmetries

- the lattice is necessary to understand the low energy (non-perturbative) features of QFTs
- operators in lattice ation have different strenght of quantum effects with the scale:

  Relevant operator: bigger effects
  Irrelevant operator: weaker effects
  Marginal: scale invariant

- At low energy physics, in the modern view irrelevant "non-renormalizable" terms are *good* (that is, they are irrelevant).
- Existence of a symmetry *protect* relevant terms from large *additive* renormalisation.

$4d$ $N = 1$ SYM with $SU(2)$ gauge group

$$\mathcal{L}_{SYM} = -\frac{1}{4}F^a_{\mu\nu}(x)F^{a\mu\nu}(x) + \frac{1}{2}\bar{\lambda}^a(x)\gamma^\mu\mathcal{D}_\mu\lambda^a(x) + m_{\tilde{g}}\bar{\lambda}\lambda\ .$$

without the mass term, the Lagrangian has:

- SUSY
- global $U(1)$ chiral symmetry a.k.a R-symmetry

- Chiral symmetry is anomalous $\rightarrow$ $U(1)$ is broken to Z_4
- Z_4 is broken *spontaneously* to Z_2 by the gluino condensate $\langle\bar{\lambda}\lambda\rangle \neq 0$ giving rise to first order phase transition
- colorless bound states are expected in this theory: confinement, where in presence of the mass term splitting in the masses occurs $\rightarrow$ two supermultiplets each consisting of three states

Q: C(Sh)ould we study these properties on the lattice ? .. numerical simulations ?

- A: at first, **Yes !!**, we c(sh)ould. it looks simpler (!) than QCD (which has several flavours)
- Q: SUSY requires supercharges Q satisfying $\{Q, \bar{Q}\} \sim P$, P are broken by the lattice, How could we have SUSY in the lattice action, and chirality ?
- A: maybe it is a bad idea to have SUSY on the lattice ! get SUSY as *accidental* symmetry in the continuum seems to be a good idea, for chirality it is obvious :-) .. don't worry..

"..Let the lattice spoil SUSY if it so wishes. "

[Curci, Veneziano 87]

- Q: we could have relevant and marginal operators that spoil SUSY in the continuum limit ?
- A: we have an emerging exact symmetry in the continuum, Thanks to chiral symmetry

- Q: could that extend to other theories ?
- A: Unfortunaly, No. for theory with scalar ϕ, the breaking relevant term is $m \mid \phi \mid^2$, no chiral symmetry, switching off the mass does not increase the symmetry

- Q: is there other trial to put SUSY on the lattice for such models ?
- A: Yes ! actually at low dimensional theories $d \leq 2$, Arkani-Hamed,Cohen, Kaplan, Katz, Sugino are **(de)contructionists** $\rightarrow$ Orbifolding method
Catterall, Karamov, Gregory, Ghadab $\rightarrow$ Q-exact SUSY

- Are you going to simulate ?
- Yes !

Go simulating !

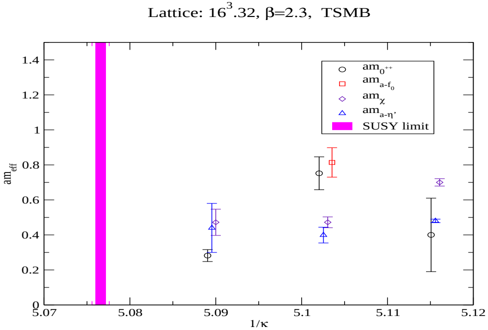
Action:	Curzi-Veneziano [previous/next talk]
fermion part:	Wilson
gauge part:	Wilson (eventually improved tlSym)
# flavours:	one flavour (effectively $N_f = \frac{1}{2}$)
Algorithm:	Two Step Multi-Boson (TSMB)[3] [Montvay 96]
Ensembles:	three points (see table below)

run	κ	ϵ	λ	n_1	n_2	n_3	n_4	n_5	# Up	offset	$A_{\rm nc}$ %
(a)	0.1955	$2.0 \cdot 10^{-5}$	4.0	40	800	300	700	760	12500	5	50-80
(b)	0.1960	$4.0 \cdot 10^{-6}$	4.0	40	1800	2000	2030	2080	23500	5	50-80
(c)	0.1965	$4.0 \cdot 10^{-6}$	4.0	40	1800	2000	2030	2080	18000	10	50-62

[3] PHMC runs are not reported here

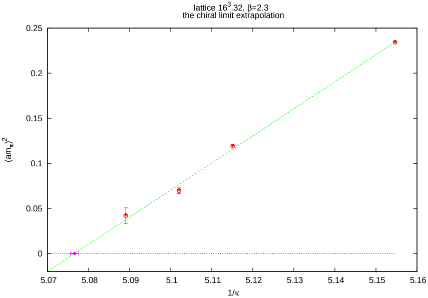
- Q: we are using Wilson formulation, do we have idea on the price we have to pay?
- A: Yes, fine-tuning and *Pfaffian* sign problem. In the continuum limit both SUSY and chirality emerge

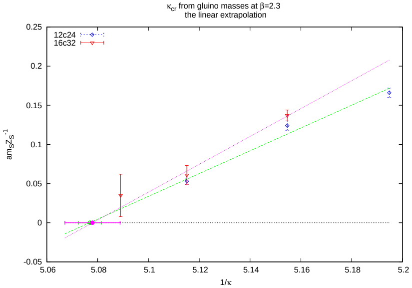
The mass spectrum



κ	$am_{a-\pi}$	0^+(glub.)	a-f_0	a-η'	$\tilde{g}g_1$	$\tilde{g}g_{\gamma 0}$
0.194[†]	0.484(1)	$<$ 0.52(1)	$<$ 0.78	0.52(2)	0.40(7)	0.43(1)
0.1955	0.345(3)	$<$ 0.4(2)	0.642(-)	0.48(1)	0.673(20)	0.700(21)
0.196	0.264(5)	$<$ 0.75(9)	0.814(84)	0.399(45)	0.424(22)	0.472(31)
0.1965	0.208(13)	$<$ 0.28(3)	-	0.44(14)	0.469(85)	0.472(75)

The chiral(SUSY) limit





→ OZI Arguments

$$(am_\pi)^2 = A\left(\frac{1}{\kappa} - \frac{1}{\kappa_{cr}}\right)$$

extrapolation: $\kappa_{cr}^{\pi} \sim 0.1969$

→ Chiral Ward-Identities: renormalized gluino mass

$$am_{\tilde{g}} Z_S = \frac{1}{2}\left(\frac{1}{\kappa} - \frac{1}{\kappa_{cr}}\right)$$

extrapolation: $\kappa_{cr}^{WI} \sim 0.1969$

Conclusion, previous problems, next alternatives

- Physical scale : Small volume $r_0/a \sim 8$, at $\beta = 2.3$ on $16c32$ lattices we have $L \sim 1$fm
- Improved gauge action .., small gauge coupling β
- Large autocorrelation, low statistics at light gluino masses
- dynamical fermions are included by PHMC algorithm: global updating, short autocorrelation , higher statistics.
- More clear picture of SYM spectrum is expected on larger lattices: $24c48$ and $32c64$; study of finite size effects.
- Refine the analysis, using new methods for spectroscopy
- fine-Tune and intensive production of new PHMC still in Runs .. Analysis are ongoing .. chiral condensate ..

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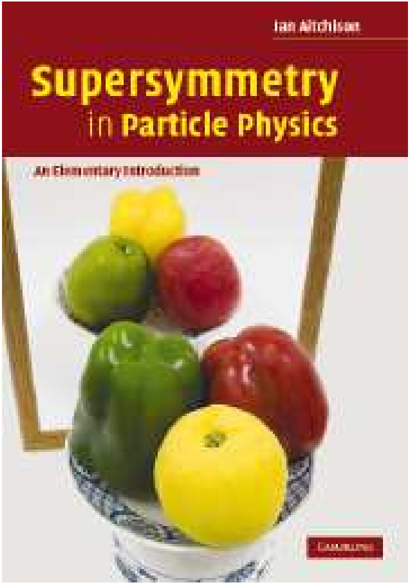
Thank You !

Latest: fruits and vegetables are SUSY partners !! ;-)



Secret to not be reported !

This was a last-minute talk .. [the author]