Precise results from lattice QCD with light quarks in the twisted-mass formulation

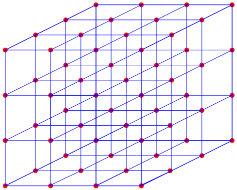
Federico Farchioni

University of Münster

DPG Frühjahrstagung Heidelberg 2007
March 8, 2007

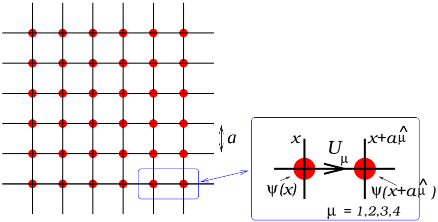
Lattice QCD in a nutshell

4d Euclidean continuum replaced by (hyper)cubic lattice:



- Invent a discretized version of the continuum QCD action:

[K. Wilson (1974)]



$$x_\mu = n_\mu a\,,\ \ n_\mu \in Z\,; \quad \text{Gauge field} \rightarrow U_\mu(x) \in SU(3)\ \text{(color)}$$

$$D_\mu[A]\psi(x) \rightarrow \nabla_\mu[U]\psi(x) \equiv \frac{1}{a}\left(U_\mu(x)\psi(x+\hat{\mu}) - \psi(x)\right)$$

- Path integral $\longrightarrow$ ordinary integral, Monte Carlo sampling possible!
- Monte Carlo simulation of the lattice theory with suitable algorithm. [M. Creutz (1980)]

  $\longrightarrow$ numerical determination of low energy properties of QCD (hadron masses, decay constants, matrix elements, etc.).

- Systematic error can be kept under control
  $\longrightarrow$ only limitation of accuracy given by the statistical uncertainty (computer power).

Four challenges in Lattice QCD

- Small finite-volume effects $\longrightarrow$ size of the box $L \geq 2$fm.
- Small discretization effects $\longrightarrow$ $a \lesssim 0.1$fm,
  improved actions: $O(a) \rightarrow O(a^2)$.
- Inclusion of dynamics of quarks: two light quarks (u, d),
  $+$ strange quark.
- Fast simulations: highly optimized simulation algorithms.

The lattice formulation has an impact on these issues!

The Wilson action

Wilson formulation in the fermionic sector for degenerate u, d:

[Wilson(1974)]

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$S^W_{quark} = \sum_x a^4 \left\{ \bar{\psi}(x) \frac{\gamma_\mu}{2} (\nabla_\mu + \nabla^*_\mu) \psi(x) \; - \; \frac{a}{2} \bar{\psi}(x) \nabla^* \nabla \psi(x) + \right.$$

$$\left. + m_0 \, \bar{\psi}(x) \psi(x) \right\}$$

Wilson term

"Core" of the action: Fermion Matrix

$$S_{quark} = \sum_{xy} \bar{\psi}(x) M_{xy} \psi(y)$$

Pros:

- Theoretically sound.
- Simple, ultralocal $\rightarrow$ cheap, ideal for parallelization.

Cos:

- Unimproved $\rightarrow$ fine (and expensive) lattices needed.
- Intricate renormalization pattern.(*)
  Example: $m_q = 0 \Leftrightarrow m_0 = m_{cr}(g_0) \neq 0$!
- Technical: Fermion matrix $M[U]_{xy}$ not protected from very small eigenvalues $\rightarrow$ simulation gets slow and unstable in the physically interesting regime of light quarks.(*)

(*) Due to explicit breaking of chiral symmetry.

Twisted Mass Lattice QCD

Cure: add a flavor-twisted mass term to the Wilson quark action.

[Frezzotti, Grassi, Sint, Weisz (1999)]

$$S_{quark}^{TM} = \sum_x a^4 \left\{ \bar\psi(x)\frac{\gamma_\mu}{2}(\nabla_\mu + \nabla^*_\mu)\psi(x) \; - \; \frac{a}{2}\,\bar\psi(x)\nabla^*\nabla\psi(x) + \right.$$

$$\left. + m_0\,\bar\psi(x)\psi(x) \; + \; i\mu_0\,\bar\psi(x)\tau_3\gamma_5\psi(x) \right\}$$

twisted mass: breaks parity and flavor!!

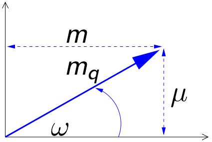
...but ...

Continuum limit still describes $N_f = 2$ QCD with quark mass:

$$m_q = \sqrt{m^2 + \mu^2}$$

and chirally rotated fields:

$$\begin{cases} \psi' = \exp\{\frac{i}{2}\omega\gamma_5\tau_3\}\,\psi \\ \bar{\psi}' = \bar{\psi}\,\exp\{\frac{i}{2}\omega\gamma_5\tau_3\} \end{cases}$$



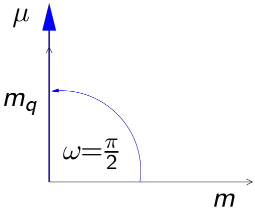
The lattice theory cannot be rotated back since the Wilson term breaks chirality $\rightarrow$ the twist changes the $O(a)$ terms!

Benefits:

- Fermion matrix M_{xy} protected from very small eigenvalues for $\mu > 0$: speed-up and smoothing of the Monte Carlo simulation $\rightarrow$ simulation of extremely light quarks possible!

$m = 0$, $\omega = \pi/2$: maximal twist.

- Automatic $O(a)$ improvement.
- Renormalization/mixing pattern can be simplified. [Frezzotti, Rossi (2003)]



Example: $F_\pi = \frac{2\mu}{M_\pi^2} |\langle 0|P^1(0)|\pi\rangle$

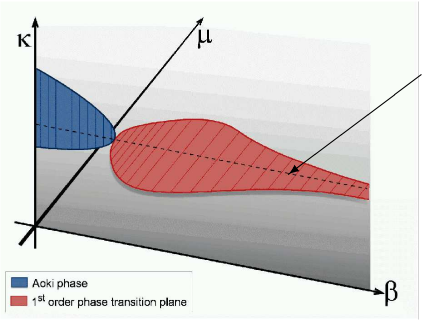
$P^a(x)$ and μ renormalize in the same way, renormalization-free determination of F_π.

No free lunch: $O(a^2)$ breaking of parity/flavor symmetry introduced by the twisted mass term.

E.g.: $M^2_{\pi^+} - M^2_{\pi^0} = c\,\Lambda^4_{QCD} a^2$, verify that $c = O(1)$!

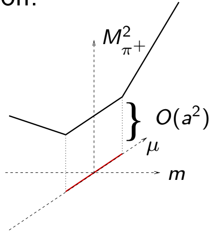
...ready to go?...

The phase structure of Wilson LQCD



[Münster; Sharpe, Wu; Scorzato (2004)]

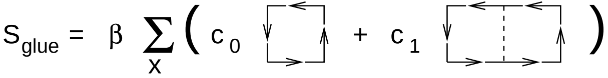
- 1st order chiral PT extends to $\mu \in [-\mu_c, \mu_c]$.
- The charged pion mass has a minimal value $(M_{\pi^+})_{min} > 0$ inside this region:



The choice of the formulation in the gluonic sector can help:

$$(M_\pi^+)^2_{min} \sim c\, a^2\, \Lambda^4_{QCD} \rightarrow \text{minimize } c!$$

Action-family:

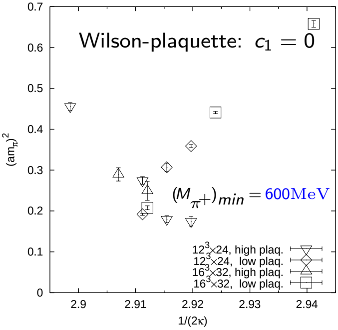


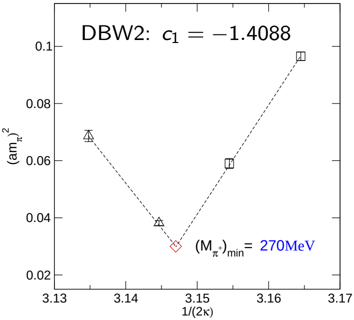
Three formulations studied @ $\mu \simeq 10\text{MeV}$, $0.1\text{fm} \leq a \leq 0.2\text{fm}$:

- Wilson-plaquette: $c_1 = 0$
- Tree-level Symanzik improved (tlSym): $c_1 = -\frac{1}{12} = -0.08\bar{3}$
- RG improved (DBW2): $c_1 = -1.4088$

[F.F. et Al. (2004-2006)]

Pion mass squared vs. *longitudinal* mass, effect of c_1:





$a \simeq 0.14\text{fm}, \ \mu \simeq 10\text{MeV}$

tlSym: intermediate situation but superior in approach to continuum $\rightarrow$ our choice.

Ready to go . . .

European Twisted Mass Collaboration

D. Alexandrou, Ph. Boucaud, Th. Chiarappa, P. Dimopoulos, F.F., R. Frezzotti, V. Gimenez Gomez, G. Herdoiza, K. Jansen, G. Koutsou, V. Lubicz, G. Martinelli, C. McNeile, F. Mescia, C. Michael, I. Montvay, G. Münster, K. Nagai, D. Palao, M. Papinutto, O. Pene, J. Pickavance, G.C. Rossi, S. Schäfer, L. Scorzato, A. Shindler, S. Simula, C. Tarantino, C. Urbach, T. Vladikas, U. Wenger

First large scale simulations

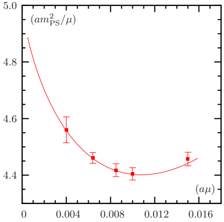
Current goals:

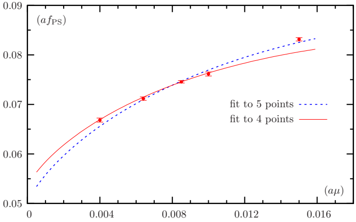
- $N_f = 2$, maximally twisted mass QCD.
- $a \simeq 0.1, 0.09, 0.07$ fm @ $L \simeq 2$ fm.
- L $\simeq 1.7, 2, 2.7$ fm, @ $a \simeq 0.09$ fm.
- 5 values of the quark mass.
- Lightest simulated pion mass $M_\pi \simeq 300\text{MeV}$.

Optimized Hybrid Monte Carlo Algorithm. [Urbach, Jansen, Shindler, Wenger (2005)]

First analyses

- $L \simeq 2$ fm, $a \simeq 0.09$ fm, $M_\pi = (300 - 570)$MeV.
- "Online" analysis: $M_{\pi^\pm}$, F_π, Sommer scale r_0.
- Smooth simulations, extremely precise results, 5-6 permille (after improved error analysis).





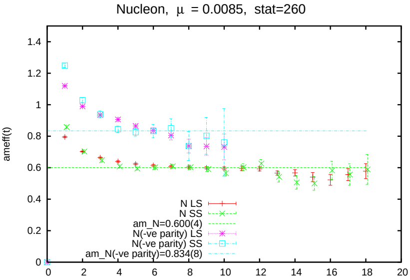
Precise determination of Gasser-Leutwyler coefficients in ChPT

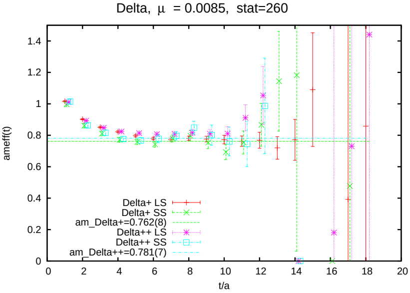
[Boucaud et Al., ETMC (2006)]

More analyses

- Baryons $\leftarrow$
- Pion form factors $\leftarrow$
- NP renorm factors of composite operators
- Overlap valence fermions
- ϵ regime
- Topology
- Rho decay
- Strange, Charm quark mass
- ...

Baryons



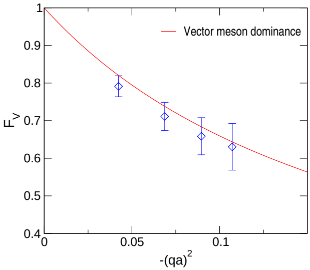


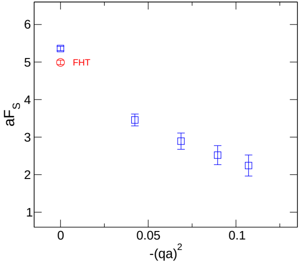
Thanks: Dina Alexandrou

- N: $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ (p=uud, n=ddu)
- $\Delta(1232)$: $I(J^P) = \frac{3}{2}(\frac{3}{2}^+)$ (Δ^{++}=uuu,Δ^{+}=uud,Δ^{0}=ddu,Δ^{-}=ddd)
- $u \leftrightarrow d$: exactly degenerate.
- $a \simeq 0.09$fm, $M_\pi^\pm \simeq 440$MeV
- Isospin breaking: $\Delta^{++} - \Delta^{+}$ ($\Delta^{-} - \Delta^{0}$); turns out to be small: $a\Delta m = 0.019(15)$.

[Quenched: Bietenholz et Al. (2004); Abdel-Rehim et Al. (2005)]

Pion form factor





Thanks: Silvano Simula

- $\langle\pi(p')|J^{\mu}_{em}|\pi(p)\rangle = (p_\mu + p'_\mu) F_V(q^2)$
- $\langle\pi(p')|\bar{q}q|\pi(p)\rangle = F_S(q^2)$
- $a \simeq 0.09\mathrm{fm}$, $M_\pi^+ \simeq 300\mathrm{MeV}$
- VMD: input $M_V = 990\mathrm{MeV}$ (meas.)
- $F_S(0)$ near to the prediction from the Feynman-Hellman theorem (FHT).

[Quenched: Abdel-Rehim et Al. (2004)]

The strange quark

The strange must be included together with the charm in a $SU(2)$ doublet: [Frezzotti, Rossi (2003)]

$$\psi_h = \begin{pmatrix} c \\ s \end{pmatrix}$$

$$S^{TM}_{h-quark} = \sum_x a^4 \left\{ \bar{\psi}_h(x) \frac{\gamma_\mu}{2} (\nabla_\mu + \nabla^*_\mu) \psi_h(x) - \frac{a}{2} \bar{\psi}_h(x) \nabla^* \nabla \psi_h(x) + \right.$$

$$\left. + m_{0h} \bar{\psi}_h(x) \psi_h(x) + i\mu_{0h} \bar{\psi}_h(x) \tau_1 \gamma_5 \psi_h(x) + \delta_0 \bar{\psi}_h(x) \tau_3 \psi_h(x) \right\}$$

mass splitting term

$$m_{c,s} = \sqrt{m_h^2 + \mu_h^2} \pm \delta$$

Algorithmic development required: optimized algorithm ready.

[Montvay, Scholz (2005)]

Comparison with a second algorithm on the way.

[Chiarappa, Frezzotti, Urbach (in progress)]

First simulations have been performed [Chiarappa, F.F. et Al. (2006)]:

- Practical approach for tuning to maximal twist:

$$m_{0l} = m_{0h} \wedge m_{PCAC}^{light} = 0$$

- Tuning μ_{0h}, δ_0 such that m_s and m_c take their physical values is feasible.

Next: light u, d ($M_\pi \simeq 300$MeV) and s at the physical point.