Numerische Methoden für komplexe Systeme, SoSe 2015

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Exercises online: http://pauli.uni-muenster.de/tp/

Exercise 1: Complex Ginzburg-Landau equation in 1D

Solve the one dimensional Ginzburg-Landau equation

$$\partial_t A = (1 + i\alpha)\frac{\partial^2 A}{\partial x^2} + A - (1 + i\beta)|A|^2 A$$

$A = A(x, t)$ using the pseudospectral method on a domain of length L. Use the *Exponential Time Differencing* method of second order (ETD2) for the time integration. Consider the following cases:

a) Plane waves:

Constants	$(\alpha, \beta) = (1, 2)$
Domain	L=[-50, 50]
Time-step	$h = 0.05$
Number of gridpoints	$N = 512$
Initial condition	$A(x, 0) =$ Noise with amplitude 0.01

b) Benjamin-Feir-Instability

Constants	$(\alpha, \beta) = (1, 2)$
Domain	L=[-50, 50]
Time-step	$h = 0.05$
Number of gridpoints	$N = 512$
Initial Condition	$A(x, 0) = \sqrt{1 - \left(\frac{20\pi}{L}\right)^2}\exp(i\frac{20\pi}{L}x) +$ Noise

c) Phase-Turbulence:

Constants	$(\alpha, \beta) = (2, -1)$
Domain	$L = [-100, 100]$
Time-step	$h = 0.05$
Number of gridpoints	$N = 512$
Initial Condition	$A(x, 0) = 1 +$ Noise with amplitude 0.01

d) Defect-Turbulence:

Constants	$(\alpha, \beta) = (2, -2)$
Domain	$L = [-100, 100]$
Time-step	$h = 0.05$
Number of gridpoints	$N = 512$
Initial Condition	$A(x, 0) = 1 +$ Noise with amplitude 0.01

e) Intermittency:

`Constants`	$(\alpha,\, \beta) = (0.5,\, -1.5), \quad (0,\, -4)$
`Domain`	$L = [-100,\, 100]$
`Time-step`	$h = 0.05$
`Number of gridpoints`	$N = 512$
`Initial Condition`	$A(x,0) = \mathrm{sech}((x + L/4)^2) + 0.8 * \mathrm{sech}((x - L/4)^2) + \mathrm{Noise}$

f) Coherent Structures:

`Constants`	$(\alpha,\, \beta) = (0,\, 1.5)$
`Domain`	$L = [-100,\, 100]$
`Time-step`	$h = 0.05$
`Number of gridpoints`	$N = 512$
`Initial Condition`	$A(x,0) =$ Noise with amplitude 0.01

Exercise 2: Complex Ginzburg-Landau equation in 2D

Solve the complex two dimensional Ginzburg-Landau equation

$$\partial_t A = (1 + i\alpha)\triangle A + A - (1 + i\beta)|A|^2 A$$

$A = A(x,\, y,\, t)$ using the pseudospectral method on the domain Ω. Use an *Exponential Time Differencing* method of second order (ETD2) for the time integration. Consider for example the case

a) "Frosen States":

`Constants`	$(\alpha,\, \beta) = (0,\, 1.5)$
`Domain`	$\Omega = [-100,\, 100] \times [-100,\, 100]$
`Time-step`	$h = 0.05$
`Number of gridpoints`	$N = 256$
`Initial Condition`	$A(x,0) = 1 +$ Noise with amplitude 0.01