Introduction to the Standard Model
Exercises 9

Deadline: Monday 20 June 2016 (12 am)
at Dr. Giudice's office (KP 301) and Ms Sonja Esch (KP 310)

Topics covered: Thrust, hadronic tensor, form factor.

1. (1.0 P) The thrust of an event, in e^+e^- annihilation, is defined as

$$T = \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \, \hat{n}|}{\sum_i |\vec{p}_i|} \ ,$$

where $\vec{p}_i$ are the momenta of the final-state particles and the maximum is obtained for the thrust axis $\hat{n}$. In the limit of the production of a perfect back-to-back $\bar{q}q$ pair the limit $T = 1$ is obtained. Prove that a perfectly symmetric many-particle configuration leads to $T = \frac{1}{2}$.

2. (2.0 P) The most general form of the hadronic tensor $W^{\mu\nu}$ is given by:

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^\mu p^\nu + \frac{W_4}{M^2} q^\mu q^\nu + \frac{W_5}{M^2}(p^\mu q^\nu + q^\mu p^\nu) \ ,$$

where q is the momentum transfer and p is the momentum of the proton. Using the Ward identity $q_\mu W^{\mu\nu} = 0$, prove that only two of the four structure functions are independent and that therefore we can write:

$$W^{\mu\nu} = W_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + \frac{W_2}{M^2} \left(p^\mu - \frac{p\, q}{q^2} q^\mu \right) \left(p^\nu - \frac{p\, q}{q^2} q^\nu \right) \ . \qquad (1)$$

3. (4.0 P) The cross section for electron proton scattering, in the laboratory frame, can be written as:

$$\left(\frac{d\sigma}{d\Omega dE'} \right) = \frac{\alpha^2}{4\pi \; m_p \; q^4} \frac{E'}{E} L^{\mu\nu} W_{\mu\nu} \ ,$$

where m_p is the proton mass, $W^{\mu\nu}$ is given by Eq. (1) and the lepton tensor $L^{\mu\nu}$ is given by:

$$L^{\mu\nu} = \frac{1}{2} \, \mathrm{Tr} \, [\not{k}' \gamma^\mu \not{k} \gamma^\nu] \ .$$

As usual the four-vectors k and k' refer to the incoming and outcoming electron, respectively. Derive the following expression:

$$\left(\frac{d\sigma}{d\Omega dE'} \right) = \frac{\alpha^2}{8\pi E^2 \sin^4 \frac{\theta}{2}} \left[\frac{m_p}{2} W_2(x, Q) \cos^2 \frac{\theta}{2} + \frac{1}{m_p} W_1(x, Q) \sin^2 \frac{\theta}{2} \right] \ .$$

4. (2.0 P) If the potential $V(r)$ has an exponential form Ae^{-mr}, with $r = |\vec{x}|$, show that the form factor $F(q^2) = \int d^3x \; e^{i\vec{q}\vec{x}} \; V(x)$ is equal to:

$$F(q^2) = \frac{K}{\left(1 + \frac{q^2}{m^2}\right)^2} \ ,$$

and determine the expression for K.