Introduction to the Standard Model
Exercises 3

Deadline: Monday 2 May 2016 (12 am)
at Dr. Giudice's office (KP 301) and Ms Sonja Esch (KP 310)

Topics covered: Photon propagator, Rutherford scattering, Bhabha scattering, vacuum polarisation.

1. (1 P) Show that the gauge dependent term of the photon propagator $(1-\xi)\frac{k_\mu k_\nu}{k^4}$ does not contribute to the determination of the amplitude $\mathcal{M}$ in the process $e^+e^- \to \mu^+\mu^-$.

2. (6 P) Consider the electron-proton scattering (with pointlike proton) $e^-(p_1)p^+(p_2) \to e^-(p_3)p^+(p_4)$. The squared amplitude $\overline{|\mathcal{M}|^2}$ for the process, summed on the final spins and averaged on the initial ones, is given by:

$$\overline{|\mathcal{M}|^2} = \frac{8e^4}{k^4}\left[p_{14}\,p_{23} + p_{12}\,p_{34} - m_p^2\,p_{13}\right] \ ,$$

where $p_{ij} = p_i p_j$, $k^\mu = p_1^\mu - p_3^\mu$ (the transfered 4-momentum) and m_p the mass of the proton. In the very high energy limit, one cannot assume that the proton is at rest, but one can neglet the mass of the electron. With these assumptions we have:

$$p_1^\mu = (E, \vec{p}_i), \quad p_2^\mu = (m_p, \vec{0}), \quad p_3^\mu = (E', \vec{p}_f), \quad p_4^\mu = p_1^\mu + p_2^\mu - p_3^\mu \ ,$$

with $|\vec{p}_i| = E$ and $|\vec{p}_f| = E'$. Determine the differential cross section in the initial reference frame (where the proton is at rest). Note that in this case the differential cross section has to be determined using:

$$\begin{aligned} F &= \sqrt{(p_1 p_2)^2 - p_1^2 p_2^2} \ , \\ dQ &= \frac{1}{(2\pi)^2}\frac{E'}{4m_p}\, dE'\, d\Omega\, \delta\left(E - E' + \frac{k^2}{2m_p}\right) \ . \end{aligned}$$

The final result is:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}\left(\cos^2\frac{\theta}{2} + \frac{E-E'}{m_p}\sin^2\frac{\theta}{2}\right) \ .$$

Hint: First of all, remove p_4 from the squared amplitude.

3. (5 P) Consider the Compton scattering $\gamma e^- \to \gamma e^-$. In this case two diagrams contribute to the process.

   i) Draw the two diagrams (a and b);
   ii) Apply the Feynman rules to determine the corresponding amplitudes ($\mathcal{M} = \mathcal{M}_a + \mathcal{M}_b$);
   iii) Calculate $|\mathcal{M}|^2$;
   iv) Sum on the final spins and average on the initial ones.

   The final results is:

$$\overline{|\mathcal{M}|^2} = e^4\,\mathrm{Tr}\left\{(\not{p}_3 + m)\left[\frac{\gamma^\nu(\not{p}_1 + \not{p}_2 + m)\gamma^\mu}{s - m^2} + \frac{\gamma^\mu(\not{p}_2 - \not{p}_4 + m)\gamma^\nu}{t - m^2}\right] \times \right.$$
$$\left.(\not{p}_2 + m)\left[\frac{\gamma_\mu(\not{p}_1 + \not{p}_2 + m)\gamma_\nu}{s - m^2} + \frac{\gamma_\nu(\not{p}_2 - \not{p}_4 + m)\gamma_\mu}{t - m^2}\right]\right\} .$$

   Moreover, calculate the term proportional to m^4 (consider only the terms, in the previous expression, which are proportional to m^4; you need to determine the trace of four gamma matrices); the result is:

$$\overline{|\mathcal{M}|^2} = \ldots + 4e^4 m^4\,\frac{p_{12}^2 + p_{24}^2 + p_{12}\,p_{24}}{p_{12}^2\,p_{24}^2} .$$

4. (2 P) In scalar QED, it is possible to show that the vacuum polarization amplitude is given by:

$$\Pi_2^{\mu\nu} = 2ie^2 \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx \frac{-2k^\mu k^\nu + g^{\mu\nu}(k^2 + x^2p^2 - m^2)}{[k^2 + p^2x(1-x) - m^2 + i\epsilon]^2} .$$

   Using the substitution $k^\mu k^\nu \to \frac{1}{d}k^2 g^{\mu\nu}$, the relation $\Gamma(2-d/2) = (1-d/2)\Gamma(1-d/2)$, and the integrals calculated in Appendix B.3 "Regularisation" (see webpage), show that:

$$\Pi_2^{\mu\nu} = -2\frac{e^2}{(4\pi)^{d/2}}\, p^2\, g^{\mu\nu}\, \Gamma\left(2 - \frac{d}{2}\right)\, \mu^{4-d} \int_0^1 dx\; x(2x-1)\Delta^{d/2-2} ,$$

   where $\Delta = m^2 - p^2x(1-x)$.