Introduction to the Standard Model
Exercises 13

Deadline: Monday 18 July 2016 (12 am)
at Dr. Giudice's office (KP 301) and Ms Sonja Esch (KP 310)

Topics covered: Charge conjugation operator, CP symmetry, fermion sector of the SM.

1. (2.0 P) Justify the reason why the operator $C = -\mathrm{i}\gamma_2$ is called charge conjugation operator. To this purpose determine the corresponding Dirac equation $(\mathrm{i}\partial\!\!\!/ - eA\!\!\!/ - m)\psi = 0$ for the field $\psi_c = C\psi$ and compare the two equations.
   Hint: remember that it is always possible to define a new Dirac representation by $\tilde{\gamma}^\mu = U\gamma^\mu U^\dagger$, where U is a unitary matrix. Prove, in our specific case, that the matrix $\tilde{\gamma}^\mu$ satisfies the Dirac algebra. Note that C is defined here in a different way compared to exercise 10, problem 5 (so it has also different properties).

2. (6.0 P) Find how the following bispinors transform under parity (P: $\psi \to \gamma_0\psi$) and under charge conjugation (C: $\psi \to -\mathrm{i}\gamma_2\psi^*$):

$$\begin{aligned} \bar{\psi}_i\psi_j \ , \\ \bar{\psi}_i\gamma_5\psi_j \ , \\ \bar{\psi}_i\gamma_\mu\psi_j \ , \\ \bar{\psi}_i\gamma_\mu\gamma_5\psi_j \ . \end{aligned}$$

   Moreover, knowing that A_μ transform as:

$$\begin{aligned} C &: \ A_\mu \to -A_\mu \ , \\ P &: \ A_0 \to A_0 \ , \quad A_i \to -A_i \ , \end{aligned}$$

   find how the following forms transform under CP:

$$\begin{aligned} \bar{\psi}_i\psi_j \ , \\ \bar{\psi}_i\gamma_5\psi_j \ , \\ \bar{\psi}_i A\!\!\!/\psi_j \ , \\ \bar{\psi}_i A\!\!\!/\gamma_5\psi_j \ . \end{aligned}$$

   HINT: for simplicity use the Weyl representation for the gammas, where $\gamma_2^* = -\gamma_2$, $\gamma_2^T = \gamma_2$ and $\gamma_0^T = \gamma_0$.

3. (1.5 P) Consider the decay of the neutral kaons in $\pi^0\pi^0$ and $\pi^+\pi^-$. Calculate the CP symmetry of the final state.

4. (1.5 P) Show that the electric charge e_0 in terms of g and g' is given by $e_0 = \frac{gg'}{\sqrt{g^2+g'^2}}$. For this, write down the covariant derivative $D_\mu = \partial_\mu - \mathrm{i}\frac{g}{2}W_\mu^a\sigma^a - \mathrm{i}g'Y_L B_\mu$ acting on the electron-neutrino doublet $(\nu_e, e^-)_L$ and look at the prefactor of the term $A_\mu e_L^-$.
   Remember that the hypercharge for this doublet has value $Y_L = -1/2$ and that the neutral fields W_μ^3 and B_μ in terms of Z_μ and A_μ read:

$$Z_\mu = \cos\theta_w W_\mu^3 - \sin\theta_w B_\mu \ , \quad A_\mu = \sin\theta_w W_\mu^3 + \cos\theta_w B_\mu \ ,$$

   with $\cos\theta_w = \frac{g}{\sqrt{g^2+g'^2}}$ and $\sin\theta_w = \frac{g'}{\sqrt{g^2+g'^2}}$.