# Introduction to the Standard Model
# Exercises 12

Deadline: Monday 11 July 2016 (12 am)
at Dr. Giudice's office (KP 301) and Ms Sonja Esch (KP 310)

**Topics covered**: Symmetry breaking, Higgs field, Glashow-Weinberg-Salam model.

1. (2.0 P) The usual $SU(2)_W \times U(1)_Y$ scalar potential for the standard model is of the form:

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2 \ ,$$

with

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \ .$$

(a) In principle, one can also have an $SU(2)_W \times U(1)_Y$ quartic invariant of the form:

$$V_1(\phi) = \lambda_1 \sum_a (\phi^\dagger \tau_a \phi)(\phi^\dagger \tau_a \phi) \ .$$

Show that this quartic term can be reduced to that in $V(\phi)$.

(b) Show that another quartic term:

$$V_2(\phi) = \lambda_2 \sum_{a,b} (\phi^\dagger \tau_a \tau_b \phi)(\phi^\dagger \tau_a \tau_b \phi)$$

is also reducible to that in $V(\phi)$.

Hint: you need to use the relation

$$\sum_a (\tau_a)_{ij} (\tau_a)_{kl} = 2\left(\delta_{jk}\delta_{il} - \frac{1}{2}\delta_{ij}\delta_{kl}\right) \ .$$

2. (1.0 P) Suppose that the fermion mass matrix in the basis of left-handed and right-handed fields is hermitian:

$$\mathcal{L} = \bar{\psi}^i_L M_{ij} \psi^j_R + h.c., \quad M^\dagger = M \ .$$

In general, the eigenvalues of $M$ obtained from unitary transformation are not always positive:

$$U M U^\dagger = M_d = \text{diag}(m_1, m_2, \ldots, m_n) \ ,$$

where $m_i$ can be negative as well as positive. Show that one can choose an appropriate biunitary transformation to diagonalise $M$ so that all diagonal elements are non-negative. (A biunitary transformation, of a matrix $A$, is given by $UAV^\dagger$ where both $U$ and $V$ are unitary).

3. (1.0 P) Explain the meaning of the upper labels of the components of the Higgs doublet:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} .$$

4. (1.0 P) Show that an arbitrary value of the Higgs field $\phi$ of length $|\phi| = v/2$ can be transformed to

$$\phi \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} .$$

by an $SU(2)$ transformation.

5. (5.0 P) Derive the mass term for the Higgs scalar $h(x)$ and the gauge fields $W^a_\mu(x)$ and $Z_\mu(x)$ from the Higgs Lagrangian assuming the Higgs mechanism:

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) + m^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 \ , \quad D_\mu = \partial_\mu - \mathrm{i}\frac{g}{2} W^a_\mu \sigma^a - \mathrm{i}\frac{g'}{2} B_\mu \ ,$$

where $\sigma^a$ are the Pauli matrices.
Define $v = \frac{m}{\sqrt{\lambda}}$, $\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$, $Z_\mu = \frac{g W^3_\mu - g' B_\mu}{\sqrt{g^2 + g'^2}}$, $A_\mu = \frac{g' W^3_\mu + g B_\mu}{\sqrt{g^2 + g'^2}}$.