Introduction to the Standard Model
Exercises 10

Deadline: Monday 27 June 2016 (12 am)
at Dr. Giudice's office (KP 301) and Ms Sonja Esch (KP 310)

Topics covered: Helicity, chirality, propagator massive boson, discrete symmetries.

1. (2.0 P) Show that a free massless neutrino can be described by a two-component spinor. Starting from the Dirac equation derive the two-component wave equations for a free massless neutrino (Weyl equations).

2. (1.0 P) Is the operator γ_5 a constant of motion for the free Dirac particle? Find the eigenvalues of this operator. Let us introduce:

$$\psi_L = \frac{1}{2}(1-\gamma_5)\psi \ , \quad \psi_R = \frac{1}{2}(1+\gamma_5)\psi \ ,$$

   where ψ is a Dirac spinor. Derive the equation of motion for these fields. Show that they are decoupled in the case of a a massless spinor. The fields ψ_R and ψ_L are known as Weyl fields.

3. (3.0 P) The Dirac spinor in momentum space can be written as,

$$u(p,\pm) = N_p \begin{pmatrix} 1 \\ \frac{\vec{\sigma}\vec{p}}{E+m} \end{pmatrix} \chi_\pm \ ,$$

   where $(\vec{\sigma}\hat{p})\chi_\pm = \pm\chi_\pm$ with $\hat{p} = \vec{p}/|\vec{p}|$. Show that the left-handed and right-handed spinors given by

$$u_L(p) = \frac{1}{2}(1-\gamma_5)u(p,-) \ , \quad u_R(p) = \frac{1}{2}(1+\gamma_5)u(p,+) \ ,$$

   are eigenstates of the helicity operator $\lambda = \vec{s}\hat{p}$ in the massless limit, where the spin operator is of the form

$$\vec{s} = \frac{1}{2}\begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} \ .$$

   Note that the same calculation should also show that the other two combinations

$$\frac{1}{2}(1+\gamma_5)u(p,-) \ , \quad \frac{1}{2}(1-\gamma_5)u(p,+) \ ,$$

   are identically zero in the same limit.

4. (2.0 P) The Lagrangian for a real vector boson field is given by

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{m^2}{2}B_\mu B^\nu \ ,$$

   where $G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. Calculate the propagator $D_{\mu\nu}(k)$ of the vector boson in momentum space.

5. (2.0 P) Under parity Dirac spinors transform as

$$\psi(t,\vec{x}) \to \psi'(t,-\vec{x}) = \gamma_0 \psi(t,\vec{x}) \ .$$

Time reversal is an antiunitary operation:

$$\psi(t,\vec{x}) \to \psi'(-t,\vec{x}) = T\psi^*(t,\vec{x}) \ .$$

The matrix T satisfies $T\gamma_\mu T^{-1} = (\gamma^\mu)^* = (\gamma_\mu)^T$ and $T^\dagger = T^{-1} = T = -T^*$.
Under charge conjugation spinors $\psi(x)$ transform as follows:

$$\psi(x) \to \psi_c(x) = C\bar{\psi}^T \ .$$

The matrix C satisfies the relations $C\gamma_\mu C^{-1} = -\gamma_\mu^T$, $C^{-1} = C^T = C^\dagger = -C$.

If $V^\mu(x) = \bar{\psi}(x)\gamma^\mu\psi(x)$ is a vector field, show that $V^\mu(x)$ is a real quantity. Find then the transformation properties of this quantity under parity P, charge conjugation C and time reversal T.

6. (0.5 P) Explain why the strong decay $\rho^0 \to \pi^-\pi^+$ is observed, but the strong decay $\rho^0 \to \pi^0\pi^0$ is not.