Introduction to the Standard Model
Exercises 1

Deadline: Monday 18 April 2016 (12 am)
at Dr. Giudice's office (KP 301) and Ms Sonja Esch (KP 310)

Topics covered: Problems to warm-up, quark model, Mandelstam variables, differential cross section.

1. (1 P) Express the mass of the proton, 1.6726×10^{-27}kg, in MeV.

2. (1 P) How large is the ratio of Coulomb force to gravitational force between proton and electron, if they are considered to be classical point particles?

3. (3 P) The pion-nucleon resonance Δ^{++} is made by three u-quarks and has spin 3/2. Therefore all three u-quarks must have spins aligned up (the orbital angular momentum is zero for the lowest state). The state is then given by:
$$|\Delta^{++}\rangle = |u\uparrow, u\uparrow, u\uparrow\rangle \ .$$
Explain why this assignment is not acceptable and how this problem is solved.

4. (5 P) In the quark model, mesons are made of a quark and an antiquark bound together. Let us consider only two flavours, up and down. We have four possible $\bar{q}q$ combinations divided in an SU(2) triplet and an isosinglet of meson. Determine in detail, using the ladder operators, the four $\bar{q}q$ bound-state wave functions. Note that, if we label the isospin doublet as (u, d) (fundamental representation of SU(2)), the antiparticle doublet is $(-\bar{d}, \bar{u})$ (conjugate representation of SU(2)). Moreover, sketch the proof in the case of three quarks and write down the wave functions of the meson octet and singlet.

5. (1 P) For the scattering process $AB \to CD$, show that
$$s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2 \ ,$$
where m_i is the rest mass of particle i.

6. (2 P) Taking $e^-e^+ \to e^-e^+$ to be the s channel process, verify that:
$$\begin{aligned} s &= 4(k^2 + m^2) \ , \\ t &= -2k^2(1 - \cos\theta) \ , \\ u &= -2k^2(1 + \cos\theta) \ , \end{aligned}$$

where θ is the center of mass scattering angle and $k = |\mathbf{k}_i| = |\mathbf{k}_f|$, where $\mathbf{k}_i$ and $\mathbf{k}_f$ are, respectively, the momenta of incident and scattered electrons in the center of mass frame. Show that the process is physically allowed provided that $s \geq 4m^2$, $t \leq 0$, and $u \leq 0$. Moreover, determine the value of t and u for forward and backward scattering.

7. (4 P) The differential cross section $d\sigma$ can be written as:

$$d\sigma = \frac{|\mathcal{M}|^2}{F} dQ \ ,$$

where dQ is the Lorentz invariant phase space factor:

$$dQ = (2\pi)^4 \delta^{(4)}(p_C + p_D - p_A - p_B) \frac{d^3 p_C}{(2\pi)^3 2E_C} \frac{d^3 p_D}{(2\pi)^3 2E_D} \ ,$$

and the incident flux is:

$$F = 4(|\mathbf{p}_A| E_B + |\mathbf{p}_B| E_A) \ .$$

In the center of mass frame for the process $AB \to CD$, show that:

$$dQ = \frac{1}{4\pi^2} \frac{p_f}{4\sqrt{s}} d\Omega \ ,$$
$$F = 4 p_i \sqrt{s} \ ,$$

and that therefore the differential cross section is:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{p_f}{p_i} |\mathcal{M}|^2 \ ,$$

where $d\Omega$ is the element of solid angle about $\mathbf{p}_C$, $s = (E_A + E_B)^2$, $\mathbf{p}_A = \mathbf{p}_B = p_i$ and $\mathbf{p}_C = \mathbf{p}_D = p_f$.