Chiral Perturbation Theory

Kai Walter

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QCD

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- Problem: sometimes no analytical prediction is possible even if the theory is accurate.

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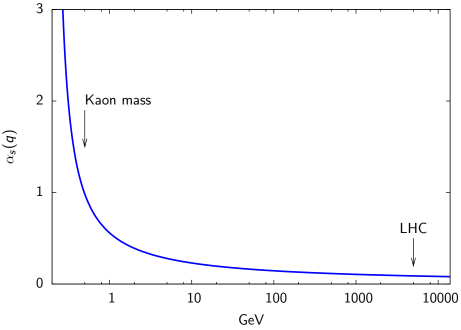

- At high energy → small coupling constant
  ⇒ perturbative regime
- At low energy $\alpha(q)_s > 1$
  ⇒ non-perturbativ regime
- Possibility to examine the non-perturbative regime:
  - Lattice QCD

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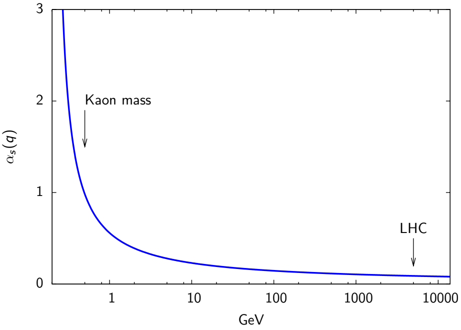

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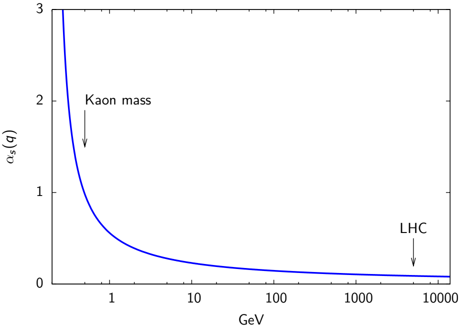

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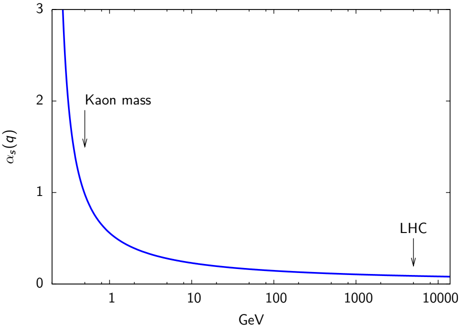

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  - **Effective Field Theory**

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Idea of EFT

Effective Field Theory is a low-energy approximation of a field theory

IDEA

We do not need quantum gravity to understand the hydrogen atom.

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Idea of EFT

Effective Field Theory is a low-energy approximation of a field theory

Idea

We do not need quantum gravity to understand the hydrogen atom. ⇒ Heavy degrees of freedom need not be included among the quantum fields of an EFT

Construct an effective Lagrangian so that heavy fields do not appear

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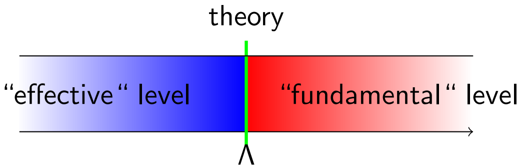
Classification of the EFT

1. Complete decoupling of heavy fields



The heavy fields with e.g. masses $> \Lambda$ are "integrated out" completely for $\ll \Lambda$: $\mathcal{L}$ depends only on light fields
Example: Fermi-Theory

EFT of type 2

- 2. Partial decoupling of the heavy fields:

Application: HQET

EFT of type 2

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  Heavy fields do not disappear completely from EFT but their high-momentum modes are integrated out

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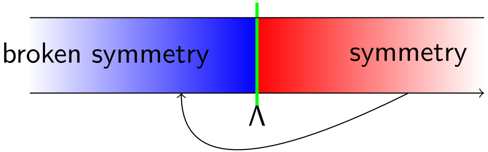
EFT OF TYPE 3

- 3. Spontaneous symmetry breaking:



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- Generally non-renormalizable
- Independent from specific physical realization (universality)

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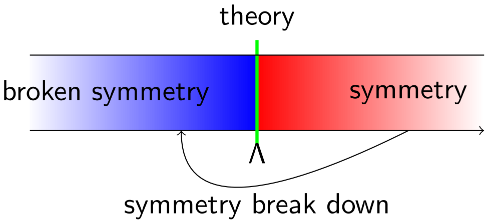

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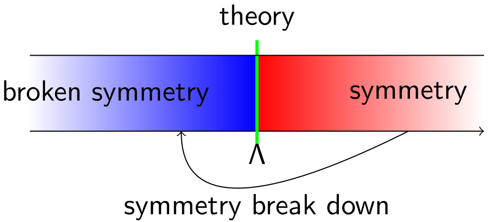

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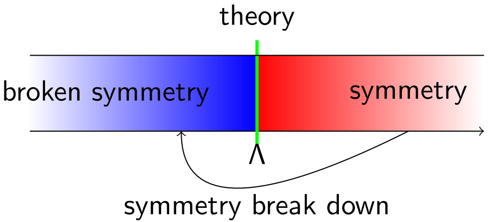

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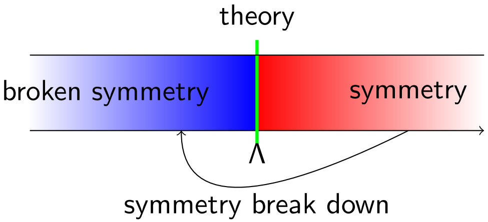

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Applications: In condensed matter physics
Electroweak symmetry break down
Chiral perturbation theory

Symmetries of QCD in light quark section

$$\mathcal{L}_{QCD} = \bar{q}_f (i\not{D} - m) q_f - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} \qquad f = 1,2,3$$

Symmetries:

- $SU(3)_c$ gauge symmetry
- $P-$, $C-$, $T-$-symmetry
- $SU(3)_F$ global symmetry (for $m_u = m_d = m_s$)
- Chiral symmetry (for $m \to 0$)

Def: (with P_L and P_R as the projection operators)

$$q_L := P_L q = \frac{1-\gamma_5}{2} q \qquad q_R := P_R q = \frac{1+\gamma_5}{2} q$$

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break chiral symmetry $\rightarrow \; -\bar{q}_{f,L}mq_{f,R} - \bar{q}_{f,R}mq_{f,L}$

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Chiral symmetry in QCD

- Continuous symmetry of massless QCD

$$SU(3)_L \times SU(3)_R \qquad \text{for } m \to 0$$

- 16 conserved currents for $m = 0$
  With the Definition of the vector and axialvector currents
  $V^{\mu,a} = \bar{q}\gamma^{\mu}\frac{\lambda_a}{2}q$, $A^{\mu,a} = \bar{q}\gamma^{\mu}\gamma_5\frac{\lambda_a}{2}q$

$$\partial_{\mu}V^{\mu,a} = i\bar{q}\left[M, \frac{\lambda_a}{2}\right]q \qquad \partial_{\mu}A^{\mu,a} = i\bar{q}\left\{M, \frac{\lambda_a}{2}\right\}\gamma_5 q$$

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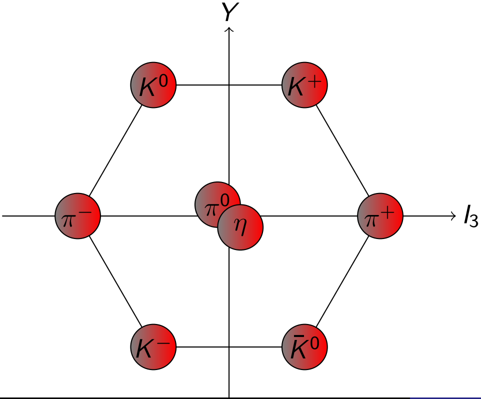

Chiral Pertubation Theory

S.Weinberg: *Physica A* ***96***,*327 (1979)*

.. if one writes down the *most general possible* Lagrangian, including *all* terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to *any given order of perturbation theory*, the result will simply be the most general possible S-matrix consistent with analyticality, perturbative unitarity, cluster decomposition and the assumed symmetriy principles. ..

- Poincar-invariant, C, P, T, isospin symmetry, chiral symmetry...
- Causality
- Conservation of the Properbility $(\sum_f |\langle f|S|i\rangle|^2 = 1)$
- local field theory

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Contruct a χPT

To construct an effective Lagrangian, we need

- The most general effective Lagrangian
- Counting scheme for perturbative description of a general Lagrangian
- Expression of Lagrangian via pion fields because degrees of freedom are no longer quarks.
- Invariance of Lagrangian under the chiral transformation for $m \longrightarrow 0$
- Mass terms that break the chiral symmetry

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Perturbativ parameter

We need a small parameter to construct a perturbative theory

momentum, mass!

the most general effective Lagrangian in momentum dimension:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \dots$$

There are no terms $\mathcal{L}_{2n+1}$ because of the Lorentz-invariance.

$$\left.\begin{array}{c} g^{\mu\nu} p_\mu p_\nu \\ \epsilon^{\mu\nu\rho\sigma} p_\mu p_\nu p_\rho p_\sigma \end{array}\right\} \text{scalar} \quad \left.\begin{array}{c} g^{\mu\nu} p_\mu p_\nu p_\rho \\ \epsilon^{\mu\nu\rho\sigma} p_\mu p_\nu p_\rho p_\sigma p_\tau \end{array}\right\} \text{vector} \dots$$

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Weinberg's counting scheme

Analysis of a given diagram under a linear rescaling of all external momenta $p_i \mapsto t \cdot p_i$

$$\Rightarrow M(tp_i) = t^D \cdot M(p_i)$$

$$\text{with } D = 2 + \sum_{n=1}^{\infty} 2(n-1)N_{2n} + 2N_L$$

N_{2n} : Number of vertices in the order $\mathcal{O}(p^{2n})$
N_L: Number of loops

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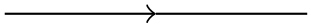
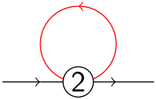
Diagram sortation via power counting scheme

$D = 2,\ \mathcal{O}(p^2)$ — $N_{2n} = 0,\ N_L = 0$



free propagator

$D = 4,\ \mathcal{O}(p^4)$ — $N_2 = 1,\ N_L = 1$



one loop
one vertex $\mathcal{O}(p^2)$

$N_4 = 1,\ N_L = 0$

no loop
one vertex $\mathcal{O}(p^4)$

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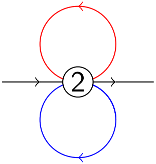
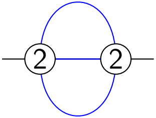
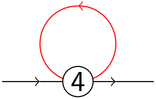
Diagram sortation via power counting scheme



$D = 6$ $N_2 = 1$ $N_L = 2$ two loops
one vertex $\mathcal{O}(p^2)$



$N_2 = 2$ $N_L = 2$ two loops
two vertices $\mathcal{O}(p^2)$



$N_4 = 1$ $N_L = 1$ one loop,
one vertex$\mathcal{O}(p^4)$

etc ...

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- Expression of Lagrangian via pion fields because degrees of freedom are no more quarks.
- Invariance of Lagrangian under the chiral transformation for $m \to 0$
- Mass terms, that break the chiral symmetry

- Expression of Lagrangian with the meson-fields $\phi_a = \begin{pmatrix} \pi^+ \\ \vdots \\ \eta \end{pmatrix}$

  Problem: transformation matrix V_L and $V_R \in SU(3)$
- Solution: parametrize the fields with Gell-Mann-matrices

$$\phi(x) = \sum_{a=1}^{8} \lambda_a \phi_a(x) = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

- Transformation of $\phi(x)$? $\rightarrow$ nonlinear!
- Solution: Definition of a new field matrix $U(x)$

$$U(x) = \exp\left(i\frac{\phi(x)}{F_0}\right)$$

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$$U'(x) = V_R U(x) V_L^\dagger$$

linear!

This is a standard procedure for implementing a symmetry transformation on Goldstone fields.
Further reading: (Coleman *Phys.Rev.177:2239-2247,1969*)
Further constituents in Lagrangian:

- $D_\mu U(x)$: Derivative of fields
- $f^R_{\mu\nu}$, $f^L_{\mu\nu}$: Field strength tensor of external fields
- $\chi = 2B \cdot \mathcal{M}$: Mass matrix with parameter B

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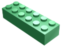
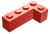
Lego-Bricks of chiral perturbation theory

-  $U(x) = \mathcal{O}(p^0)$
-  $D_\mu U(x) = \mathcal{O}(p)$
-  $f^L_{\mu\nu} = \mathcal{O}(p^2)$
-  $f^R_{\mu\nu} = \mathcal{O}(p^2)$
-  $2B \cdot M = \mathcal{O}(p^2)$

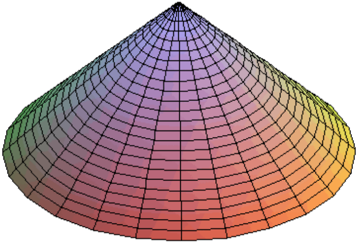
Contruct a χPT

To construct an effective Lagrangian, we need

- The most general effective Lagrangian
- Counting scheme for perturbative description of a general Lagrangian
- Expression of Lagrangian via pion fields because degrees of freedom are no longer quarks.
- Invariance of Lagrangian under the chiral transformation for $m \to 0$
- Mass terms, that break the chiral symmetry

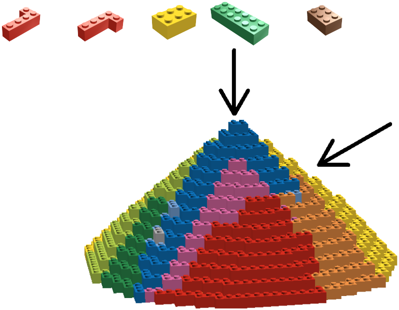
we have lego-bricks

and we know how the theory looks like (Symmetries)



Construction of an effective Lagrangian

we have lego-bricks



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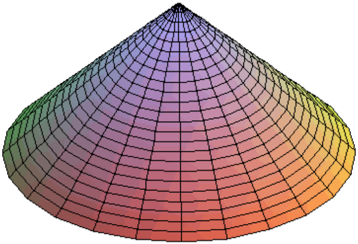

approximate!

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Transformation of Lego-Bricks

Transformation of the pion-fields $U(x)$ and $D_\mu U(x)$:

$$U'(x) = V_R U(x) V_L^\dagger \qquad [D_\mu U(x)]' = V_R [D_\mu U(x)] V_L^\dagger,$$

of the external field

$$(f_{\mu\nu}^R)' = V_R f_{\mu\nu}^R V_R^\dagger, \qquad (f_{\mu\nu}^L)' = V_L f_{\mu\nu}^L V_L^\dagger,$$

and of the mass matrix χ with spurion analysis:

$$\chi' = V_R \chi V_L^\dagger$$

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Transformation of lego-bricks

element	G	C	P	order
$U(x)$	$V_R U(x) V_L^\dagger$	U^T	$U^\dagger$	$\mathcal{O}(p^0)$
$D_\mu U(x)$	$V_R [D_\mu U(x)] V_L^\dagger$	$(D_\mu U)^T$	$(D_\mu U)^\dagger$	$\mathcal{O}(p^1)$
χ	$V_R \chi V_L^\dagger$	χ^T	$\chi^\dagger$	$\mathcal{O}(p^2)$
$f^R_{\mu\nu}$	$V_R f^R_{\mu\nu} V_R^\dagger$	$-(f^L_{\mu\nu})^T$	$f_L^{\mu\nu}$	$\mathcal{O}(p^2)$
$f^L_{\mu\nu}$	$V_L f^L_{\mu\nu} V_L^\dagger$	$-(f^R_{\mu\nu})^T$	$f_R^{\mu\nu}$	$\mathcal{O}(p^2)$

- $\mathrm{Tr}(UU^\dagger)$, $\mathrm{Tr}(UU^\dagger)\mathrm{Tr}(UU^\dagger)\ldots,$
- $\mathrm{Tr}(D_\mu U(D^\mu U)^\dagger), \mathrm{Tr}(D_\mu U(D^\mu U)^\dagger)\mathrm{Tr}(D_\nu U(D^\nu U)^\dagger)\ldots$
- $\mathrm{Tr}(\chi U^\dagger)$, $\mathrm{Tr}(U\chi^\dagger), \ldots$
- $\mathrm{Tr}(f^R_{\mu\nu} f_R^{\mu\nu})$, $\mathrm{Tr}(U f^L_{\mu\nu} U^\dagger f_R^{\mu\nu})$

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$f_{\mu\nu}^R$	$V_R f_{\mu\nu}^R V_R^\dagger$	$-(f_{\mu\nu}^L)^T$	$f_L^{\mu\nu}$	$\mathcal{O}(p^2)$
$f_{\mu\nu}^L$	$V_L f_{\mu\nu}^L V_L^\dagger$	$-(f_{\mu\nu}^R)^T$	$f_R^{\mu\nu}$	$\mathcal{O}(p^2)$

Possible combination

- $\mathrm{Tr}(UU^\dagger)$, $\mathrm{Tr}(UU^\dagger)\mathrm{Tr}(UU^\dagger)\ldots,$
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Lowest order chiral Lagrangian

$\mathcal{O}(p^2)$

$$\mathrm{Tr}(D_\mu U (D^\mu U)^\dagger)$$
$$\mathrm{Tr}(\chi U^\dagger),\ \mathrm{Tr}(U\chi^\dagger)$$
$$\mathrm{Tr}(U f^L_{\mu\nu} U^\dagger) = \mathrm{Tr}(f^L_{\mu\nu}) = 0$$

the chiral Lagrangian at lowest order

$$\mathcal{L}_2 = \frac{F_0^2}{4}\mathrm{Tr}[D_\mu U (D^\mu U)^\dagger] + \frac{F_0^2}{4}\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)$$
$$= \frac{F_0^2}{4}\mathrm{Tr}[D_\mu U (D^\mu U)^\dagger] + \frac{F_0^2 B_0}{2}\mathrm{Tr}(\mathcal{M} U^\dagger + U\mathcal{M}^\dagger)$$

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The chiral Lagrangian at NLO

$\mathcal{O}(p^4)$: $\quad \mathrm{Tr}(D_\mu U (D_\nu U)^\dagger)\mathrm{Tr}(D^\mu U (D^\nu U)^\dagger), \mathrm{Tr}(f^R_{\mu\nu} f_R^{\mu\nu}), \mathrm{Tr}(U f^L_{\mu\nu} U^\dagger f_R^{\mu\nu})$

and so on ...

Chiral Lagrangian at NLO

$$
\begin{aligned}
\mathcal{L}_4 = \quad & L_1 \left\{ \mathrm{Tr}[D_\mu U (D^\mu U)^\dagger] \right\}^2 + L_2 \mathrm{Tr}[D_\mu U (D_\nu U)^\dagger] \mathrm{Tr}[D^\mu U (D^\nu U)^\dagger] \\
+ \quad & L_3 \mathrm{Tr}[D_\mu U (D^\mu U)^\dagger D_\nu U (D^\nu U)^\dagger] + L_4 \mathrm{Tr}[D_\mu U (D^\mu U)^\dagger] \mathrm{Tr}(\chi U^\dagger + U \chi^\dagger) \\
+ \quad & L_5 \mathrm{Tr}[D_\mu U (D^\mu U)^\dagger (\chi U^\dagger + U \chi^\dagger)] + L_6 \left[\mathrm{Tr}(\chi U^\dagger + U \chi^\dagger) \right]^2 \\
+ \quad & L_7 \left[\mathrm{Tr}(\chi U^\dagger - U \chi^\dagger) \right]^2 + L_8 \mathrm{Tr}(U \chi^\dagger U \chi^\dagger + \chi U^\dagger \chi U^\dagger) \\
- \quad & i L_9 \mathrm{Tr}[f^R_{\mu\nu} D^\mu U (D^\nu U)^\dagger + f^L_{\mu\nu} (D^\mu U)^\dagger D^\nu U] + L_{10} \mathrm{Tr}(U f^L_{\mu\nu} U^\dagger f_R^{\mu\nu}) \\
+ \quad & H_1 \mathrm{Tr}[f^R_{\mu\nu} f_R^{\mu\nu} + f^L_{\mu\nu} f_L^{\mu\nu}] + H_2 \mathrm{Tr}[\chi \chi^\dagger]
\end{aligned}
$$

The chiral Lagrangian at NLO

$\mathcal{O}(p^4)$: $\mathrm{Tr}(D_\mu U(D_\nu U)^\dagger)\mathrm{Tr}(D^\mu U(D^\nu U)^\dagger), \mathrm{Tr}(f^R_{\mu\nu} f^{\mu\nu}_R), \mathrm{Tr}(U f^L_{\mu\nu} U^\dagger f^{\mu\nu}_R)$

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Chiral Lagrangian at NLO

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- \; & iL_9 \mathrm{Tr}[f^R_{\mu\nu} D^\mu U(D^\nu U)^\dagger + f^L_{\mu\nu}(D^\mu U)^\dagger D^\nu U] + L_{10} \mathrm{Tr}(U f^L_{\mu\nu} U^\dagger f^{\mu\nu}_R) \\
+ \; & H_1 \mathrm{Tr}[f^R_{\mu\nu} f^{\mu\nu}_R + f^L_{\mu\nu} f^{\mu\nu}_L] + H_2 \mathrm{Tr}[\chi\chi^\dagger]
\end{aligned}$$

Chiral Lagrangian at NNLO

Chiral Lagrangian at NNLO

Chiral Lagrangian at NNLO

Chiral Lagrangian at NNLO

too many terms!

NUMBER OF THE PARAMETER IN χPT

Order	2 flavour		3 flavour	
	parameter	number	parameter	number
p^2	F, B	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2
p^6	c_i^r	52+4	C_i^r	90+4
...				

there are two types of terms:

- Terms with low energy constant (LECs)
- Contact terms
  contain no pion-fields $\Rightarrow$ no physical relevance
  required for the renormalization of the loop-contribution

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Pion masses at LO

Lagrangian at leading order:

$$\mathcal{L}_2 = \frac{F_0^2}{4}\mathrm{Tr}[\partial_\mu U(\partial^\mu U)^\dagger] + \frac{F_0^2 B_0}{2}\mathrm{Tr}(\mathcal{M}U^\dagger + U\mathcal{M}^\dagger)$$

$m = m_u = m_d$: mass of the up- and down- quark
m_s: mass of the strange-quark
expansion of $U(x)$ in $\mathcal{O}(\phi^2)$

$$U(x) = \exp\left(i\frac{\phi}{F_0}\right) = 1 + i\frac{\phi}{F_0} - \frac{\phi^2}{2F_0^2}\dots \qquad \text{with } \phi = \lambda_a\phi_a$$

$$\Rightarrow \mathcal{L}_2 \quad =$$

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Lagrangian at leading order:

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$$\Rightarrow \mathcal{L}_2 \quad = \quad \frac{1}{2}(\partial_\mu\phi_a)(\partial^\mu\phi_a)$$

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Pion masses at LO

Lagrangian at leading order:

$$\mathcal{L}_2 = \frac{F_0^2}{4}\mathrm{Tr}[\partial_\mu U(\partial^\mu U)^\dagger] + \frac{F_0^2 B_0}{2}\mathrm{Tr}(\mathcal{M}U^\dagger + U\mathcal{M}^\dagger)$$

$m = m_u = m_d$: mass of the up- and down- quark
m_s: mass of the strange-quark
expansion of $U(x)$ in $\mathcal{O}(\phi^2)$

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$$\begin{aligned}\Rightarrow \mathcal{L}_2 \quad = \quad & \frac{1}{2}(\partial_\mu\phi_a)(\partial^\mu\phi_a) \\ & - mB_0\left(2\pi^+\pi^- + \pi^0\pi^0\right) - (m+m_s)B_0K^+K^- \\ & - (m+m_s)B_0K^0\bar{K}^0 - \frac{1}{3}(m+2m_s)B_0\eta^2\end{aligned}$$

Pion masses at LO

$$M_\pi^2 = 2B_0 m \qquad M_K^2 = B_0(m + m_s) \qquad M_\eta^2 = \frac{2}{3}B_0(m + 2m_s)$$

These masses satisfy the Gell-Mann-Okubo relation:

$$4M_K^2 = 3M_\eta^2 + M_\pi^2 \qquad \checkmark$$

Theoretical prediction of the quark masses is not possible because of B_0, but the ratio:

$$\frac{M_K^2}{M_\pi^2} = \frac{m+m_s}{2m} \Rightarrow \frac{m_s}{m} = 25.9 \qquad \frac{M_\eta^2}{M_\pi^2} = \frac{2m_s+m}{3m} \Rightarrow \frac{m_s}{m} = 24,3$$

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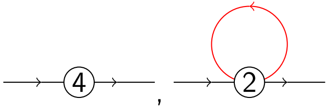
Pion masses at NLO



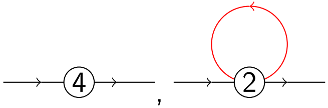
$$\mathcal{L}_{D=4} = \mathcal{L}_4^{2\phi} + \mathcal{L}_2^{4\phi}$$

Mass at NLO: $M^2 = M_0^2 + \Sigma(p^2)$

- Contact contribution:
  taylor expansion of chiral Lagrangian $\mathcal{L}_4$ in $\mathcal{O}(\phi^2)$

$$\Sigma_c = a_\phi + b_\phi p^2$$

Pion masses at NLO



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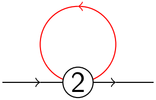

- Loop contribution:
  Taylor expansion of $\mathcal{L}_2$ in $\mathcal{O}(\phi^4)$ + loop integral of the form:

$$\int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - M^2}$$

Renormalization with the method of dimensional regularization + redefinition of the low-energy constants

$$\Sigma(p^2) = A_\phi + B_\phi p^2$$

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Pion masses at NLO

Pion mass at NLO

$$M^2_{\pi,4} = M^2_{\pi,2} \left\{ 1 + \frac{M^2_{\pi,2}}{32\pi^2 F^2} \ln\left(\frac{M^2_{\pi,2}}{\mu^2}\right) - \frac{M^2_{\eta,2}}{96\pi^2 F^2} \ln\left(\frac{M^2_{\eta,2}}{\mu^2}\right) \frac{16}{F^2}\left[(2m + m_s)B(2L^r_6 - L^r_4) + mB(2L^r_8 - L^r_5)\right] \right\}$$

μ: renormalization scale, $m_q^2 \ln(m_q)$: chiral logarithm

μ is not a new parameter!!

$$\frac{dM^2_{\pi,4}}{d\mu} = 0$$

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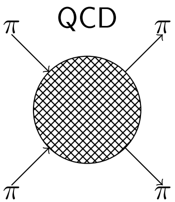
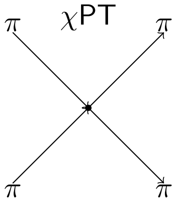
Application 2: π-π-scattering at LO





loops, gluon propagator . . . including nonperturbative interactions

described by a contact interaction
easy to calculate the scattering amplitude

Lagrangian at LO

$$\mathcal{L}_2 = \frac{F^2}{4}\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{F^2}{4}\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)$$

$$U(x) = \exp\left(i\frac{\phi(x)}{F}\right), \qquad \phi = \sum_{a=1}^{3} \tau_a \phi_a = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

Lagrangian must contain four fields: Expand $\mathcal{L}_2$ in $\mathcal{O}(\phi^4)$

$$\Rightarrow \mathcal{L}_2^{4\phi} = \frac{1}{6F^2}\left(\vec{\phi}\cdot\partial_\mu\vec{\phi}\cdot\vec{\phi}\cdot\partial^\mu\vec{\phi} - \vec{\phi}^2\cdot\partial_\mu\vec{\phi}\cdot\partial^\mu\vec{\phi}\right) + \frac{M_\pi^2}{24F^2}(\vec{\phi}^2)^2$$

SCATTERING AMPLITUDE

$$\mathcal{L}_2^{4\phi} = \frac{1}{6F^2}\left(\delta_{ab}\delta_{cd}\phi_a\partial_\mu\phi_b\phi_c\partial^\mu\phi^d - \delta_{ab}\delta_{cd}\phi_a\phi_b\partial_\mu\phi_c\partial^\mu\phi_d\right) + \frac{M_\pi^2}{24F^2}\delta_{ab}\delta_{cd}\phi_a\phi_b\phi_c\phi_d$$

to calculate the scattering amplitude

1. Fourier trasformation ($\partial_\mu\phi_i \rightarrow ip_i\phi_i$):

$$-\delta_{ab}\delta_{cd}p_b p_d\phi_a\phi_b\phi_c\phi_d$$

2. Symmetrize the vertex:

$$\frac{1}{12}\left[(p_a+p_b)(p_c+p_d)\delta_{ab}\delta_{cd} + (p_a+p_c)(p_b+p_d)\delta_{ac}\delta_{bd} + (p_a+p_d)(p_b+p_c)\delta_{ad}\delta_{bc}\right]\phi_a\phi_b\phi_c\phi_d$$

$$\Rightarrow \mathcal{M} = i\left[\delta_{ab}\delta_{cd}\frac{s-M_\pi^2}{F^2} + \delta_{ac}\delta_{bd}\frac{t-M_\pi^2}{F^2} + \delta_{ad}\delta_{bc}\frac{u-M_\pi^2}{F^2}\right]$$

for on-shell pion

SCATTERING AMPLITUDE

$$\mathcal{L}_2^{4\phi} = \frac{1}{6F^2}\left(\delta_{ab}\delta_{cd}\phi_a\partial_\mu\phi_b\phi_c\partial^\mu\phi^d - \delta_{ab}\delta_{cd}\phi_a\phi_b\partial_\mu\phi_c\partial^\mu\phi_d\right) + \frac{M_\pi^2}{24F^2}\delta_{ab}\delta_{cd}\phi_a\phi_b\phi_c\phi_d$$

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SCATTERING AMPLITUDE

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1. Fourier trasformation ($\partial_\mu\phi_i \longrightarrow ip_i\phi_i$):

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for on-shell pion

SCATTERING LENGTH

Prediction of the s-wave scattering length:

$$a_0^0 = \frac{7M_\pi^2}{32\pi F^2} = 0.156, \qquad a_0^2 = -\frac{M_\pi^2}{16\pi F^2} = -0.045$$

For $F = 93.2$ MeV and $M_\pi = 139.57$ MeV
Calculation until order p^6 done by Bijnens et al.

$$a_0^0 = \underbrace{0.156}_{\mathcal{O}(p^2)} + \underbrace{0.044}_{\mathcal{O}(p^4)} + \underbrace{0.017}_{\mathcal{O}(p^6)} = 0.217$$

data from experiment (M. Kermani et al. [CHAOS Collaboration])

$$a_0^0 = 0.216 \pm 0.013(\text{stat}) \pm 0.008(\text{syst})$$

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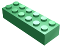
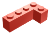
Conclusion

- χPT is a EFT with spontaneous breakdown of chiral symmetry.
- Low-energy QCD described by the nonlinear realization of the pion-fields $U(x) = \exp\left(i\frac{\lambda_a \phi_a}{F^2}\right)$.
- There are some lego bricks of the χPT.

 $U(x)$  $D_\mu U(x)$  $f^L_{\mu\nu}$  $f^R_{\mu\nu}$  $2B \cdot M$

- Weinberg's power counting scheme for sortation of Feynman diagrams.
- Advantages:
  - Analytical examination of the non-perturbative regime of QCD
  - No complicated calculations
  - Universality of Lagrangian
- Disadvantage
  - A lot of low-energy constants

Thank you for your attention!