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Confined Subspaces: a Schematic Model [1]

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Abstract. An essential feature of confining systems is the existence of channels into which probability flux can enter only for short spacetime intervals, but where it can never, irrespective of the total energy involved, propagate to asymptotic distances. It is demonstrated in a schematic quantum-mechanical model that this situation, which is unknown in ordinary scattering theory, does arise in a representation obtained through a non-unitary similarity transformation, and therefore involving a nontrivial metric. In the "confined" channels of this representation, propagation is described by Green's functions having a complex conjugate pair of poles in momentum space instead of the usual real-axis pole. On the asymptotically accessible subspace, the S matrix is unitary, but finite-time unitarity for the total system becomes visible only when using the nontrivial metric. It is suggested that field-theory propagators with complex-conjugate pairs of singularities as obtained in several QCD studies, which seem to contradict finite-time unitarity, can in fact be imbedded into a unitary framework along the line illustrated by this model. A further conjecture is that interacting QCD fields differing from the perturbative ones by a non-unitary similarity transformation form an essential ingredient for a theory of confinement.

[1]Dedicated to my colleague Professor Achim Weiguny on the occasion of his retirement.

1 Confined and Accessible Subspaces

If one reflects on the observational basis of our notion of confinement in strong interactions, one realizes that prior to any detailed interpretation in terms of bags, flux tubes, infinitely rising potentials or other (essentially classical) pictures, confinement is first and foremost a statement about asymptotic states and the hadronic S matrix. This point has been emphasized e.g. by Seiler [1]. The confining field system has no continuum states describing the appearance of single quarks or gluons at asymptotic, i.e. macroscopic, space-time distances from a reaction event. However there is another statement whose empirical basis nowadays is just about as solid: one has convincing evidence for the existence of intermediate states in which a $q - \bar{q}$ or $q - \bar{q}-$plus-gluons configuration, coupled to singlet total color, propagates "almost freely" for short space-time intervals of order $1/\Lambda$ with a "confinement scale" Λ – this is the now universally accepted interpretation of jet events. Confining systems are therefore also distinguished by the existence of channels, or subspaces of their state space, with the peculiar property that the component of the total state vector in such a subspace, and the associated share of total probability, always decay quickly at asymptotic distances: after a brief interval $\sim 1/\Lambda$, the state vector gets rotated back into the asymptotically accessible subspace, where states describe the asymptotic appearance of individually colorless fragments in free relative motion. Particle configurations corresponding to such "confined subspaces" then can neither be prepared nor detected by devices at macroscopic distances. Nevertheless these short-lived states can by no means be dismissed as "unphysical" just because they are asymptotically inaccessible; in jet experiments they are entirely physical if transitory configurations whose constituents in fact grow more particle-like with increasing initial energy, permitting experimental determinations of their energies and spins. They just display the unusual feature of being "flux-repellent": their special dynamics forces any probability flux that may have entered back into the asymptotically accessible channels.

Ordinary scattering theory has known such a flux pattern in the case of *energetically closed channels*; they too can be populated only "virtually", i.e. at non-asymptotic times. However in scattering theory there is always a finite threshold energy beyond which a given channel "opens up" and starts propagating flux to macroscopic distances. By contrast, a confined channel has no threshold and never opens up in this sense. It is then of interest to inquire about how such a peculiar behavior can be imbedded into a normal unitary scattering theory.

The present paper does not approach this question in the full field-theory context of QCD; it is restricted to a study of a non-relativistic, quantum-mechanical, coupled-channel model. However, while in a quantum-mechanical system with finite number of degrees of freedom the confinement phenomenon must always be imposed by hand, we try to avoid doing this too directly through infinitely rising potentials or other classical mechanisms, if only because the status of all such mechanisms in fundamental QCD must presently be characterized as unsettled. Instead we demonstrate that the pattern of confined-vs.-accessible subspaces can and does emerge when a "normal" coupled-channel system is subjected to a suitable non-unitary similarity transformation,

$$| \Psi \rangle \longrightarrow | \Psi'' \rangle = T \; | \Psi \rangle \; , \tag{1.1}$$

$$\mathcal{O} \;\longrightarrow\; \mathcal{O}'' = T \, \mathcal{O} \, T^{-1} \; , \tag{1.2}$$

of its states $| \Psi \rangle$ and observables $\mathcal{O}$ respectively, where the operator

$$T \;=\; e^{iB} \, e^{A} \; , \tag{1.3}$$

$$A \;=\; A^{\dagger} \qquad , \qquad B \;=\; B^{\dagger} \; , \tag{1.4}$$

consists of an Hermitian but non-unitary factor $\exp(A)$ followed by a unitary factor $\exp(iB)$, both mixing the channels of the original system. Since

$$\langle \Psi_1 \mid \Psi_2 \rangle \;=\; \langle T^{-1} \Psi_1'' \mid T^{-1} \Psi_2'' \mid \rangle \;=\; \langle \Psi_1'' \mid (T T^{\dagger})^{-1} \, \Psi_2'' \rangle \;=\; \langle \Psi_1'' \mid M \, \Psi_2'' \rangle \; , \tag{1.5}$$

the new representation calls for the use of a nontrivial metric operator,

$$M \;=\; (T T^{\dagger})^{-1} \;=\; e^{iB} \, e^{-2A} \, e^{-iB} \; , \tag{1.6}$$

in the calculation of probability amplitudes. The imposition of confinement then takes a considerably more indirect form, which nevertheless may be suggestive of what is going on in the "true" theory: it proceeds by *postulating* that it is the channels of the transformed system, rather than of the original one, that define the asymptotic preparation-and-detection possibilities realized in nature. Quantum mechanics cannot produce, from within its own theoretical framework, cogent reasons why only this identification should be viable. However, as we shall discuss, there is a serious possibility that quantum field theory may do better in this respect and produce an unambiguous dynamical preference.

In Sect. 2, we describe the setup of the quantum-mechanical model. In Sect. 3, we introduce the two-step similarity transform (1.1/1.2), evaluate the transformed Hamiltonian according to Eq. (1.2), and discuss the metric operator (1.6) to be used with the transformed representation. In Sect. 4 we verify, by studying the effective Hamiltonians for subspaces, that the transformed system indeed possesses a confined and an asymptotically accessible subspace. An interesting result here will be that propagation within the confined subspace is described by Green's functions with a complex-conjugate pair of poles in momentum representation, rather than the single real-axis pole that normally describes free propagation: QCD propagators exhibiting complex singularities have been obtained in several studies from different starting points [2], [3], [4], [5], [6], [7], and, insofar as they produce spatio-temporal propagation limited to an inverse confinement scale, may be interpreted as providing a description of confinement not based on classical mechanisms. They have so far been viewed with mistrust mainly because they appear to violate finite-time unitarity, or in other words, Hermiticity of the Hamiltonian, which would seem to restrict Green's-function singularities to the spectrum of the latter, and therefore to the real axis. The present model points to a way of removing this obstacle, by suggesting that such propagators are legitimate in the framework of a theory with a nontrivial metric. This aspect, along with further discussion and conjecture on the relation with field theory, will be the subject of Sect. 5.

2 Definition of the Model

The model we study is an ordinary non-relativistic two-particle system, with relative-coordinate and relative-momentum operators $\mathbf{X}$ and $\mathbf{P}$, which can exist in three orthonormal eigenstates of an internal degree of freedom, so its state space has a three-channel structure. The internal triplet has nothing to do with the color-$SU(3)$ degree of freedom of QCD; nevertheless, as a convenient device for handling the purely algebraic aspects of a three-channel problem, we declare the three intrinsic states to be eigenstates

$$| \; \frac{1}{2}; +\frac{1}{2} \; \rangle = | \, s \, \rangle, \qquad | \; \frac{1}{2}; -\frac{1}{2} \; \rangle = | \, c \, \rangle, \qquad | \; 0; \; 0 \; \rangle = | \, 0 \, \rangle \tag{2.1}$$

of the operators $\mathbf{\Sigma}^2$ and Σ_3 for a purely formal $SU(2)$ degree of freedom called sectospin. The corresponding orthogonal projectors are denoted Π_s, Π_c, Π_0 so that

$$\Pi_s \; + \; \Pi_c \; + \; \Pi_0 \; = \; \mathbf{1}_\Sigma \, , \tag{2.2}$$

the identity in sectospin space. The singlet and doublet together may be viewed as spanning a triplet-representation space of secto-$SU(3)$, on which linear combinations of the nine dyadics $\mid i\rangle\langle j \mid$ act as a set of $U(3)$ generators. We emphasize that the dynamics studied below will have no $SU(2)$ or $U(3)$ symmetry. The sectospin assignments are merely a formal way of implementing orthogonality of three subspaces; their main role will be to facilitate use of the standard tabulations of $SU(3)$ commutation and anti-commutation relations, and for this purpose we shall repeatedly mention identifications of our channel-coupling operators with triplet-representation $SU(3)$ generators in the standard Gell-Mann convention.

A general state of this system takes the form

$$\mid \Psi(t) \rangle \;= \mid \psi_s(t) \rangle \otimes \mid s \rangle + \mid \psi_c(t) \rangle \otimes \mid c \rangle + \mid \psi_0(t) \rangle \otimes \mid 0 \rangle, \qquad (2.3)$$

where the $\mid \psi_i \rangle$ are orbital-motion states, and scalar products are formed according to

$$\langle \Phi \mid \Psi \rangle \;=\; \langle \phi_s \mid \psi_s \rangle \;+\; \langle \phi_c \mid \psi_c \rangle \;+\; \langle \phi_0 \mid \psi_0 \rangle \,. \qquad (2.4)$$

The Hamiltonian acting on this space has the form

$$H \;=\; K(\mathbf{P})\,\mathbf{1}_\Sigma \;+\; V(\mathbf{X}) \Big(\mid 0 \rangle\langle c \mid + \mid c \rangle\langle 0 \mid \Big)\,, \qquad (2.5)$$

where $K(\mathbf{P}) = \frac{1}{m}\mathbf{P}^2$ is a non-relativistic kinetic energy for equal masses, counted from a threshold energy normalized to zero, while $V(\mathbf{X})$ is the operator for a transition potential,

$$V(\mathbf{X}) \;=\; \frac{\beta}{R}\,, \qquad\qquad R \;=\; |\mathbf{X}|\,, \qquad (2.6)$$

with a coupling constant β of dimension energy $\times$ length. (The sectospin operator in Eq. (2.5) is the generator $2F_6$ of standard $SU(3)$ convention). The Hamiltonian (2.5) is obviously Hermitian with respect to the scalar product (2.4).

Since the $\mid s \rangle$ channel fails to appear in the interaction term of Eq. (2.5), its dynamical role, for the time being, is a trivial one: the stationary orbital amplitudes $\mid \tilde{\psi}(E) \rangle$ defined by

$$\mid \psi_n(t) \rangle = \int_{-\infty}^{+\infty} \frac{dE}{2\pi\hbar}\, \exp(-i\frac{E}{\hbar}t) \;\mid \tilde{\psi}_n(E) \rangle\,, \qquad (n = s, c, 0) \qquad (2.7)$$

obey the coupled-channel equations

$$\begin{pmatrix} K - E & 0 & 0 \\ 0 & K - E & V(\mathbf{X}) \\ 0 & V(\mathbf{X}) & K - E \end{pmatrix} \begin{pmatrix} \mid \tilde{\psi}_s(E) \rangle \\ \mid \tilde{\psi}_c(E) \rangle \\ \mid \tilde{\psi}_0(E) \rangle \end{pmatrix} \;=\; 0\;, \qquad (2.8)$$

in which $|\,\tilde{\psi}_s\,\rangle$ decouples and either evolves freely or, unless fed by the initial conditions, remains zero at all times. It will nevertheless play an important role, as an auxiliary degree of freedom, in the transformed system. We will refer to $|\,s\,\rangle$ as a *shadow* state or channel; its interpretation will become clearer when discussing the relation with the field-theory case in Sect. 5.

The system defined by Eqs. (2.3 / 2.4) and Eq. (2.8) may be viewed as a highly schematized model of a $B+\bar{B}$ heavy-mesons channel (sector $|\,0\,\rangle$) coupled to (the color-singlet component of) the heavy-quark-antiquark channel $b+\bar{b}$ (sector $|\,c\rangle$). Since the two systems perturbatively have very nearly the same absolute thresholds ($\approx 2m_b$) and reduced masses ($\approx \frac{1}{2}m_b$), use of a single kinetic energy K in Eq. (2.5) is appropriate. Also, the leading perturbative process driving the $B+\bar{B} \longleftrightarrow b+\bar{b}$ transition is single exchange of the light u quark, which perturbatively is almost massless on the scale of m_b, so that a Coulomb-like transition potential is reasonable. For simplicity, we abstain from modelling the short-range, van-der-Waals like direct potential in the $|\,0\,\rangle$ channel, which perturbatively is of higher order, so $B+\bar{B}$ interaction is envisaged as proceeding only through virtual jumps into the $b+\bar{b}$ channel and back. However, since the latter channel perturbatively has a direct Coulomb-type potential from single exchange of a massless gluon between the b and $\bar{b}$, we shall occasionally remark on the extension

$$H \longrightarrow H + U_c(\mathbf{X}) \,, \qquad U_c(\mathbf{X}) = -\frac{\gamma}{R}\,|\,c\,\rangle\langle\,c\,| \tag{2.9}$$

to Eq. (2.5), where γ is another coupling of the same mass dimension and order of magnitude as β.

The above interpretations refer to the *perturbative* solution of QCD. For this reason, the representation in which H has the form of Eq. (2.5) and the scalar product is given by Eq. (2.4) will be referred to as the "representation modelling the perturbative phase", or loosely, "the perturbative phase". The spectrum in this phase contains free, asymptotically detectable b and $\bar{b}$ quarks. Indeed the system of Eqs. (2.8), even when fed at $t \to -\infty$ by an incoming packet in the 0 channel only, will by normal coupled-channel dynamics generate a nonzero c-channel amplitude $|\,\psi_c\,\rangle$ at $t \to +\infty$ corresponding to production of free $b+\bar{b}$ pairs. Associated with this will be an effective one-channel potential $W_0(E+i\epsilon)$ in the 0 channel whose negative-definite absorptive part,

$$\mathrm{Im}\, W_0(E+i\epsilon) = -\pi\theta(E)\; V(\mathbf{X})\,\delta(E-K(\mathbf{P}))\,V(\mathbf{X}) \,, \tag{2.10}$$

describes the leaking of probability into the c channel.

The interpretation fixes orders of magnitude for the parameters in Eq. (2.5): we have $m \approx m_b$ and $\beta \approx \hbar c \cdot \alpha_s(2m_b)$, where $\alpha_s(2m_b)$ is the strong coupling constant at the scale of the absolute threshold energy $2m_b$. Thus,

$$\frac{mc^2}{\mathrm{GeV}} \approx 5 \, ; \qquad\qquad \frac{\beta}{\hbar c} \approx 0.2 \, . \tag{2.11}$$

3 Similarity Transform and Metric

On the above system we now perform a similarity transformation by the operator $\exp A$, the first factor in Eq. (1.3), with an Hermitian generator

$$A \; = \; \left[\mathrm{arsinh} \left(\frac{\Lambda}{\beta} R \right) \right] \cdot \Big(\; | \, 0 \rangle \langle s \, | \; + \; | \, s \rangle \langle 0 \, | \; \Big) \; = \; A(\mathbf{X}) \; , \tag{3.12}$$

causing mixing with the shadow channel (the sectospin operator in Eq. (3.12) is $2F_4$ in standard $SU(3)$ convention), and involving a new energy scale Λ which corresponds to the nonperturbative confinement scale of the field-theory problem. In the context of our above interpretation, this implies the orders of magnitude,

$$\frac{\Lambda}{E_0} \;\; \approx \;\; 1 \qquad \left(E_0 \; = \; mc^2 \left(\frac{\beta}{\hbar c} \right)^2 \right) \; , \tag{3.13}$$

$$\frac{\Lambda}{mc^2} \;\; \approx \;\; \frac{1}{25} \, , \tag{3.14}$$

the latter quantity being the only truly small parameter in the problem. Here E_0 gives the order of magnitude of the Rydberg energy formally associated with the coupling β and mass m.

The Baker-Campbell-Hausdorff (BCH) commutator series for the first-stage transformed Hamiltonian,

$$H' \; = \; e^{A} \, H \, e^{-A} \; , \tag{3.15}$$

can be worked out by using secto-$SU(3)$ relations such as

$$\Big(\; | \, 0 \rangle \langle s \, | \; + \; | \, s \rangle \langle 0 \, | \; \Big)^2 \;\; = \;\; \Pi_0 \; + \; \Pi_s \, , \tag{3.16}$$

$$\Big[\; | \, 0 \rangle \langle s \, | \; + \; | \, s \rangle \langle 0 \, | \; , \; | \, 0 \rangle \langle c \, | \; + \; | \, c \rangle \langle 0 \, | \; \Big] \;\; = \;\; | \, s \rangle \langle c \, | \; - \; | \, c \rangle \langle s \, | \tag{3.17}$$

and can be summed in straightforward fashion. (Alternatively, one may use the closed form of the transformation, Eq. (3.28) below, directly in Eq.

(3.15)). The result, in the matrix form analogous to Eqs. (2.8), reads

$$\Big(\langle i \mid H' \mid j \rangle \Big) \;=\; \begin{pmatrix} K + U_s'(\mathbf{X}) & \Lambda & i\,Y(\mathbf{X},\mathbf{P}) \\ -\Lambda & K & F'(\mathbf{X}) \\ i\,Y(\mathbf{X},\mathbf{P}) & F'(\mathbf{X}) & K + U_s'(\mathbf{X}) \end{pmatrix} . \tag{3.18}$$

Thus H' contains a transformed coupling potential between the 0 and c channels,

$$F'(\mathbf{X}) \;=\; \sqrt{\Big(\frac{\beta}{R}\Big)^2 \,+\, \Lambda^2}\;, \tag{3.19}$$

and, as the essential new feature, a constant, anti-Hermitian coupling term of strength Λ in the s–c (sectospin-doublet) subspace,

$$\Lambda \cdot \Big(\;\mid s\,\rangle\langle\,c\mid \,-\, \mid c\,\rangle\langle\,s\mid\; \Big). \tag{3.20}$$

(The sectospin generator here is our $2i\Sigma_2$, or $2iF_2$ in $SU(3)$ parlance). In addition there are two spatially short-ranged terms arising from single and double commutation of the kinetic energy K with the generator (3.12) : another non-Hermitian coupling between the s and 0 channels, which after some rewriting assumes the form

$$[A, K] \;=\; i\,Y(\mathbf{X},\mathbf{P}) \cdot \Big(\;\mid 0\rangle\langle s\mid \,+\, \mid s\rangle\langle 0\mid\; \Big)\,, \tag{3.21}$$

$$Y(\mathbf{X},\mathbf{P}) \;=\; \Big(\frac{\Lambda}{E_0}\Big) \cdot \big[\, D\,y(R) + y(R)\,D\,\big]\,, \tag{3.22}$$

with D denoting the dimensionless dilatation generator,

$$D(\mathbf{X},\mathbf{P}) \;=\; \frac{1}{2\hbar}\,(\mathbf{X}\cdot\mathbf{P} + \mathbf{P}\cdot\mathbf{X})\,, \tag{3.23}$$

and $y(R)$ a function of screened-Coulomb form,

$$y(R) \;=\; \frac{\beta}{R\,\sqrt{1 + (\frac{\Lambda}{\beta}R)^2}}\;, \tag{3.24}$$

and a Hermitian, short-range, direct potential in both the 0 and s channels,

$$\frac{1}{2!}\Big[A, [A, K]\Big] \;=\; U_s'(\mathbf{X})\,(\,\Pi_0 + \Pi_s\,)\,, \tag{3.25}$$

$$U_s'(\mathbf{X}) \;=\; -\Big(\frac{\Lambda^2}{E_0}\Big)\frac{1}{1 + (\frac{\Lambda}{\beta}R)^2}\;. \tag{3.26}$$

These two terms will lead to only minor quantitative modifications of the subsequent development. Lastly, we note that a direct c-channel potential of type (2.9), when included, commutes with the generator (3.12) and therefore emerges unaffected from this stage of the transformation:

$$e^{A}\,(U_c \mid c\rangle\langle c \mid)\, e^{-A} \;=\; U_c \mid c\rangle\langle c \mid \;. \tag{3.27}$$

The first-stage transformation itself can be written, by using relation (3.16), in the closed form

$$\exp A(\mathbf{X}) \;=\; \mathbf{1} + \left[\sqrt{1+\left(\frac{\Lambda}{\beta}R\right)^2} - 1\right]\,(\Pi_0{+}\Pi_s) + \left(\frac{\Lambda}{\beta}R\right)\Big[\;\mid 0\rangle\langle s \mid \;+\; \mid s\rangle\langle 0 \mid\;\Big]\,. \tag{3.28}$$

At small distances, it approaches unity, which is reminiscent of the asymptotic-freedom property of the ‘true’ theory.

We remark that the transform inverse to Eq. (3.15) – the retrieval of the manifestly Hermitian, original Hamiltonian (2.5) from the non-Hermitian one through construction of the transformation $\exp(-A)$ – may be viewed as an application of the methods described in Ref. [8]. Such a construction, in general, turns out to have its limitations – the non-Hermiticity “must not be too strong” to be amenable to such a mapping. In the present context this restriction is hidden in the inequalities $\Lambda < F'$ and $y(R) < F'$ that are seen to hold between the non-Hermitian and Hermitian coupling strengths in Eq. (3.18).

Comparison of Eq. (3.18) with Eq. (3.42) below would seem to indicate that after this first transformation step, the *decoupled c–s* subspace, with the $\mid \psi_0\rangle$ component switched off, already exhibits the “confining” properties we are trying to model, but the full system, in the end, would not. The reason is subtle. Those properties, we anticipate, arise from the fact that the anti-Hermitian coupling term (3.20) has a coefficient – the constant Λ – which at large interparticle separation does not decrease to zero. We will call such terms *persistent*; they clearly modify the asymptotic-propagation properties previously determined by the kinetic energy K alone, and in the decoupled c–s subspace would indeed produce the “damped” propagation limited to space-time intervals of order Λ^{-1}. (From the point of view of the full theory, they model properties of the nonperturbative vacuum state). Indeed the decoupled c–s sector of Eq. (3.18), with the c–channel potential (3.27) added, is *precisely* the model Hamiltonian studied in [9], where confinement in heavy-quarkonium states produced by the persistent terms alone, and without infinitely rising potentials, was used to obtain semi-quantitative fits to bottomonium and charmonium spectra. However, at this stage the effect is still counteracted by the presence of another asymptotically constant function, the transformed c-0 channel coupling (3.19). By decomposing this function as

$$F'(\mathbf{X}) = \Lambda \mathbf{1} + V'(\mathbf{X}) \tag{3.29}$$

with a short-range remainder potential,

$$V'(\mathbf{X}) = \sqrt{\Lambda^2 + \left(\frac{\beta}{R}\right)^2} - \Lambda = \frac{\beta}{R} \cdot \frac{1}{\frac{\Lambda}{\beta} R + \sqrt{1 + (\frac{\Lambda}{\beta} R)^2}}, \tag{3.30}$$

we can write the full matrix of the persistent terms in $H' - E$, which defines the negative inverse of the free Green's-function operator for the transformed three-channel system:

$$\Big(\langle i \mid H'_{(pers)} - E\mathbf{1} \mid j \rangle \Big) = \begin{pmatrix} K - E & \Lambda & 0 \\ -\Lambda & K - E & \Lambda \\ 0 & \Lambda & K - E \end{pmatrix} . \tag{3.31}$$

Its determinant, whose zeroes encode the singularities of the Green's function, is seen to be

$$\begin{aligned} \Delta'_{(pers)}(E) &= (K - E)\,[\,(K - E)^2 + \Lambda^2\,] \\ &\qquad - \Lambda\,[\,\Lambda(K - E)\,] \\ &= (K - E)^3 \,. \end{aligned} \tag{3.32}$$

The first line, arising from the c–s subdeterminant, looks like Eq. (3.43) below and is what would meet our needs, but it gets sabotaged by the second line, which arises from the persistent portion of the 0–c coupling term,

$$\Lambda \Big[\mid 0 \rangle\langle c \mid + \mid c \rangle\langle 0 \mid \Big], \tag{3.33}$$

in Eq. (3.31). The result in the third line is then still the same as for the untransformed system, and says that despite the deceptive appearance of the c–s submatrix, the system will still behave asymptotically as a three-channel scattering system, with three normal $(E - K)^{-1}$ Green's functions, in a somewhat exotic representation. Without dwelling on this point, we note that indeed a study of the effective 0–channel equation for the Hamiltonian (3.18) would reveal that this channel develops a term $\Lambda^2\,(E - K)^{-3}$ in its free Green's function, giving it pathological scattering properties that in the end would overrun the apparent "confining" properties of the s–c sector.

We thus seek a further transformation step that will remove the unwanted term (3.33) while leaving the desirable term (3.20) essentially intact. Since this is now a modification of the Hermitian part of H' alone, it can obviously be achieved by a *unitary* transformation,

$$H'' = e^{iB} H' e^{-iB} . \tag{3.34}$$

A generator B with the desired properties is

$$B = \frac{\Lambda}{2}\Theta \cdot \Big[\, |\, 0 \rangle\langle\, c\, | + |\, c\rangle\langle\, 0\, | \, \Big], \tag{3.35}$$

where Θ denotes an Hermitian operator on orbital degrees of freedom which is canonically conjugate to the kinetic energy K in the sense that

$$\Big[\frac{1}{2}\Theta, K \Big] = i\,\mathbf{1} . \tag{3.36}$$

(The sectospin part, we recall, is the $2F_6$ of standard $SU(3)$ notation). Transformation of K by Eqs. (3.34/3.35) then results in the terminating BCH series

$$K'' \equiv e^{iB} K e^{-iB} = K - \Lambda \Big[\, |\, 0 \rangle\langle\, c\, | + |\, c\rangle\langle\, 0\, | \, \Big] \tag{3.37}$$

and therefore cancels the undesirable term (3.33), since the latter clearly commutes with this transformation.

An operator with the property (3.36) exists and has the form,

$$\Theta(\mathbf{P},\mathbf{X}) = \frac{1}{2} \, (K^{-1}D + DK^{-1}), \tag{3.38}$$

involving again the dilatation generator (3.23). By using the commutation relation $[\, D, \mathbf{P}^2 \,] = 2i\mathbf{P}^2$, one verifies relation (3.36) immediately. Again there is a kind of asymptotic-freedom property, since $\Theta \to 0$ at large momenta.

In coordinate space, the operator K^{-1} of Eq. (3.38) is defined through the Green's function $-(4\pi|\mathbf{x}' - \mathbf{x}|)^{-1}$ of the Laplacian; this leads to the representation

$$\langle\, \mathbf{x}' \,|\, \Theta \,|\, \mathbf{x} \,\rangle = \frac{m}{4\pi\hbar^2} \, \frac{i\,(\mathbf{x}'^2 - \mathbf{x}^2)}{|\, \mathbf{x}' - \mathbf{x}\,|^3} . \tag{3.39}$$

Thus Θ acts in the manner of a nonlocal potential; it is a non-persistent operator. It is then straightforward to verify that the transformation of the non-Hermitian coupling (3.20) under (3.34/3.35), though nontrivial, does not change its content of persistent terms : on the r.h.s. of the relation

$$e^{iB}\Big[\Lambda\cdot\Big(\ |\,s\,\rangle\langle\,c\,|\ -\ |\,c\,\rangle\langle\,s\,|\ \Big)\Big]\,e^{-iB}\ =$$
$$\Lambda C_\Lambda\Big(\ |\,s\,\rangle\langle\,c\,| - |\,c\,\rangle\langle\,s\,|\ \Big)\ -\ \Lambda S_\Lambda\,i\Big(\ |\,0\,\rangle\langle\,s\,| + |\,s\,\rangle\langle\,0\,|\ \Big) \tag{3.40}$$

with

$$C_\Lambda\ =\ \cos\Big(\frac{1}{2}\Lambda\Theta\Big)\,,\qquad\qquad S_\Lambda\ =\ \sin\Big(\frac{1}{2}\Lambda\Theta\Big)\,, \tag{3.41}$$

only the unit term in the expansion of the cosine operator is persistent. Therefore the matrix of persistent terms, i.e. the negative inverse of the Green's-function operator $\mathcal{G}''(E)$ for asymptotic propagation, now becomes,

$$-\ \big[\,\mathcal{G}''(E)\,\big]^{-1}\ =\ \begin{pmatrix} K\,-\,E & \Lambda & 0 \\ -\Lambda & K\,-\,E & 0 \\ 0 & 0 & K\,-\,E \end{pmatrix}\,, \tag{3.42}$$

and in contrast to Eq. (3.32) now has a determinant,

$$\Delta''_{(pers)}(E)\ =\ (\,K-E\,)\,[\,(K-E)^2\,+\,\Lambda^2\,]\,, \tag{3.43}$$

indicating a qualitative change in the asymptotic-propagation characteristics of the system: the square-bracketed operator from the c–s sector has no nullspace at real energies E, and therefore will permit no incoming or outgoing wave packets in free asymptotic motion.

A price one pays for the relative simplicity of the mechanism of Eq. (3.37) is that most of the non-persistent, interaction terms become nonlocal, and somewhat unwieldy. In detail, we have

$$E\mathbf{1}\ -\ H''\ =\ \big[\,\mathcal{G}''(E)\,\big]^{-1}\ -\ \mathcal{U}''(\mathbf{X},\!\mathbf{P})\ , \tag{3.44}$$

where the interaction term $\mathcal{U}''$ has matrix representation,

$$\Big(\ \langle i\,|\,\mathcal{U}''(\mathbf{X},\!\mathbf{P})\,|\,j\rangle\ \Big)\ =\ \begin{pmatrix} U'_s & Z'' & i\,Y'' \\ -Z''^\dagger & U''_c & V'' \\ i\,Y''^\dagger & V''^\dagger & U''_0 \end{pmatrix}\ . \tag{3.45}$$

Like Eq. (3.42), this displays a very special and restricted *pattern of Non-Hermiticity*: the latter resides completely in the non-Hermitian s–0 and s–c coupling terms,

$$iY''\ =\ i\,(\,Y\,C_\Lambda\ -\ \Lambda\,S_\Lambda\,)\,, \tag{3.46}$$
$$Z''\ =\ \Lambda\,(C_\Lambda-\mathbf{1})\ +Y\,S_\Lambda\ , \tag{3.47}$$

which together with their *negative* adjoints in the 0–s and c–s positions form the entire anti-Hermitian part of the matrix operator, while diagonal elements and the complete lower-right (c–0) submatrix, with coupling term

$$V'' \;=\; \frac{1}{2}\left[e^{\frac{i}{2}\Lambda\Theta}V'(\mathbf{X})e^{-\frac{i}{2}\Lambda\Theta} + e^{-\frac{i}{2}\Lambda\Theta}V'(\mathbf{X})e^{\frac{i}{2}\Lambda\Theta}\right] + i\left[S_\Lambda U_s' C_\Lambda - C_\Lambda U_c' S_\Lambda\right] \tag{3.48}$$

and its adjoint, remain Hermitian. The new diagonal, Hermitian potentials are,

$$U_0'' \;=\; C_\Lambda U_s' C_\Lambda + S_\Lambda U_c S_\Lambda + \frac{1}{2}\left[e^{\frac{i}{2}\Lambda\Theta}V'(\mathbf{X})e^{-\frac{i}{2}\Lambda\Theta} + e^{-\frac{i}{2}\Lambda\Theta}V'(\mathbf{X})e^{\frac{i}{2}\Lambda\Theta}\right]\,, \tag{3.49}$$

$$U_c'' \;=\; S_\Lambda U_s' S_\Lambda + C_\Lambda U_c C_\Lambda + \frac{1}{2}\left[e^{\frac{i}{2}\Lambda\Theta}V'(\mathbf{X})e^{-\frac{i}{2}\Lambda\Theta} + e^{-\frac{i}{2}\Lambda\Theta}V'(\mathbf{X})e^{\frac{i}{2}\Lambda\Theta}\right]\,. \tag{3.50}$$

Here we have already included the effects of the original c–channel potential (3.27).

By combining Eq. (3.28) with the analogous closed form for the second transformation step,

$$\exp(\pm iB) \;=\; \mathbf{1} + (C_\Lambda - \mathbf{1})\left[\Pi_0 + \Pi_c\right] \pm i\,S_\Lambda\left[\,|\,0\rangle\langle c\,| + |\,c\rangle\langle 0\,|\,\right], \tag{3.51}$$

one may find the total transformation T, which we write as a set of transformation equations for the orbital amplitudes $|\,\psi_n''(t)\,\rangle$ defined in analogy with Eq. (2.3):

$$|\,\psi_s''\rangle \;=\; \sqrt{1+(\frac{\Lambda}{\beta}R)^2}\;|\,\psi_s\rangle \qquad + \;(\frac{\Lambda}{\beta}R)\;|\,\psi_0\,\rangle\,, \tag{3.52}$$

$$|\,\psi_c''\rangle \;=\; i\,S_\Lambda(\frac{\Lambda}{\beta}R)\;|\,\psi_s\rangle + C_\Lambda\;|\,\psi_c\rangle + i\,S_\Lambda\sqrt{1+(\frac{\Lambda}{\beta}R)^2}\;|\,\psi_0\,\rangle\,, \tag{3.53}$$

$$|\,\psi_0''\rangle \;=\; C_\Lambda(\frac{\Lambda}{\beta}R)\;|\,\psi_s\rangle + S_\Lambda\;|\,\psi_c\rangle + C_\Lambda\sqrt{1+(\frac{\Lambda}{\beta}R)^2}\;|\,\psi_0\,\rangle\,. \tag{3.54}$$

It is clear that the initial condition of preparing an incoming wave packet in e.g. the $|\,\psi_0''(t)\,\rangle$ amplitude means something entirely different from the corresponding preparation for the untransformed $|\,\psi_0(t)\,\rangle$ amplitude. To realize the $|\,\psi''_0(t)\,\rangle$- packet initial condition in the original system would require, according to Eq. (3.54), a very complicated, coherent, and fine-tuned preparation in all three of the original channels that is practically impossible to achieve. Inasmuch as the channels of a given representation define the natural possibilities of asymptotic preparation and detection, these possibilities have now been changed – and, as we will see in a moment, restricted – in a qualitative way.

Finally, the metric operator M for the new representation, calculated from Eq. (1.6), can be given a compact form by introducing a hybrid quantity Ω which is both a state in sectospin space and an operator on orbital motion:

$$\Omega \;=|\,s\,\rangle\cdot\sqrt{1+\Big(\frac{\Lambda}{\beta}R\Big)^2}\;+\;|\,c\,\rangle\cdot\Big[-i\,S_\Lambda\Big(\frac{\Lambda}{\beta}R\Big)\Big]\;+\;|\,0\,\rangle\cdot\Big[-C_\Lambda\Big(\frac{\Lambda}{\beta}R\Big)\Big]\;. \tag{3.55}$$

Then

$$M\;=\;\mathbf{1}\;+\;2\Big(\Omega\Omega^\dagger-\;|\,s\,\rangle\langle\,s\,|\;\Big)\,. \tag{3.56}$$

In matrix form, the nontrivial term is

$$\Big(\;\langle i\,|\;(\Omega\Omega^\dagger-\;|\,s\,\rangle\langle\,s\,|\,)\;|\,j\rangle\;\Big)\;=$$

$$\begin{pmatrix} (\frac{\Lambda}{\beta}R)^2 & i\sqrt{1+(\frac{\Lambda}{\beta}R)^2}(\frac{\Lambda}{\beta}R)S_\Lambda & -\sqrt{1+(\frac{\Lambda}{\beta}R)^2}(\frac{\Lambda}{\beta}R)C_\Lambda \\ -iS_\Lambda(\frac{\Lambda}{\beta}R)\sqrt{1+(\frac{\Lambda}{\beta}R)^2} & S_\Lambda(\frac{\Lambda}{\beta}R)^2S_\Lambda & iS_\Lambda(\frac{\Lambda}{\beta}R)^2C_\Lambda \\ -C_\Lambda(\frac{\Lambda}{\beta}R)\sqrt{1+(\frac{\Lambda}{\beta}R)^2} & -iC_\Lambda(\frac{\Lambda}{\beta}R)^2S_\Lambda & C_\Lambda(\frac{\Lambda}{\beta}R)^2C_\Lambda \end{pmatrix}. \tag{3.57}$$

By construction, the norm formed with this metric, being identical with the original norm that evolved under a manifestly Hermitian Hamiltonian, is conserved,

$$\frac{d}{dt}\,\langle\,\Psi''(t)\,|\,M\,|\,\Psi''(t)\,\rangle\;=\;0\,. \tag{3.58}$$

The "naive" norm formed from the $|\,\psi''_n\,\rangle$'s is therefore *not* conserved; rather, upon abbreviating $\Xi=\Omega\Omega^\dagger-\;|\,s\,\rangle\langle\,s\,|$, we have

$$\frac{d}{dt}\left(\sum_{n=s,c,0}\langle\,\psi''_n(t)\,|\,\psi''_n(t)\,\rangle\right)\;=\;\langle\,\Psi''(t)\,|\;\frac{2}{i\hbar}\Big[(H'')^\dagger\Xi\;-\;\Xi H''\Big]\;|\,\Psi''(t)\,\rangle\,. \tag{3.59}$$

The square bracket can be worked out but provides no special insight.

4 Effective Subspace Equations

The Green's-function operator for the transformed system is obtained by forming the inverse of the operator matrix (3.42):

$$\mathcal{G}''(E) \;=\; \begin{pmatrix} \mathcal{G}_+(E) & \mathcal{G}_-(E) & 0 \\ -\mathcal{G}_-(E) & \mathcal{G}_+(E) & 0 \\ 0 & 0 & \mathcal{G}_0(E+i\epsilon) \end{pmatrix} . \tag{4.1}$$

Only the ordinary Green's operator for the 0 channel,

$$\mathcal{G}_0(E+i\epsilon) \;=\; (\,E+i\epsilon\,-\,K\,)^{-1}\;, \tag{4.2}$$

needs an $i\epsilon$ prescription for E real and > 0; it ensures that the 0 channel continues to permit free incoming and outgoing wave packets at asymptotic distances, and thus remains an *asymptotically accessible subspace.* By contrast, the two functions appearing in the sectospin-1/2 (s–c) subspace are completely nonsingular on the real E axis:

$$\mathcal{G}_+(E) \;=\; \frac{E-K}{(E-K)^2+\Lambda^2} \;=\; \frac{1}{2}\left(\frac{1}{E-K+i\Lambda}+\frac{1}{E-K-i\Lambda}\right)\;, \tag{4.3}$$

$$\mathcal{G}_-(E) \;=\; \frac{\Lambda}{(E-K)^2+\Lambda^2} \;=\; \frac{i}{2}\left(\frac{1}{E-K+i\Lambda}-\frac{1}{E-K-i\Lambda}\right)\;. \tag{4.4}$$

In fact, these functions have the usual Green's-function pole at real E replaced by a pole pair at complex-conjugate positions, with imaginary parts $\pm i\Lambda$. Their Fourier transforms to the time domain,

$$G_+(t-t') \;=\; \frac{1}{2i\hbar}\, e^{-\frac{\Lambda}{\hbar}|t-t'|}\,\epsilon(t-t')\, e^{-\frac{i}{\hbar}K(t-t')}\;, \tag{4.5}$$

$$G_-(t-t') \;=\; \frac{1}{2\,\hbar}\, e^{-\frac{\Lambda}{\hbar}|t-t'|}\, e^{-\frac{i}{\hbar}K(t-t')}\;, \tag{4.6}$$

where $\epsilon(\tau)$ denotes the step function $\theta(\tau)-\theta(-\tau)$, show explicitly the nature of the asymptotic propagation in the c–s sector: they cause the components $|\,\psi''_{c,s}(t)\,\rangle$ of the state in this sector to always decrease, with time constant $\hbar/\Lambda$, for both $t\to+\infty$ and $t\to-\infty$. In the new representation, the c–s sector has become a *confined subspace.* For this reason, we refer to this representation as the "representation modelling the nonperturbative or confining phase". It is obvious without calculation that, as a consequence, S-matrix elements to, from, and within this subspace will vanish. The transformation has effectively reduced the space on which a nonzero S matrix obtains.

To minimize inessential technical complication, we write equations for the formal elimination of the confined subspace only to zeroth order in the internal 2×2 potential matrix (the non-persistent terms) for that subspace,

$$\Big(\langle i \mid \mathcal{U}_1''(\mathbf{X},\mathbf{P}) \mid j \rangle \Big) \;=\; \begin{pmatrix} U_s' & Z'' \\ -Z''^{\dagger} & U_c'' \end{pmatrix} . \tag{4.7}$$

Inclusion of $\mathcal{U}_1''$, on which we comment briefly at the end of this section, will not change the qualitative conclusions we are going to draw about the effective equations in the accessible and confined subspaces.

At this order, the formal solution for the s- and c-channel orbital components in terms of $\mid \tilde{\psi}_0''(E) \,\rangle$ then takes the form,

$$\begin{pmatrix} \mid \tilde{\psi}_s''(E) \,\rangle \\ \mid \tilde{\psi}_c''(E) \,\rangle \end{pmatrix} \;=\; \mathcal{G}_1''(E) \;\cdot \begin{pmatrix} iY'' \mid \tilde{\psi}_0''(E) \,\rangle \\ V'' \mid \tilde{\psi}_0''(E) \,\rangle \end{pmatrix} , \tag{4.8}$$

where $\mathcal{G}_1''$ is the sectospin-1/2 submatrix of the Green's operator (4.1),

$$\mathcal{G}_1''(E) \;=\; \begin{pmatrix} \mathcal{G}_+(E) & \mathcal{G}_-(E) \\ -\mathcal{G}_-(E) & \mathcal{G}_+(E) \end{pmatrix} . \tag{4.9}$$

Like (4.7), it again displays the special non-Hermiticity pattern of having a purely anti-Hermitian nondiagonal part while its diagonal part is purely Hermitian. Note that since the inverse of $\mathcal{G}_1''$ – the s–c submatrix of Eq. (3.42) – has no nullspace, there are no "solutions to the homogeneous equation" allowed in (4.8) – in a sense, the transformed system has severely restricted its own range of admissible initial conditions.

The effective one-channel equation for the orbital amplitude in the 0 subspace may now be derived by using Eq. (4.8) to formally eliminate the confined subspace, and takes the form

$$\big\{ E \,-\, \big[\, K \,+\, U_0'' \,+\, W_0''(E) \,\big] \big\} \;\mid \tilde{\psi}_0''(E) \,\rangle \;=\; 0 \;, \tag{4.10}$$

featuring a new energy-dependent effective potential

$$W_0''(E) \;=\; V''^{\dagger}\, \mathcal{G}_+(E)\, V'' - Y''^{\dagger}\, \mathcal{G}_+(E)\, Y'' + i \Big[Y''^{\dagger}\, \mathcal{G}_-(E)\, V'' \,-\, V''^{\dagger}\, \mathcal{G}_-(E)\, Y'' \Big] \;, \tag{4.11}$$

which due to Eqs. (4.3 / 4.4) is completely nonsingular, and therefore develops no absorptive part of type (2.10), on the real energy axis. Moreover, in W_0'', the non-Hermitian coupling terms are seen to have paired off in such a way as to render this potential *Hermitian* at fixed E:

$$(\, W_0''(E) \,)^{\dagger} \;=\; W_0''(E) \;. \tag{4.12}$$

This is the situation that "normally" exists for coupling to closed channels, i.e. other channels at energies below their thresholds, but the new feature here is that it prevails at *all* energies. Just as in the closed-channel scenario, this implies that although the naive one-channel norm $\langle \psi''_0 \mid \psi''_0 \rangle$ is not a constant locally in time, it is conserved between *asymptotic* times: by adapting the usual derivation for the time development of a norm, one has

$$\frac{\partial}{\partial t} \langle \psi''_0 \mid \psi''_0 \rangle = -\frac{1}{\hbar} \mathrm{Im} \left\{ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{dE' dE}{(2\pi\hbar)^2} \exp\left(i \frac{E' - E}{\hbar} t \right) \right. \\ \times \left. \langle \tilde{\psi}''_0(E') \mid W''_0(E') - W''_0(E) \mid \tilde{\psi}''_0(E) \rangle \right\} , \qquad (4.13)$$

which vanishes upon integration over all times as the exponential is turned into a $2\pi\hbar\delta(E' - E)$, so that

$$\| \psi''_0(t = +\infty) \| - \| \psi''_0(t = -\infty) \| = 0 . \qquad (4.14)$$

As a consequence, the one-channel S or T matrix for the asymptotically accessible 0 channel, extracted "naively" – i.e., without any reference to the norm (3.56) – from the $\mid \tilde{\psi}''_0(E) \rangle$ states, obeys *elastic unitarity.* (Formally, the imaginary part of the T matrix at real E still arises exclusively from the $i\epsilon$ in the Green's function (4.2), with no contribution from the potential). Unitarity at finite times for the total system, on the other hand, becomes manifest only upon using the nontrivial metric, as emphasized in connection with Eq. (3.59).

For contrast, one may look at the effective equation for the confined subspace; to this end one solves formally for $\mid \tilde{\psi}''_0(E) \rangle$ in terms of the other two components, and under the only natural initial condition now remaining, that of an incoming free packet in the infinite past:

$$\mid \psi''_0(t) \rangle \longrightarrow e^{-\frac{i}{\hbar}Kt} \mid \phi''_0 \rangle \quad (t \longrightarrow -\infty) ; \qquad \langle \phi''_0 \mid \phi''_0 \rangle = 1 . \qquad (4.15)$$

By using the result in the s and c equations one then obtains the effective equations for the sectospin-1/2 subspace,

$$\left\{ \left[\mathcal{G}''_1(E) \right]^{-1} - \mathcal{W}''_1(E + i\epsilon) \right\} \cdot \begin{pmatrix} \mid \tilde{\psi}''_s(E) \rangle \\ \mid \tilde{\psi}''_c(E) \rangle \end{pmatrix} = \begin{pmatrix} iY'' \mid \tilde{\xi}''_0(E) \rangle \\ V'' \mid \tilde{\xi}''_0(E) \rangle \end{pmatrix} , \qquad (4.16)$$

with an energy-dependent effective-potential matrix,

$$\mathcal{W}''_1(E + i\epsilon) = \begin{pmatrix} -Y''\Gamma_0(E + i\epsilon)Y''^\dagger & iY''\Gamma_0(E + i\epsilon)V''^\dagger \\ iV''\Gamma_0(E + i\epsilon)Y''^\dagger & V''\Gamma_0(E + i\epsilon)V''^\dagger \end{pmatrix} \qquad (4.17)$$

containing the 0–channel Green's operator,

$$\Gamma_0(E+i\epsilon) \;=\; \left[\; E+i\epsilon \;-\; \left(K+U_0''\right)\;\right]^{-1} \;. \tag{4.18}$$

The inhomogeneous term expresses the fact that the sectospin-1/2 subspace is fed by the initial packet evolved under the influence of U_0'':

$$|\,\tilde{\xi}_0''(E)\,\rangle \;=\; \lim_{\epsilon\to 0}\,\left\{\; i\epsilon\,\Gamma_0(E+i\epsilon)\,2\pi\hbar\delta(E-K)\;\;|\,\phi_0''\,\rangle\;\right\}\;. \tag{4.19}$$

The essential point about the potentials of Eq. (4.17) is that in contrast to the W_0'' of Eq. (4.11), they do have absorptive parts, arising from the discontinuity of the Green's function (4.18) across its "normal" scattering cut on the positive real E axis. These absorptive parts describe the fact that the confined channels must leak their short-lived share of total probability back to the asymptotically accessible space. We reemphasize that while the direction of this leakage is qualitatively what one expects, it does not imply quantitative conservation of the naive norm at finite times.

None of these qualitative features is changed when the interaction matrix (4.7) within the s–c subspace is taken into account, and we therefore devote only the briefest comment to this case. The Green's-operator matrix (4.9) is now replaced by the matrix

$$\Gamma_1(E) \;=\; \mathcal{G}_1''(E) \;+\; \mathcal{G}_1''(E)\,\mathcal{U}_1''\,\mathcal{G}_1''(E) \;+\; \mathcal{G}_1''(E)\,\mathcal{U}_1''\,\mathcal{G}_1''(E)\,\mathcal{U}_1''\,\mathcal{G}_1''(E) \;+\; \ldots\;. \tag{4.20}$$

One checks easily that each term of Eq. (4.20) preserves the special non-Hermiticity pattern mentioned above for $\mathcal{U}_1''$ and $\mathcal{G}_1''(E)$. The effective 0-channel potential (4.11), now formed with the appropriate $\Gamma_1(E)$ elements instead of the $\mathcal{G}_+(E)$ and $\mathcal{G}_-(E)$, therefore continues to be Hermitian at fixed E. Since all $\mathcal{G}_1''(E)$ factors in Eq. (4.20) are nonsingular on the positive real E axis, Γ_1 has no branch cut there, and W_0'' continues to have no absorptive part.

The only new feature that may occur is for $\Gamma_1(E)$ to develop isolated singularities in E. It was demonstrated in Ref. [9] that as long as the non-Hermiticity in the *decoupled s–c* subspace is not too strong, a limited number of poles may indeed occur on the real E axis, signalling the formation of bound states (in our interpretation, meson states of the bottomonium type) which for the decoupled system are stable. Since that system, as we have seen, has no threshold, these poles need not be restricted to $E<0$. If a meson pole does occur at an energy $E=\epsilon_b>0$, it is still embedded, from the point of view of the total system, into the continuous spectrum of the 0 channel, and will turn into a resonance visible in that channel when the channel-coupling terms iY'' and V'' in Eq. (3.45) are switched on. We do not dwell on the treatment of such resonances, which apart from minor peculiarities (connected to the non-Hermiticity in the s–c sector) is entirely conventional, and somewhat out of the main line of interest in this paper. We only note, as an unfamiliar feature, that due to the second term in Eq. (3.46) for the s–0 coupling potential Y'', the widths of these resonances have terms proportional to the "confinement scale" Λ that remain nonzero even when the "perturbative" couplings β, γ are switched off completely.

5 Remarks on the Relation with Field Theory

We have seen that division of the state space of a quantum-mechanical scattering system into "asymptotically accessible" and "permanently confined" subspaces can occur, provided the representation whose channels define the asymptotic preparation and detection possibilities of the system is one in which Hermiticity of the Hamiltonian holds only with respect to a nontrivial metric. A lesson which emerged along the way, and which may be relevant for the interpretation of certain results in field theory, is that Green's functions with conjugate pairs of singularities in the complex energy plane – which at first sight seem to contradict unitarity since the singularities of the resolvent for an Hermitian evolution generator should be entirely on the real energy axis – may nevertheless play a legitimate role in such a context, since unitarity must now be understood with respect to the nontrivial metric. A further lesson of practical relevance is that as long as one is interested only in S-matrix elements between asymptotically accessible channels, it may not be necessary to actually determine the complicated metric operator, since S-matrix (asymptotic) unitarity holds even with respect to the "naive" scalar product.

The interpretation we have given of the model alludes to the perturbative vs. the full nonperturbative solution of the QCD problem. The slight conceptual mismatch in this interpretation will be noted – we really discussed one solution to a certain Hamiltonian in two different representations, whereas on the field-theory side we spoke of *two* solutions, of different quality, to the same Lagrangian. While this mismatch marks the limits of our quantum-mechanical modelling, it also leads on to two interesting aspects. First, due to its infinite number of degrees of freedom, field theory is known to naturally accommodate representations connected by non-unitary transformations [10], and in particular the "dressing transformation" T mapping the noninteracting field operators $\Phi^{(0)}$ of perturbation theory onto their fully interacting and renormalized counterparts,

$$\Phi(x) \;=\; T\,\Phi^{(0)}(x)\,T^{-1}\;, \tag{5.1}$$

can and in general will be a non-unitary similarity transformation. The above results then lead to the conjecture that the dressing transform for QCD may be of a kind, exemplified by our model transformation, which creates short-lived propagation in channels containing unbound gluons and quarks, turning them into "confined subspaces", while leaving probability conservation between asymptotic times intact on the subspace of asymptotically accessible configurations.

Second, while in quantum mechanics there is obvious arbitrariness in declaring one representation to be the defining one for the possibilities of asymptotic measurement, in field theory there is a clear preference for the nonperturbative solution (which contains the α_s-nonanalytic Λ scale) over the perturbative solution (which misses out on Λ) to be the one that makes contact with experiment and defines the possible channels of asymptotic preparation and detection. There exists, therefore, the distinct and intriguing possibility that field theory may remove the arbitrariness of singling out the "confining" representation.

Finally, the role played by the Λ scale in the model transformation, Eqs. (3.12) and (3.35), points toward an interpretation of the "shadow" channel: it is a channel which decouples in the perturbative treatment, and starts participating in the dynamics only when and if that nonperturbative scale is allowed to develop. We do have a glimpse of the Λ-scale content of the nonperturbative QCD solution through the operator-product expansion: there, the perturbative form of an amplitude is corrected by terms containing powers Λ^n, and these powers arise in the form of vacuum expectations of local composite operators of suitable mass dimensions ("vacuum condensates"). Such condensates vanish identically in perturbation theory, where composite operators can be defined only through free-field normal ordering with respect to the perturbative vacuum. The s channel, and its mixing with the other channels through transformations that reduce to the identity at $\Lambda = 0$, may therefore be characterized as describing schematically the *effect of nonvanishing vacuum condensates* on QCD amplitudes.

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