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In collaboration with:
F. Farchioni, A. Ferling, I. Montvay, G. Münster, E.E. Scholz, J. Wuilloud

Münster, March 27th 2008
N=1 SU(2) SYM Theory on the Lattice with Light Dynamical Wilson Gluinos

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Outline:

- Intro & Low energy features of N=1 SYM in the continuum
- N=1 SYM on the Lattice
- Numerical results & SYM Spectrum
- Summary & Conclusion
Motivations for SUSY

- Stabilization of the *hierarchy problem* in SM
  - In SM $SU(3)_c \times SU(2)_L \times U(1)_Y$ EW and Strong couplings do not match

- Discovery of SUSY in *LHC* is an “indirect” discovery of the *Higgs boson*

- SUSY is a necessary ingredient in *String Theory*

- What is the dark matter made of? LSP (light susy particle)

  …
Motivations for SUSY

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--- the goal of this study ---

Investigation of *low-energy* dynamics of strongly coupled SUSY gauge theories
SUSY is fascinating!
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SUSY world:
Unification!

Running gauge Couplings:
Is SUSY a symmetry of the nature?

\[ m_{\tilde{e}} \neq m_e \]

SUSY must be broken $\rightarrow$ soft breaking

\[ m_{\tilde{g}} \sim O(1 \text{ TeV}) \]
\[ \mathcal{L}_{YM} = -\frac{1}{4} F_{\mu \nu}^a(x) F^{a \mu \nu}(x) + \frac{i}{2} \bar{\lambda}^a(x) \gamma^\mu D_\mu \lambda^a(x) - \frac{m \tilde{g}}{2} \bar{\lambda}^a \lambda^a \]

Equivalence to One flavor QCD
At large \( N_c \)

Presence of anomalous global chiral symmetry:
\( U(1)_\lambda \leftrightarrow R\)-symmetry

Spontaneous discrete chiral symmetry breaking:
\( Z_{2N_c} \rightarrow Z_2 \)
Low energy features of N=1 SYM

Confinement \[\rightarrow\] Colorless bound states

(perturbation theory cannot be applied!)

\[\chi_H\]
- Spin-1/2
- Gluino-Glue

\[a-\eta' / a-f_0\]
- Spin-0
- Gluino-Gluino

\[\chi_L\]
- Spin-1/2
- Gluino-Glue

\[0^{++} / 0^{-+}\]
- Spin-0
- Glue-Glue

\[m_{\tilde{g}} \neq 0\]

Higher supermultiplet

Lower supermultiplet

\[m_{\tilde{g}} = 0\]

[ Veneziano & Yankielowicz 82, Farrar et al. 98 ]
Non-perturbative methods, Lattice regularization

\[ a : \text{ lattice spacing} \]
\[ x : \text{ lattice site} \]
\[ U_\mu(x) : \text{ gauge fields (links)} \]
\[ \psi(x) : \text{ fermion fields} \]

Lattice volume: \( L \times L \times L \times T \)
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Lattice volume: \( L \times L \times L \times T \)

Problems!!

\[ \{Q, \bar{Q}\} \sim P_\mu \]

- No infinitesimal translations
- SUSY is broken by space-time discretization
- Naive discretization of SYM action leads to \textit{The doubling problem}: no balance between Fermionic and bosonic degrees of freedom

[Diagram showing lattice regularization with arrows and notation for \( U_\mu(x) \) and \( \psi(x) \)]
Lattice action

\[ S_{\text{lattice}} = S_g + S_{\tilde{g}} \xrightarrow{a \to 0} S_{\text{SYM}} + \mathcal{O}(a) \]

\[
S_g^W = \frac{2N_c}{g_0^2} \sum_x \sum_{\mu \nu} \left[ 1 - \frac{1}{N_c} \text{Re} \text{Tr} U_{\mu \nu} \right]
\]

\[
S_{\tilde{g}}^W = a^4 \sum_x \frac{1}{4a} \sum_{\mu = \pm 1}^{\pm 4} \left[ r \tilde{\lambda}_x^a \lambda_x^a - \tilde{\lambda}_{x+a\hat{\mu}}^a (r + \gamma_\mu) V^{ab}_\mu (x) \lambda^b (x) \right] + \frac{m_0}{2} \tilde{\lambda}_x^a \lambda_x^a
\]

\[ = -\frac{1}{2} \sum_{xy} a^4 \tilde{\lambda}_y Q_{yx} \lambda_x \]

SUSY and chiral limit

Tuning \( m_0 \to m_{0\text{cr}}(g_0) \leftrightarrow m_{\tilde{g}} = 0 \)

[ Curci & Veneziano 87 ]
Path integral (euclidean):

$$\langle O \rangle = \frac{1}{Z} \int [DU] O[U] \sigma e^{-S_g(U)} + \frac{1}{2} \ln[\det Q]$$

Importance Sampling (Monte Carlo)

$$\{U^{(i)}, i = 1 \ldots N\}$$

$$\approx \frac{1}{N} \sum_{i=1}^{N} \frac{\sigma^{(i)} O[U^{(i)}]}{\sum_{i=1}^{N} \sigma^{(i)}}$$

**Problems!!**

- Finite $a$ (lattice spacing) effects: $O(a)$-improvements $\rightarrow O(a^2)$

- Finite volume effects $L$

- Small dynamical fermion mass $\rightarrow$ slowing down

**Algorithms:**

- TSMB [Montvay 96]
- TS-PHMC [Montvay & Scholz 05]

**Finite volume effects**

- $L$

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Observables

Update
- Extremal eigenvalues
- Autocorrelation-times
- Wilson loops, Polyakov loops
- Physical scale $r_0 = 0.5 \text{ fm}$
- Static quark potential

Analysis
- Correction factors
- Pfaffian sign
- Correlation functions
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Run history

Lattice: $24^4 \beta = 1.6 \, \kappa = 0.1575 \, \text{tlSym TS-PHMC}$

Static quark potential

Lattice: $16^3 \times 32 \, \beta = 1.6 \, \text{PHMC-tlsym}$
Mass determination

- Construct interpolating operators:
- Smearing techniques: Jacobi/APE (variational method)
- Time-slice correlation functions
- Effective mass plateaux: $t$-range
- Fitting function
- Error from jackknife/linearization
- $X^2$ correlated fit
- Choose $t$-range to minimize $X^2$

Spin: $0,1/2$ bound states: $O_{JPC}$

$$S_t = \frac{1}{\sqrt{V_s}} \sum_{\vec{x}} O[U](\vec{x}, t)$$

$$C(\Delta t) = \langle S_t S_{t+\Delta t} \rangle - \langle S_t \rangle \langle S_{t+\Delta t} \rangle$$

$$am_{\text{eff}} = \log \frac{C(t)}{C(t+1)}$$

$$C(t) \to a_0^2 + \sum_{n=1} a_n^2 e^{-E_n t} \pm a_n^2 e^{-E_n (T-t)}$$

$$m = E_1$$
Disconnected

SET / IVST

Connected (a-π)
Bound states masses

Spin-0 Pseudo-scalar Adjoint meson \(a-\eta'\)

Spin-0 Scalar Adjoint meson \(a-f_0\)

Lattice: \(24^3 \times 48\) \(\beta=1.6\) TS-PHMC action: tISym+Stout, Glueball 0^+

Spin-0 Scalar glueball
Spin-1/2
Gluino-Glue

Bound states masses

Gluino-Glue

$\text{Tr}_{\gamma_0}$

$\kappa_s = 0.155$

$\kappa_s = 0.157$

$\kappa_s = 0.1575$

$\text{Tr}_l$

$\text{am}_{eff}$

$\tilde{t}_i$
Chiral ~ SUSY limit

Lattice: $24^3 \cdot 48 \beta = 1.6$ TS-PHMC
(unstout)

$$k_{cr}^{Wls} \sim 0.2027$$

SUSY Ward-Identities
(renormalized gluino mass)

$$am_{\tilde{g}} Z_S = \frac{1}{2} \left( \frac{1}{k} - \frac{1}{k_{cr}} \right)$$

$$ (am_\pi)^2 \sim A \left( \frac{1}{k} - \frac{1}{k_{cr}} \right)$$

$k_{cr}$: Critical hopping parameter.

OZI-arguments
Chiral transition

Spontaneous discrete chiral symmetry breaking

\[ Z_4 \longrightarrow Z_2 \]

Two vacua:

\[ \langle \bar{\lambda} \lambda \rangle_+ > 0 ; \langle \bar{\lambda} \lambda \rangle_- < 0 \]

At zero gluino mass

Renormalized condensate:

\[ \langle \bar{\lambda} \lambda \rangle_{R(\mu)} = Z(a\mu)\langle \bar{\lambda} \lambda \rangle - b_0(a\mu) \]

First order phase transition
Bound states spectrum

Spectrum of SU(2) Super-Yang-Mills on the Lattice
action: tlSym (gauge) + Wilson (gauginos), Algorithm: TS-PHMC

- Lattice simulation $16^3.32$ and $24^3.48$, $a \sim 0.1$ fm

- $m_{a-\pi} \sim 460$ MeV

- $m_{\tilde{g}\tilde{g}} = 1580$ MeV

- $m_{\tilde{g}\tilde{g}} = 760$ MeV

- Possible mixing in the Scalar channel ($f_0 - 0^+$)

SUSY + $O(a)$ effects
The first *quantitative* results of low-energy spectrum of SU(2) SYM

Large physical volume $L > 2 \, \text{fm}$ (required for spectrum studies)

*Finite size effects* under control

*Higher statistics* for analysis were collected

Efficient algorithm for *dynamical* simulations: *TS-PHMC*

Small gluino mass $m \sim 126 \, \text{MeV}$

Is Gluino-Gluino and Gluino-Glue *mass splitting* an $O(a)$ effect? !?

Answer: *extrapolation* to continuum limit

Next: apply recent *QCD* methods of spectroscopy to SYM
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**Thank you!**