Introduction to Supersymmetry at the NLC

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This document is geared toward knowledgeable undergraduates or first-year graduate students who might be interested in joining our group.

High energy physics is the study of elementary particles that make up the world around us. Our group is interested in physics at energies higher than ever before achieved at any electron-positron collider. We are investigating the physics opportunities at the Next Linear Collider (NLC), to be built in the next decade. In particular, we are focussing on one particular class of theories "beyond the Standard Model" called supersymmetry (SUSY).

This document begins with a brief introduction of the Standard Model; then it discusses problems with the Standard Model and how supersymmetry fixes them; then it goes on to discuss the proposed Next Linear Collider; and finally it finishes with a discussion of some of the studies we are performing in our group.

1 The Standard Model

The Standard Model (SM) describes all the elementary particles in nature, and the forces between them.

The spins of particles are quantized in multiples of half of Planck's constant $\hbar$. Particles whose spins are half-integer multiples of Planck's constant have very different behavior from those whose spins are integer multiples of $\hbar$. The former are called fermions[1], and they make up all the matter that we see around us; the latter are called bosons, and they act as the carriers of the forces by which particles of matter interact with each other. The next two sections describe these particles.

[1] For those who know about statistical mechanics, fermions follow Fermi-Dirac statistics; the Pauli exclusion principle applies to them, so that you never have two fermions in the same state. Bosons follow Bose-Einstein statistics; they love to be in the same state as their neighboring bosons (like the photons in a laser beam).

The masses and other properties of all these particles can be found in the Particle Data Book.

1.1 The particles

Table 1 lists the fermions—the particles that make up matter in the world around us. All these particles have spins of $\frac{1}{2}\hbar$.

Generation	Leptons		Quarks	
1	e	ν_e	d	u
2	μ	ν_μ	s	c
3	τ	ν_τ	b	t

Table 1: The fermions; all have half-integer spins.

We can make the following observations about the fermions:

- The quarks usually interact via the strong force; leptons never do.
- There are three charged massive leptons: the electron, the muon and the tau. Each has a corresponding neutral massless partner called a neutrino.
- There are six quarks; the "down", "strange" and "bottom" quarks have an electric charge of $-1/3$ that of the proton, while the "up", "charmed" and "top" quarks have an electric charge of $+2/3$ that of the proton.
- For each of these particles, there is a corresponding antiparticle with exactly the same mass, but opposite electric charge, lepton and baryon numbers, intrinsic parity, etc. For example, the partner of the electron (e^-) is the positron (e^+), the partner of the tau neutrino (ν_τ) is the tau anti-neutrino ($\overline{\nu}_\tau$), and the partner of the up quark (u) is the up anti-quark ($\bar{u}$). This makes for twelve particles and twelve antiparticles.
- A particle that consists of quarks is called a hadron. A bound state of three quarks is called a baryon (for example, the proton is a uud state, the neutron is a udd state). A meson is a bound state of a quark and and antiquark (for example, a charged pion (π^+) is a $u\bar{d}$ state). The strong force is so strong that it's impossible to pull a quark free of a hadron.

- The fermions come in three "generations"; a particle in generations 2 or 3 are identical in almost every respect to the corresponding particle in the first generation, except that it is more massive (and they have different lepton and baryon numbers—see Appendix B). All the matter that we know about comes from the first generation—the massive particles of the second and third generation quickly decay into the lighter first generation particles. (It is currently a mystery why there are two extra "unnecessary" generations of particles.)

1.2 The forces

Table 2 lists the bosons—the particles that propagate forces. The spins are all integer multiples of $\hbar$.

Force	Particle	Spin
electromagnetic	γ (photon)	1
weak	$W^{\pm}$, Z^0	1
strong	g (gluon)	1
gravity	G (graviton)	2

Table 2: The bosons, and the forces that they are associate with.

A number of interesting points about the bosons:

- The electromagnetic force couples only to particles that have an electric charge.
- The weak force couples to every particle, but in general it is only seen in the radioactive decay of particles as in the decay of a neutron to a proton (which might be more properly described as the decay a d quark to a u quark, as described in Appendix A).
- The strong force holds the quarks inside the baryons and mesons.
- Although gravity is mentioned in this table, it is not really part of the Standard Model; it is more properly described by Einstein's general theory of relativity. The rest of the Standard Model is built upon quantum mechanics, and so far no one has come up with a consistent theory that combines both relativity and quantum mechanics. However, see section 1.3 for more about this.

1.3 Unification

To the best of our knowledge, the Standard Model agrees well with experimental data; however, high energy physicists are still unhappy with it since it is so ad hoc. For example, there are several dozen different parameters that describe the model, all of which cannot be predicted *a priori*, but must be measured. (Think of them as a few dozen different knobs that could be turned—each setting of the knobs would give you a completely different universe.) These parameters include the masses of the particles and the coupling strengths of the forces.

For many years, theorists have been working on trying to unify the forces; that is, to show that all four forces of nature can be derived from a single force (put another way, there could be a single force in nature, and the forces that we see are low-energy approximations to this single unified force). A major step towards unification occurred in the late '60's when Glashow, Weinberg and Salam showed that the electromagnetic force and the weak force are just two facets of the *electroweak* force.

In the process of unifying these two forces, a new boson called the Higgs boson (h^0) had to be introduced. Among other things, this boson gives masses to all the particles. This particle is the only one in the Standard Model that hasn't yet been observed experimentally (the theoretical situation isn't completely clear, either—you will see below that supersymmetry requires more than one Higgs boson).

The successful unification of the electromagnetic and weak forces has given particle physicists hope that all the other forces, including gravity, may be unified. Superstrings are a popular candidate these days, and supersymmetry is a natural consequence of many superstring theories.

There is another reason why supersymmetry is popular these days. We know that the coupling strengths of the forces change with energy; for example, the strength of the electromagnetic force at ordinary energies is $\alpha_{EM}(0) = 1/137$; but at the energies of LEP or SLC (90 GeV) the strength increases to $\alpha_{EM}(90\text{ GeV}) = 1/128$. If the forces really are unified at some high energy scale, then we would expect the electromagnetic, weak and strong forces to have the same strength at this unification scale. However, the top part of Figure 1 shows that the Standard Model coupling strengths extrapolated to very high energies do not converge at a single point. However, the introduction of supersymmetry (bottom plot) causes the coupling strengths to converge at a single point.

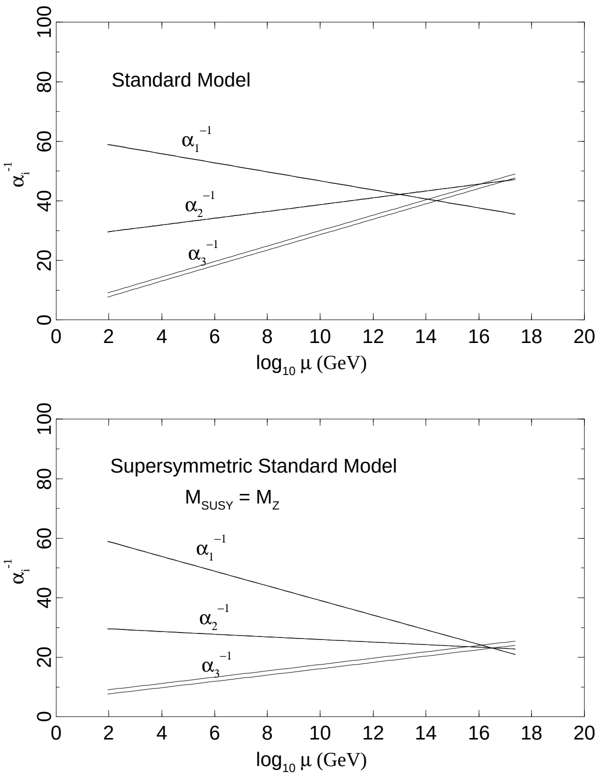

Figure 1: The convergence of the U(1), SU(2) and SU(3) coupling constants. (Note that the y-axis is the inverse of the coupling strength.) [Langacker]

2 Supersymmetry

2.1 Problems with the SM

One major problem with the Standard Model is something called the Higgs divergence problem. Figure 2 shows a Feynman diagram involving a Higgs boson (if you aren't familiar with Feynman diagrams and how to use them for calculations, now would be a good time to review appendix A). As was mentioned earlier, the Higgs boson gives masses to particles; this diagram is one of many that contributes to the Higgs boson's own mass. There are infinitely many such diagrams, involving more than one such fermion loop, and if one tries to calculate the Higgs mass correction, one gets a value which diverges to infinity. This is not very good.

Supersymmetry can solve this problem.

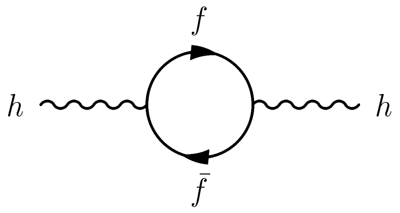

Figure 2: A Higgs boson dissociating into a virtual fermion-antifermion pair.

2.2 Supersymmetry

Supersymmetry postulates that for every Standard Model particle there is a corresponding supersymmetric particle (or "sparticle") which has a spin that is different by 1/2 unit. For example, the spin-1/2 electron will have a spin-0 supersymmetric partner called a "selectron", and the spin-1 photon has a spin-1/2 partner named a "photino"[2].

The existence of particles with exactly the same properties as the Standard Model particles, except for different spins, helps solve the divergence problem mentioned in the last section. For every diagram like Figure 2 there is a diagram that looks like Figure 3; these diagrams have the same vertices and coupling constants, and hence the same magnitude for the amplitude. But since the particle spins are different, the amplitude has the opposite sign. So when calculating the cross section using the prescription in appendix A, the amplitudes cancel yielding a finite interaction probability.

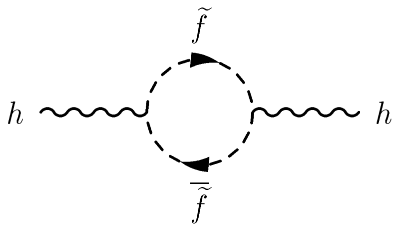

Figure 3: A Higgs boson dissociating into a virtual sfermion-antisfermion pair; this diagram cancels the one in figure 2.

[2]The notation is to prepend an "s-" to the names of the fermions to get the names of their partners, and to append "-ino" to the names of the bosons to get the names of their partners. When writing the symbols, we put a tilde above the particle's symbol to denote the supersymmetric partner (so $\tilde{\nu}_e$ is the electron sneutrino).

We know that supersymmetry cannot be an exact symmetry—if it were, there would be particles with exactly the properties of the electrons (including mass) as the electron, except for the spin. But a charged spin-0 particle with exactly the same mass of the electron would certainly have been seen already. If supersymmetry is broken, then the sparticles may have much greater masses than ordinary particles (though they must have masses less than about 1 TeV in order for the cancellation of Figure 3 to work).

2.3 R parity

In our studies we assume that R-parity is conserved (if that statement doesn't make sense, take a look at appendix B for more about conservation laws). The practical impact of this statement is that every supersymmetric interaction must involve two supersymmetric particles. In other words, all supersymmetric particles are produced in pairs, and if a SUSY particle decays, it must decay into another SUSY particle (plus any number of normal particles). This means that the lightest supersymmetric particle (LSP) cannot decay, since there aren't any supersymmetric particles that it can decay into. In our studies, we usually assume that the LSP is the lightest neutralino, $\widetilde{\chi}_1^0$.

2.4 The sparticles

The minimal supersymmetric standard model (MSSM) is the Standard Model with the fewest changes such that supersymmetry can be incorporated. (There is a slight change in the Higgs sector: instead of a single Higgs boson: there must be five: h^0, H^0, H^+, H^- and A^0.) We then postulate a superpartner for every Standard Model particle with the same coupling strengths. Unfortunately, this leads to a lack of predictive power—just as the masses in the Standard Model all the particle masses are arbitrary and must be measured, the same is true in the MSSM. In our studies, we work with more constrained models (for example, mSUGRA, a minimal-supergravity inspired model).

Table 3 shows the supersymmetric particles compared to the Standard Model particles.

We can make a number of observations:

- There are four neutralinos $\widetilde{\chi}_i^0$; each is a mixture of the photino, zino, and the two neutral Higgsinos.
- There are two charginos $\widetilde{\chi}_i^+$; each is a mixture of the wino and charged Higgsino.

Standard Model	Supersymmetry
γ, Z^0, h^0, H^0	$\widetilde{\chi}_1^0$, $\widetilde{\chi}_2^0$, $\widetilde{\chi}_3^0$, $\widetilde{\chi}_4^0$
W^+, H^+	$\widetilde{\chi}_1^+$, $\widetilde{\chi}_2^+$
e^-, ν_e, μ^-, ν_μ, ν_τ	$\widetilde{e}_R^-$, $\widetilde{e}_L^-$, $\widetilde{\nu}_e$, $\widetilde{\mu}_R^-$, $\widetilde{\mu}_L^-$, $\widetilde{\nu}_\mu$, $\widetilde{\nu}_\tau$
τ^-	$\widetilde{\tau}_1$, $\widetilde{\tau}_2$
u, d, s, c	$\widetilde{u}_R$, $\widetilde{u}_L$, $\widetilde{d}_R$, $\widetilde{d}_L$, $\widetilde{s}_R$, $\widetilde{s}_L$, $\widetilde{c}_R$, $\widetilde{c}_L$
b	$\widetilde{b}_1$, $\widetilde{b}_2$
t	$\widetilde{t}_1$, $\widetilde{t}_2$

Table 3: The Standard Model and supersymmetric particles.

- In the Standard Model, we don't bother distinguishing between left- and right-handed fermions, since they have the same mass and can be easily converted into each other by flipping the spins. Their supersymmetric partners are spin-0, so the sfermions appear as two different states possibly with different masses.

- Exception: in the Standard Model, there are only left-handed neutrinos, so you get only one sneutrino from each generation.

- Another exception: because the tau lepton and the bottom and top quarks are massive, their left- and right-handed partners can mix to form states known as $\widetilde{\tau}_1$, etc. (These states would be characterized by different branching ratios and anomalously low masses.)

3 The NLC

So to find supersymmetry, it is necessary to build a collider that can create particles with many hundreds of GeV in mass. We have been studying the possibility of building an electron-positron collider which will be able to reach energies as high as 1.5 TeV. For comparison, the LEP I and SLC colliders run at 90 GeV (the energy to create Z^0's), and LEP II exceeds 160 GeV (the energy needed to make W pairs).

3.1 Hadron and lepton colliders

The Large Hadron Collider will begin operation in a few years. It will be a proton-proton collider with energies as high as 14 TeV. One might ask why one would want to build an electron-positron collider as well.

Historically, hadron colliders such as the LHC and lepton colliders such as the NLC have been complementary. Hadron collider can go to much higher energies, while lepton colliders tend have much cleaner beams and lower backgrounds. Some of the advantages of the NLC will include:

- Large electron polarization
- Clean beams that allow complete detector coverage
- A beam energy that can be tuned to optimize the analysis

3.2 The NLC program

We expect that the NLC will have three phases, shown in table 4. In addition to the machine parameters show in the table, we expect to be able to polarize the electron beam, which has a significant effect on the rates of some interactions.

Phase	Energy	Luminosity
I	500 GeV	$5 \times 10^{33}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$
II	1 TeV	$1 \times 10^{34}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$
III	1.5 TeV	$1 \times 10^{34}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$

Table 4: Three phases of NLC running.

4 Current Research Topics

In general, we want to be able to answer three questions about supersymmetry at the NLC:

- If supersymmetry exists, will we be able to use the NLC to determine if there is physics beyond the Standard Model?
- Will we be able to distinguish between supersymmetry and any other competing theory?
- Will we be able to make precision measurements of the supersymmetric parameters, to determine exactly which model of supersymmetry occurs in nature?

There are many possible forms that supersymmetry can take. For our studies, theorists have suggested a number of points (which we number from 1 to 5) that should be studied. These five points have very distinct characteristics, and if the NLC can answer the above three questions for all the points, we may have a good chance at performing precision measurements no matter what form SUSY takes in nature. The five points in question are:

1. Chargino production
2. Slepton production
3. Low-mass Higgs, charginos and sleptons
4. Higgsinos and a variety of low-mass Higgs
5. Low mass stop squarks

4.1 Data selection

If one were to build the NLC detector and turn on the beam, one would see a great number of events (an "event" is a single electron-positron collision that produces signals in the detector) from the Standard Model processes. Somewhere buried in all this data, there may be events from the production of supersymmetric particles. The point of our current research is to simulate data corresponding to each of these five scenarios as well as an appropriate amount of Standard Model background, and then devise methods of using the detector to separate the SUSY signal from the background.

4.2 Projects

We have a number of ongoing projects in our group, including:

- For each of the five SUSY parameter sets, generate simulated data with a simple detector model, and see how well the detector works. More specifically, this involves:
  - Generating simulated data at different energies and different electron polarizations, both from the SUSY parameter set and the Standard Model
  - Looking at both the SUSY and SM data, find a set of "cuts" that remove the SM events but keep supersymmetry events (the cuts will be different for each of the different sparticles we are interested in)

- Measuring the masses of the sparticles by looking at the characteristics of their decay products (for example, in two-body decays one looks at the endpoints of the energy spectrum of the daughter particles). Of course, we know the true masses of the sparticles (since we generated them), so the point of the exercise is to find the size of error in the mass measurement method
    - Determining the coupling constants between the sparticles (by looking at the rates of reactions when varying the electron polarization)

- The above work uses a simple model of an NLC detector, based on our assumptions on how such a detector ought to work. We are also trying to verify these assumptions by using a much more detailed detector simulation program (called GEANT). With results from this simulation, we will be able to generate much more realistic events.

A Feynman Diagrams

Since we use several of them in this document, it is worthwhile explaining Feynman diagrams a bit more detail. Figure 4 shows an example where a neutron is decaying into a proton, an electron and an electron antineutrino. The neutron and proton are both bound states of three quarks (*udd* for the neutron and *uud* for the proton). As an exercise, verify that the electric charges work out right.

In this diagram, time flows from the left to the right. You can see that the decay of the neutron is really just the decay of a d quark into a u quark, with the other two quarks being unaffected.

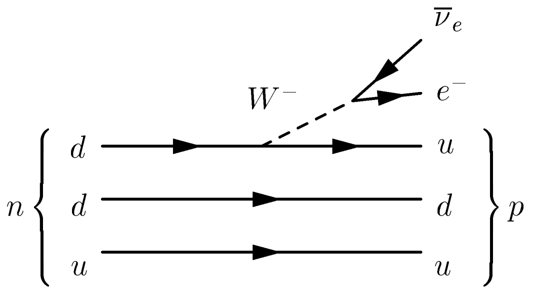

Figure 4: The decay of a neutron into a proton, electron and an electron antineutrino.

We conventionally draw fermions with solid lines, photons with wavy lines, gluons with spiral lines, and all other bosons with dashed lines. Arrows on the fermion lines indicate whether it is a particle or an antiparticle, with particle lines pointing in the direction of positive time.

In addition to the purely graphical display of how particles interact, Feynman diagrams are used to calculate the cross sections (or interaction probability or rate) for particle interactions. At every vertex (point where three particles meet) there is coupling constant (the coupling constant for the electromagnetic force is α, also known as the fine structure constant, which has a value of 1/137 (at low energies)). One multiplies all the coupling constants (and *propagator* terms that depend on the momenta and masses of the intermediate particles) to calculate the amplitude for the reaction. The rate of a reaction occurring is proportional to the square of the amplitude.

For a given set of initial state particles, there may be several reactions that produce identical final states. Figure 5 shows an example for the reaction $e^+e^- \rightarrow \tilde{e}_R^+\tilde{e}_R^-$, which can occur either via a photon or Z, or via a neutralino. When this is the case, quantum mechanics requires that one calculates the probability of the reaction by calculating the amplitude of each diagram, adding the amplitudes and then squaring the sum. It is possible for the amplitudes to cancel—depending on the masses and spins of the particles involved, the probability might end up being larger or smaller than the probability if only one diagram contributed. We use this argument in section 2.2 to explain why supersymmetry solves the Higgs divergence problem.

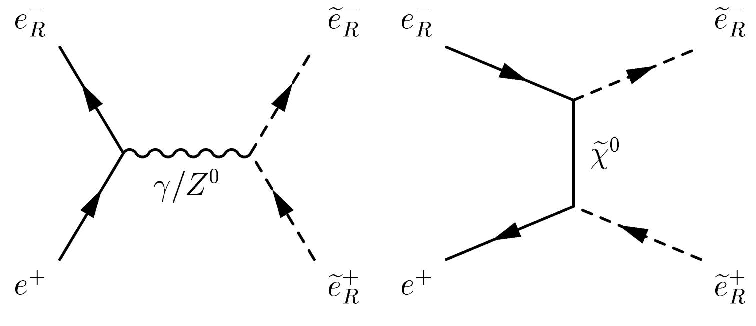

Figure 5: The production of pairs of right-handed selectrons via two different modes.

B Conservation Laws

In addition to the conserved quantities that you are familiar with, such as energy, momentum, angular momentum and electric charge, there are other quantities that are also conserved. For example, it is an observed fact that the total number of electrons plus electron neutrinos minus positrons minus electron antineutrinos in the universe is a constant.

Consider Figure 6. It shows the decay of a muon via the weak force: $\mu^- \to e^- \nu_\mu \overline{\nu}_e$. We can assign a number called the "electron number" to all particles, such that the electron and its neutrino are +1, the positron and the electron antineutrino are −1, and all other particles are 0. We can assign a similar number to muons and their neutrinos. Table 5 shows that these two quantum numbers are conserved; that is, the electron and muon number at the beginning is the same as the total of the electron and muon numbers in the final state. These quantities are generically called "lepton number."

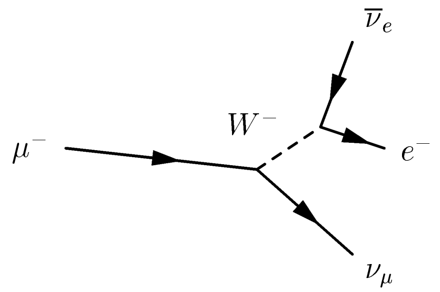

Figure 6: The decay of a muon into a muon neutrino, an electron and an electron antineutrino.

	μ^-	e^-	ν_μ	$\overline{\nu}_e$
electric charge	−1	−1	0	0
muon number	+1	0	+1	0
electron number	0	+1	0	−1

Table 5: The quantum numbers in the decay of a muon. Note that the sum of the last three columns (all the decay products) is the same as the first column (the initial particle).

If you stare at Figure 6 long enough, you can see that the electric charge "flows" from the muon through the W^- to the electron, the muon number flows from the muon to the muon neutrino and the electron number flows backwards along the antineutrino and then forwards through the electron. Thus, all the quantum numbers are conserved at every interaction vertex.

In addition to the electron, muon and tau numbers there is also a baryon number[3]; quarks have $+1/3$, antiquarks have $-1/3$ and all other particles have 0 (in other words, baryons have $+1$ and mesons and all other non-baryons have 0). Thus the total number of quarks plus antiquarks in the universe is constant[4]. If a baryon is created in an interaction, an antibaryon must be created at the same time to balance baryon number; but any number of mesons can be created without violating conservation laws.

We postulated in section 2.3 that there exists a quantum number known as R-parity. Unlike the additive quantum numbers discussed above, this is a multiplicative quantum number, where all normal particles have $R = +1$ and SUSY particles[5] have $R = -1$ (after reading the next paragraph and staring at Figure 7 you should see that it has to be defined this way, since the bosons have supersymmetric partners, but they don't have antiparticles in the way that the fermions do). We postulate that at every vertex, the *product* of R for each particle must be $+1$.

Figure 7 shows the production of a pair of neutralinos. There are two vertices in the graph; each vertex involves one SM particle and two SUSY particles; thus the R for each vertex is $(+1)(-1)(-1) = +1$.

A practical consequences of R-parity conservation is that every SUSY interaction must involve two SUSY particles. SUSY particles must be created in pairs, and a SUSY particle must decay into another SUSY particle and Standard Model particles. It also means that the lightest SUSY particle (LSP) cannot decay, since there is no lighter SUSY particle that it can decay into.

You may wish to browse through all the other diagrams in this paper, and verify that all these conservation laws are respected.

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[3]Note that there aren't separate numbers for the six flavors of quarks, since the weak force is allowed to cross generations in the quark sector (i.e. $c \to u\overline{u}d$ is allowed, but the corresponding $\mu^- \to \nu_e e^- \overline{\nu}_e$ never happens).

[4]It is not known why the lepton numbers and baryon number are conserved; it is just an observed fact of nature. Since these conservation laws are ad hoc, there are experiments that look for proton decay—i.e. baryon number violation—in the hopes of finding physics beyond the Standard Model.

[5]R is sometimes defined as $R = (-1)^{3B+L+2S}$, where B is baryon number, L is lepton number and S is spin. Verify that all particles have $R = +1$ and all sparticles have $R = -1$.

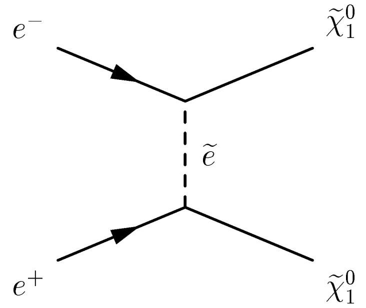

Figure 7: The production of a pair of neutralinos.