Forschungsseminar Quantenfeldtheorie

Eichfixierung in Higgs-Modellen

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11. Juni 2001

Literatur:

- E.S. Abers, B.W. Lee, *Gauge Theories*, Phys. Rep. **C9** (1973) 1
- B.W. Lee, J. Zinn-Justin, *Spontaneously Broken Gauge Symmetries I-IV*, Phys. Rev. **D5** (1972) 3121,3137,3155 & Phys. Rev. **D7** (1973) 1049
- B.W. Lee, *Renormalizable Massive Vector-Meson Theory - Peturbation Theory of the Higgs Phenomenon* Phys. Rev **D5** (1972) 823
- T. Kugo, *Eichtheorie*, Springer (1997)
- J.C. Taylor, *Gauge Theories of the Weak Interaction* Cambridge University Press (1976)
- L.H. Ryder, *Quantum Field Theory, 2nd ed.* Cambridge University Press (1996)
- M.E. Peskin, D.V. Schröder, *Introduction to Quantum Field Theory* Reseus Books (1995)
- einige vorläufige Ergebnissse aus der Vorbereitung der eigenen Diplomarbeit (work in progress)

Spontane Symmetriebrechung

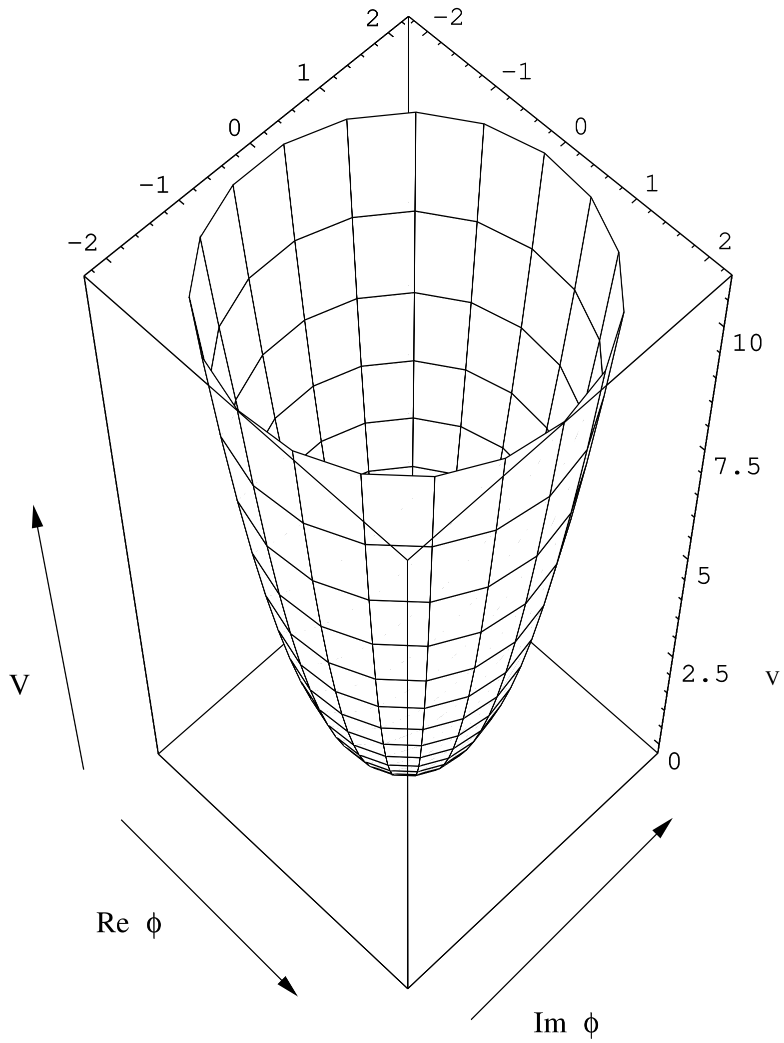
Vakuum respektiert nicht mehr die Symmetrie der zugrunde-liegenden Theorie

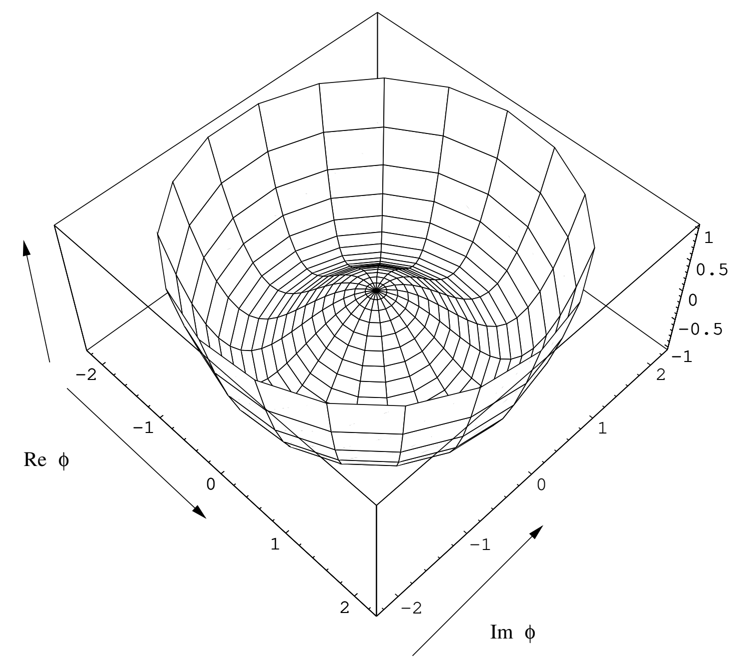
Einfaches Beispiel: ϕ^4-Theorie mit globaler $U(1)$-Symmetrie

$$\mathscr{L} = \partial_\mu \phi \partial_\mu \phi^* - \underbrace{m^2 |\phi|^2 - \lambda |\phi|^4}_{V(|\phi|)}$$

Invariant unter $\phi \longrightarrow \phi' = e^{i\alpha}\phi$

Minimum des Potentials $V(|\phi|)$ hängt von $\mathrm{sgn}(m^2)$ ab





Entwicklung um Vakuum im Falle Spontan gebrochener Symmetrie ($m^2 < 0$) um Vakuumerwartungswert $a \in \mathbb{R}$:

$$\phi(x) \;=\; [\rho(x) \;+\; a]\, e^{i\theta(x)}$$

$$V \;=\; \lambda\rho^4 \;+\; 4a\lambda\rho^3 \;+\; 4\lambda a^2\rho^2 - \lambda a^4$$

$$m_\rho^2 \;=\; 4\lambda a^2$$

kein Massenterm für $\theta(x) \Rightarrow$ Goldstone-Boson

Allgemeiner Fall:

- Symmetrie Gruppe der Lagrange-Dichte: G
- Vakuum gebrochen bis auf Untergruppe H von G
- $\Rightarrow$ Anzahl der Goldstone-Bosonen $= \mathrm{dim} G/H$

Higgs-Mechanismus

bisher nur globale Symmetrieen betrachtet, nun lokale Symmetrien

allgemein:

- Skalare Felder $\phi_i \in \mathbb{R}$
- $\mathscr{L}(\phi_i)$ sei invariant unter G

$$G : \phi_i(x) \longrightarrow (1 + i\alpha^a(x)t^a)_{ij}\phi_j(x)$$

$$t^a_{ij} = iT^a_{ij}$$

- kovariante Ableitung

$$\begin{aligned} D_\mu\phi &= (\partial_\mu - igA^a_\mu t^a)\,\phi \\ &= (\partial_\mu + gA^a_\mu T^a)\,\phi \end{aligned}$$

- Kinetischer Term

$$\begin{aligned} \frac{1}{2}(D_\mu\phi_i)^2 &= \frac{1}{2}(\partial_\mu\phi_i)^2 + gA^a_\mu(\partial_\mu\phi_i T^a_{ij}\phi_j) \\ &\quad + \frac{1}{2}g^2 A^a_\mu A^b_\mu (T^a\phi)_i (T^b\phi)_i \end{aligned}$$

- Vakuumerwartungswert des Feldes

$$< \phi_i > = (\phi_0)_i$$

- Entwicklung

$$\phi_i' = \phi_i + (\phi_0)_i$$

- $\Rightarrow$
  - Massenterm

$$\frac{1}{2} g^2 A_\mu^a A_\mu^b (T^a \phi_0)_i (T^b \phi_0)_i$$

$$m_{ab}^2 = g^2 (T^a \phi_0)(T^b \phi_0)$$

    * postitiv semidefinite Massenmatrix
    * $T^a \phi_0 = 0 \quad \rightarrow$ entsprechendes Eichboson massenlos
  - Beitrag der Goldstone-Bosonen aus $g A_\mu^a (\partial_\mu \phi_i T_{ij}^a \phi_j)$

$$g A_\mu^a \partial_\mu \phi_i (T^a \phi_0)_i + \ldots$$

nur Beitrag parallel zu $T^a \phi_0$ Goldstone-Bosonen

Abelsches Higgs-Modell, $D = 2$

$$
\begin{aligned}
\mathscr{L} &= \mathscr{L}_G + \mathscr{L}_\phi \\
\mathscr{L}_G &= \frac{1}{4} F_{\mu\nu} F_{\mu\nu} \\
\text{mit: } F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \;\; A_\mu(x) \in \mathbb{R} \\
\mathscr{L}_\phi &= |D_\mu \phi|^2 \; + \; V(|\phi|) \\
& \quad \text{mit } \phi(x) \in \mathbb{C} \\
D_\mu &= \partial_\mu \; - \; i e A_\mu \\
V(|\phi|) &= m^2 |\phi|^2 \; + \; \frac{g}{6} |\phi|^4
\end{aligned}
$$

Lokale $U(1)$-Symmetrie, d.h. invariant unter:

$$
\begin{aligned}
\phi(x) &\longrightarrow \phi'(x) \; = \; e^{-i\alpha(x)} \, \phi(x) \\
A_\mu &\longrightarrow A'_\mu(x) \; = \; A_\mu(x) - \frac{1}{e} \partial_\mu \alpha(x) \\
F_{\mu\nu} &\longrightarrow F'_{\mu\nu} \; = \; F_{\mu\nu}
\end{aligned}
$$

Minimum des Potentials

$$V(|\phi|) \;=\; m^2|\phi|^2 \;+\; \frac{g}{6}|\phi|^4$$

- Symmetrische Phase $m^2 > 0$
  - $\phi_0 = 0$
  - Eichfeld masselos
- Higgs-Phase $m^2 < 0$
  - $|\phi_0|^2 = -\frac{3m^2}{g} = \frac{3\widetilde{m}^2}{2g}$
    $(m^2 = -\frac{\widetilde{m}^2}{2}, \widetilde{m}^2 > 0)$
  - Eichfeld erhält Masse

Quantisierung von Eichfeldern

- Pfadintegralmethode: $\int \mathcal{D}\phi \mathcal{D}A_\mu \, e^{-\int d^2x \, \mathscr{L}(\phi(x), A_\mu(x))}$
- Eichinvarianz unter Transformation G

$$\begin{aligned} A_\mu &\rightarrow A_\mu^g \\ \phi &\rightarrow \phi^g \end{aligned}$$

besagt

$$\mathscr{L}(\phi(x), A_\mu(x)) \;=\; \mathscr{L}(\phi^g(x), A_\mu^g(x))$$

- Eichorbit von (A, ϕ) sind sämtliche Konfigurationen (A^g, ϕ^g)
- da in $\int \mathcal{D}A_\mu$ alle Konfigurationen enthalten, auch der gesamte Eichorbit $\int \mathcal{D}g \Longrightarrow$ divergenter Anteil
- wähle durch Eichbedingung je einen Repräsentanten eines Eichorbits aus: **Eichfixierung** (gauge-fixing)

Unitäre Eichung

- Skalarfeld: $\phi(x) \;=\; \rho(x)\, e^{i\omega(x)}$, $\rho(x), \omega(x) \in \mathbb{R}$
- Kovariante Ableitung

$$\begin{aligned} D_\mu\phi &= [\partial_\mu\rho \,+\, i(\partial_\mu\omega - eA_\mu)\rho]\, e^{i\omega} \\ |D_\mu\phi|^2 &= (\partial_\mu\rho)^2 \,+\, \rho^2(\partial_\mu\omega - eA_\mu)^2 \end{aligned}$$

- Eichtransformation

$$\begin{aligned} \omega(x) &\longrightarrow \omega'(x) \,=\, \omega(x) \,-\, \alpha(x) \\ \rho(x) &\longrightarrow \rho'(x) \,=\, \rho(x) \\ A_\mu(x) &\longrightarrow A'_\mu(x) \,=\, A_\mu(x) \,-\, \frac{1}{e}\partial_\mu\alpha(x) \\ F_{\mu\nu} &\longrightarrow F'_{\mu\nu} \,=\, F_{\mu\nu} \end{aligned}$$

- Lagrangedichte

$$\begin{aligned} \mathscr{L} &= \underbrace{\frac{1}{4}F_{\mu\nu}F_{\mu\nu}}_{\mathscr{L}_{G,\mathrm{quad}}} + \underbrace{(\partial_\mu\rho)^2 \,+\, m^2\rho^2}_{\mathscr{L}_{\rho,\mathrm{quad}}} \\ &\quad + (\partial_\mu\omega)^2\rho^2 \,+\, e^2A_\mu^2\rho^2 \,-\, 2e(\partial_\mu\omega)A_\mu\rho^2 \,+\, \frac{g}{6}\rho^4. \end{aligned}$$

- **Eichfixierung** $\alpha(x) \equiv \omega(x)$ damit:

$$
\begin{aligned}
\rho'(x) &= \rho(x) \\
\omega'(x) &= 0 \\
A'_\mu(x) &= A_\mu - \frac{1}{e}\partial_\mu \omega(x) \equiv B_\mu \\
F'_{\mu\nu} &\equiv \partial_\mu B_\nu - \partial_\nu B_\mu
\end{aligned}
$$

- Potential

$$
\begin{aligned}
V(\rho) &= -\frac{\widetilde{m}^2}{2}\rho^2 + \frac{g}{6}\rho^4 \\
&= \frac{g}{6}\left(\rho^2 - \frac{v^2}{2}\right)^2 - \mathrm{const.} \\
& \qquad \frac{v^2}{2} = \frac{3\widetilde{m}^2}{2g}
\end{aligned}
$$

Entwickle $\rho(x)$ um Vakuumerwartungswert

$$
\rho(x) = \frac{1}{\sqrt{2}}\Big(v + \sigma(x)\Big)
$$

$$
\begin{aligned}
\mathscr{L} \;=\; & \underbrace{\frac{1}{2}(\partial_\mu \sigma)^2 + \frac{\widetilde{m}^2}{2}\sigma^2}_{\mathscr{L}_{\sigma,\mathrm{quad}}} + \underbrace{\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{2}e^2 v^2 B_\mu^2}_{\mathscr{L}_{\mathrm{G,\ quad}}} \\
& \underbrace{+\frac{1}{2}e^2\sigma^2 B_\mu^2 + e^2 v \sigma B_\mu^2 + \frac{gv}{3!}\sigma^3 + \frac{g}{4!}\sigma^4}_{\mathscr{L}_{\mathrm{int}}}
\end{aligned}
$$

- Masse des σ-Feldes: $m_0^2 = \widetilde{m}^2$
- Masse des B_μ-Feldes: $m_{0,v}^2 = e^2 v^2$

Feynman Regeln, Unitäre Eichung

- Propagatoren
  - Skalarfeldpropagator:

$$\Delta_\sigma \;=\; \frac{1}{m_0^2 + k^2} \;=\;$$

  - Eichfeldpropagator:

$$\Delta_{\mu\nu} \;=\; \frac{1}{m_{v,0}^2 + k^2}\left(\delta_{\mu\nu} \;+\; \frac{k_\mu k_\nu}{m_{v,0}^2}\right) \;= \mu$$

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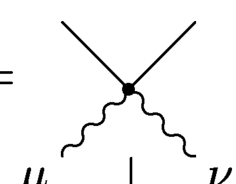
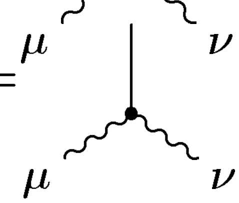
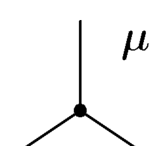
- Vertizes
  - $\sigma^2 B_\mu^2$-Vertex: $-2e^2\delta_{\mu\nu} \;=$



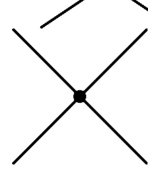
  - σB_μ^2-Vertex: $-2e^2 v\delta_{\mu\nu} \;=$



  - σ^3-Vertex: $-gv \;=$



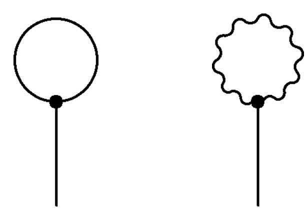
  - σ^4-Vertex: $-g \;=$

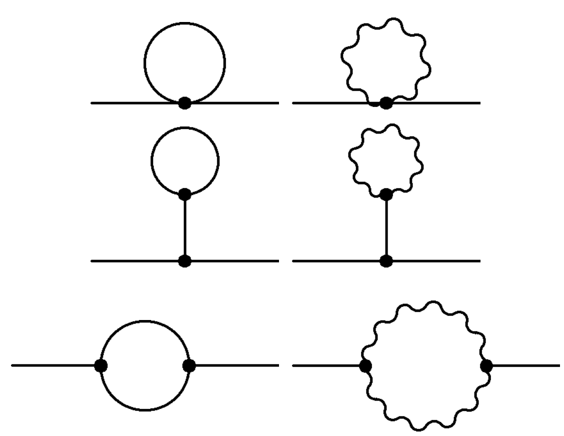


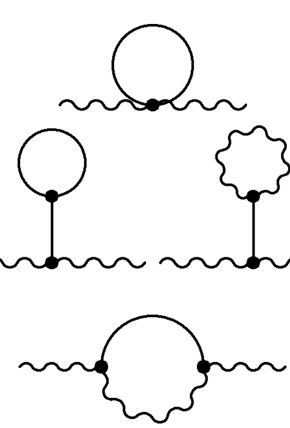
- Integration über innere Impulse $\frac{1}{(2\pi)^2}\int d^2 k_i$
- Impulserhaltung an jedem Vertex: $\delta(\sum_i k_i)$
- Division durch S

$$S = 2^l \left(\prod_i v_{i,\sigma}!\right) \left(\prod_i v_{i,B}!\right) r$$

1-Loop Graphen, Unitäre Eichung







Allgemeine Eichfixierung: R_ξ-Eichungen

- ϕ_i Skalarfelder ($i = 1, \ldots , K$)
- Transformation unter G (i.a. reduzibel)

$$\phi \longrightarrow (1 + iu^\alpha L_\alpha)\phi$$

mit u^α: infinitesimale Parameter und L_α: Darstellende der Generatoren der Gruppe G

- Eichfixierungsfunktion:

$$F_\alpha(A_\mu, \phi) - a_\alpha = 0$$

mit:

- a_α beliebige Funktion der Raum-Zeit-Koordinaten
- $F_\alpha(A_\mu^g, \phi^g) - a_\alpha = 0$ besitzt eine eindeutige Lösung g (gegeben: A_μ, ϕ)

- Identität:

$$1 = \int \mathcal{D}g \, \delta(F(A^g)) \det\left(\frac{\delta F(A^g)}{\delta g}\right)$$

-

$$F_\alpha(A_\mu^g, \phi^g) = F_\alpha(A_\mu, \phi) + M_{Fab} u_b + \mathcal{O}(u^2)$$

mit $M_{Fab} = \left.\frac{\delta F_\alpha(A_\mu^g,\phi^g)}{\delta u_\beta}\right|_{u=0}$

- Identität in Funktionalintegral einsetzen

$$\int \mathcal{D}g \det(M_F)\delta(F_\alpha(A_\mu^g,\phi^g) - a_\alpha) \int \mathcal{D}A_\mu \mathcal{D}\phi e^{-S}$$

Eichinvarianz, $\mathcal{D}A_\mu^g = \mathcal{D}A_\mu$

$$\left(\int \mathcal{D}g\right) \int \mathcal{D}A_\mu \mathcal{D}\phi \det(M_F)\delta(F_\alpha(A_\mu,\phi) - a_\alpha)\, e^{-S}$$

- a_α beliebig, Gauß-Integration $\int \mathcal{D}a_\alpha e^{-\frac{1}{2}\int d^D x\, a_\alpha^2}$
- Erzeugendes Funktional nun:

$$Z_F[J, J_\mu] \;=\; \int \mathcal{D}A_\mu \mathcal{D}\phi \det M_F\, e^{-\int d^D x\, (\mathscr{L} + \frac{1}{2}F_\alpha^2 - J_\mu A_\mu - J\phi)}$$

- $\mathscr{L}_{GF} \;=\; \frac{1}{2}F_\alpha^2$

Offene Fragen: Unabhängigkeit der Theorie von der gewählten Eichung, Eichfixierungsfunktion, $\det M_F$

Berechnung von $\det$: Geist-Felder

$$\det M_F \;=\; \int \mathcal{D}\bar{c}\mathcal{D}c\; e^{-\int d^n x\; \bar{c}(x)\; M_F\; c(x)}$$

- $c(x), \bar{c}(x)$ komplexe Grassmannfelder
- Geist-Felder, da Dynamik eines Skalaren Feldes, jedoch keine c-Zahlen
- $\Rightarrow$ **unphysikalische Teilchen, nur in Schleifenbeiträgen**
- Hinzufügen eines (Fadeev-Popov-) Geistterms zu Lagrangedichte

$$\mathscr{L}_{FP} \;=\; \bar{c} M_F c$$

Wahl von F_α

$$\mathscr{L} = \frac{1}{4}F^\alpha_{\mu\nu}F_{\alpha\mu\nu} + \frac{1}{2}(D_\mu\phi)^2 + V(\phi)$$

•

$$\begin{aligned} F^\alpha_{\mu\nu} &= \partial_\mu A^\alpha_\mu - \partial_\nu A^\alpha_\nu + C_{\alpha\beta\gamma}A^\beta_\mu A^\gamma_\nu \\ D_\mu\phi &= (\partial_\mu + iL_\beta A^\beta_\mu)\phi \\ & \quad \text{Strukturkonstanten:}[L_\alpha, L_\beta] = iC_{\alpha\beta\gamma}L_\gamma \end{aligned}$$

- $V(\phi)$ invariant, Polynom der Ordnung 4 in ϕ
- Vakuumerwartungswert v, damit $\phi = v + \phi'$

Entwicklung um v:

$$\begin{aligned}\mathscr{L} &= \frac{1}{4}F^{\alpha}_{\mu\nu}F_{\alpha\mu\nu} + \frac{1}{2}(v, L_\alpha L_\beta v)A^{\alpha}_{\mu}A^{\beta}_{\mu} + \frac{1}{2}(D_\mu\phi')^2 \\ &\quad -i(v, L_\alpha \partial_\mu \phi')A^{\alpha}_{\mu} + (v, L_\alpha L_\beta \phi')A^{\alpha}_{\mu}A^{\beta}_{\mu}) \\ &\quad +V(\phi' + v)\end{aligned}$$

$$\begin{aligned}V(\phi' + v) &= V(v) + \phi'_i \frac{\partial V(v)}{\partial v_i} + \ldots \\ &\quad + \frac{1}{4!}\phi'_i\phi'_j\phi'_k\phi'_l \frac{\partial^4 V(v)}{\partial v_i \partial v_j \partial v_k \partial v_l}\end{aligned}$$

- Minimum: $\frac{\partial V(v)}{\partial v_i} = 0$
- Eichbosonen-Massenmatrix: $(\mu^2)_{\alpha\beta} = (v, L_\alpha L_\beta, v)$
- P Projektor auf Goldstone-Bosonen, dann

$$(M^2)_{ij} = (1 - P)_{ik} \frac{\partial^2 V(v)}{\partial v_k \partial v_j}$$

$$\mathscr{L}_{quad} = \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{1}{2}A_\mu^\alpha(\boldsymbol{\mu}^2)_{\alpha\beta}A_\mu^\beta$$
$$+\frac{1}{2}(\partial_\mu\phi')^2 + \frac{1}{2}\phi_i'(M^2)_{ij}\phi_j' - iA_\mu^\alpha(v, L_\alpha\partial_\mu\phi')$$

Wähle nun Eichfixierungsfunktion so, daß Kopplung zwischen A_μ und ϕ' verschwindet, und $\mathscr{L}_{quad}$ nicht singulär:

$$F_\alpha = \sqrt{\xi}\left[\partial_\mu A\boldsymbol{\mu}^\alpha - \frac{i}{\xi}(v, L^\alpha\phi')\right]$$

$$\frac{1}{2}F^2 = \frac{\xi}{2}(\partial_\mu A_\mu^\alpha)^2 + \frac{1}{2\xi}(v, L^\alpha\phi')^2 - i(\partial_\mu A_\mu^\alpha)(v, L^\alpha\phi')$$

Renormierte S-Matrix unabhängig von ξ

benutze Ward-Takahashi-Identität für Z_F

Notationen:

- ϕ_a Skalar- und Eichfelder
- $a = \begin{cases} \alpha = 1, \ldots, N & \text{Erzeugende von } G \\ i = 1, \ldots, K & \text{Skalarfelder} \end{cases}$
- $$\begin{aligned} \phi_a^g &= \phi_a + [\Gamma_{ab}^\alpha \phi_b + \Lambda_a^\alpha]\, u_\alpha + \mathcal{O}(u^2) \\ \text{mit z. Bsp.: } \Gamma_{ab}^\alpha &= iC_{a\alpha b}\delta(x_a - x_b) \\ \Lambda_a^\alpha &= -\delta_{\alpha a}\frac{\partial}{\partial x_a}\delta(x_a - x_\alpha) \end{aligned}$$
- $Z_F[J] = \int \mathcal{D}\phi \, \det M_F \, e^{-S[\phi]+\frac{1}{2}F_\alpha^2+J_a\phi_a}$
- $F_\alpha(\phi^g) = F_\alpha(\phi) + (M_F)_{\alpha\beta}u_\beta + \mathcal{O}(u^2)$

der Weg zur WT-Id.:

- $\phi_a \longrightarrow \phi_a^g \equiv \phi' = \phi_a + (\Gamma_{ab}^{\beta}\phi_b + \Lambda_a^{\beta})\boldsymbol{u}_\beta$
- Einschränkung: $(M_F)_{\alpha\beta}\boldsymbol{u}_\beta \;=\; \lambda_\alpha \;=\; \mathrm{const.}$
- $\Rightarrow \begin{cases} \boldsymbol{u}_\alpha \;=\; \left(M_F^{-1}(\phi)\right)_{\alpha\beta}\lambda_\beta \;=\; \boldsymbol{u}_\alpha(\phi) \\ F_\alpha(\phi') \;=\; F_\alpha(\phi) \;+\; \lambda_\alpha \;+\; \mathcal{O}(\lambda^2) \end{cases}$
- $\mathcal{D}\phi\,\det M_F(\phi) \;=\; \mathcal{D}\phi'\,\det M_F(\phi')$ (ohne Beweis, s. Zinn-Justin/Lee IV)

- Änderung der Integrationsvariablen ändert Wert nicht $\Rightarrow$ $\frac{\delta}{\delta\lambda_\alpha} Z_F \; = \; 0$
-

$$
\begin{aligned}
0 &= \int \mathcal{D}\phi \det M_F e^{-S+\frac{1}{2}F_\alpha^2+J_a\phi_a} \frac{\delta}{\delta\lambda_\alpha}\{S+\frac{1}{2}F_\alpha^2+J_a\phi_a\} \\
0 &= \int \mathcal{D}\phi \det M_F e^{-S+\frac{1}{2}F_\alpha^2+J_a\phi_a} \{F_\alpha + J_a(\Gamma^\beta_{ab}\phi_b + \Lambda^\beta_a)(M_F^{-1})_{\beta\alpha}\} \\
0 &= \left\{ F_\alpha \left[\frac{\delta}{\delta J}\right] + J_a \left(\Gamma^\beta_{ab}\frac{\delta}{\delta J_b} + \lambda^\beta_a\right) \left(M_F^{-1}\left[\frac{\delta}{\delta J}\right]\right)_{\beta\alpha} \right\} Z_F[J]
\end{aligned}
$$

Ward-Takahashi-Identität

$$
\begin{aligned}
\left(M_F^{-1}\left[\frac{\delta}{\delta J}\right]\right)_{\beta\alpha} Z_F[J] &= ? \\
Z_{\alpha\beta}[J] &:= \int \mathcal{D}\phi \mathcal{D}c \mathcal{D}\bar{c}\; c_\alpha \bar{c}_\beta e^{-S+\frac{1}{2}F_\alpha^2+\bar{c}_\alpha (M_F)_{\alpha\beta} c_\beta + J_a\phi_a} \\
\left(M_F\left[\frac{\delta}{\delta J}\right]\right)_{\alpha\beta} Z_{\beta\gamma}[J] &= \int \mathcal{D}\phi \mathcal{D}c \mathcal{D}\bar{c} (M_F)_{\alpha\beta} c_\beta \bar{c}_\gamma e^{-S+\frac{1}{2}F_\alpha^2+\bar{c}_\alpha (M_F)_{\alpha\beta} c_\beta + J_a\phi_a} \\
&= \int \mathcal{D}\phi \mathcal{D}c \mathcal{D}\bar{c} \bar{c}_\gamma \left(-\frac{\partial}{\partial \bar{c}_\alpha}\right) e^{-S+\frac{1}{2}F_\alpha^2+\bar{c}_\alpha (M_F)_{\alpha\beta} c_\beta + J_a\phi_a} \\
\left(M_F\left[\frac{\delta}{\delta J}\right]\right)_{\alpha\beta} Z_{\beta\gamma}[J] &= \delta_{\alpha\gamma} Z_F[J] \\
\left(M_F^{-1}\left[\frac{\delta}{\delta J}\right]\right)_{\beta\alpha} Z_F[J] &= Z_{\beta\alpha}[J]
\end{aligned}
$$

$$F_\alpha \longrightarrow F_\alpha \;+\; \Delta F_\alpha$$

$$\begin{aligned}
\Delta Z &= Z_{F+\Delta F} \;-\; Z_F \\
&= \int \mathcal{D}\phi \, \det M_F \, e^{-S+\frac{1}{2}F_\alpha^2+J_a\phi_a} \left\{ F_\alpha \Delta F_\alpha \;+\; \frac{\delta \Delta F_\alpha}{\delta \phi_a} (\Gamma^\beta_{ab}\phi_b + \Lambda^\beta_a)(M_F^{-1})_{\beta\alpha} \right\}
\end{aligned}$$

$$\begin{aligned}
\det M_{F+\Delta F} &= \det \left\{ \left(\frac{\delta F_\alpha}{\delta \phi_a} + \frac{\delta \Delta F_\alpha}{\delta \phi_a} \right) \left(\Gamma^\beta_{ab}\phi_b + \Lambda^\beta_a \right) \right\} \\
&= \det \left\{ M_F \;+\; \frac{\delta \Delta F_\alpha}{\delta \phi_a} \left(\Gamma^\beta_{ab}\phi_b + \Lambda^\beta_a \right) \right\} \\
&= \det M_F \left(1 \;+\; \frac{\delta \Delta F_\alpha}{\delta \phi_a} \left(\Gamma^\beta_{ab}\phi_b + \Lambda^\beta_a \right) (M_F^{-1})_{\beta\alpha} \;+\; \mathcal{O}((\Delta F)^2) \right)
\end{aligned}$$

benutze $\Delta F_\alpha\left[\frac{\delta}{\delta J}\right]$ W.T. mit:

$$\Delta F_\alpha\left[\frac{\delta}{\delta J}\right]\; J_a\, e^{J_a\phi_a} \;=\; e^{J_a\phi_a}\left\{J_a\Delta F_\alpha(\phi) \;+\; \frac{\delta\Delta F_\alpha(\phi)}{\delta\phi_a}\right\}$$

$$\begin{aligned} 0 \;&=\; \int \mathcal{D}\phi \det M_F\, e^{S+\frac{1}{2}F_\alpha^2+J_a\phi_a} \;\cdot \\ &\cdot\left\{F_\alpha\Delta F_\alpha + \left[J_a\Delta F_\alpha(\phi) + \frac{\delta\Delta F_\alpha(\phi)}{\delta\phi_a}\right]\left(\Gamma^\beta_{ab}\phi_b + \Lambda^\beta_a\right)(M_F^{-1})_{\beta\alpha}\right\} \end{aligned}$$

$\Longrightarrow$

$$\Delta Z \;=\; \int \mathcal{D}\phi \det M_F\, e^{-S+\frac{1}{2}F_\alpha^2+J_a\phi_a} \;\cdot\; J_a\Delta F_\alpha(\phi)\left(\Gamma^\beta_{ab}\phi_b + \Lambda^\beta_a\right)(M_F^{-1})_{\beta\alpha}$$

1. Ordnung in ΔF:

$$Z_{F+\Delta F} \;=\; \int \mathcal{D}\phi \det M_F e^{-S+\frac{1}{2}F_\alpha^2+J_a\{\phi_a+(\Gamma^\alpha_{ab}\phi_b+\Lambda^\alpha_a)(M_F^{-1})_{\alpha\beta}\Delta F_\beta\}}$$

$$J_a\phi_a \;\xrightarrow{F\to F+\Delta F}\; J_a\{\phi_a + (\Gamma^\alpha_{ab}\phi_b + \Lambda^\alpha_a)(M_F^{-1})_{\alpha\beta}\Delta F_\beta\}$$

- S-Matrix Elemente: amputierte n-Punkt-Greenfunktionen, $p^2 = m^2$
- nur Pole der äußeren Beine tragen bei $\rightarrow$ Selbstenergiebeiträge
- $S_{F+\Delta F} \;=\; \prod_l \left(\frac{Z^R_{F+\Delta F}}{Z^R_F}\right)^{\frac{1}{2}}_l S_F$
- $$\boldsymbol{S} \;=\; \frac{\boldsymbol{S_F}}{\prod_l (\boldsymbol{Z_F})_l^{\frac{1}{2}}}$$

  unabhängig von der gewählten Eichfixierung

BRS-Symmetrie

- $\mathscr{L} + \mathscr{L}_{GF} + \mathscr{L}_{FP}$ nicht mehr eichinvariant
- gesucht: WT-Identitäten, aus invariantem $\mathscr{L}$
- Symmetrie im Falle der Eichfixierung: **B**ecchi, **R**ouet, **S**tora-Symmetrie[1]

- BRS-Trafo: $\boldsymbol{\delta_B}$
- $u_\alpha(x)$ Transformationsparameter, dann

$$u_\alpha(x) \to \lambda c_\alpha(x)$$

  mit: $c_\alpha(x)$ Geistfeld, λ Grassmannwertige Konstante
-

$$\begin{aligned} \delta\phi &= ig\lambda c_\alpha T_\alpha \phi \\ \delta A_\mu &= \lambda D_\mu c_\alpha \end{aligned}$$

- $\delta = \lambda \boldsymbol{\delta_B}$
- beachte: $\boldsymbol{\delta_B}(FG) = (\boldsymbol{\delta_B} F)G + (-)^{|F|} F(\boldsymbol{\delta_B} G)$
- Forderung: ${\boldsymbol{\delta_B}}^2 = 0 \Rightarrow \boldsymbol{\delta_B} c_\alpha = -\frac{1}{2} g f_{\alpha\beta\gamma} c_\beta c_\gamma$

[1] auch: BRST-Sym., +Tyutin

- Antigeistfeld $\bar{c}$, Nakanishi-Lautrup-Feld B

$$\boldsymbol{\delta_B} \bar{c}_\alpha \;=\; B_\alpha \;\Rightarrow \boldsymbol{\delta_B} B_\alpha = 0$$

- $F_\alpha(A, \phi, c, \bar{c}, B)$ mit verschwindender Geistzahl
- $\mathscr{L}_{GF+FP} \;=\; -\boldsymbol{\delta_B}(\bar{c}_\alpha F_\alpha)$
- $\boldsymbol{\delta_B}(\mathscr{L} + \mathscr{L}_{GF+FP}) = 0$
- $F_\alpha(A, \phi, B) \Rightarrow \begin{array}{c} \mathscr{L}_{GF} = B_\alpha F_\alpha \\ \mathscr{L}_{FP} = \bar{c}_\alpha \boldsymbol{\delta_B} F_\alpha \end{array}$

Anwendungen:

- Konstruktion von Eichtermen
- WT-Identitäten
- Unitarität der **physikalischen** S-Matrix
  - erhaltene BRS-Ladung $Q_B, Q_B^2 = 0$
  - $\mathcal{H}$ in $\mathcal{H}_0$, $\mathcal{H}_1$ und $\mathcal{H}_2$

$$\begin{aligned} \psi_1 \in \mathcal{H}_1 \quad &: \quad Q\psi_1 \neq 0 \\ \psi_2 \in \mathcal{H}_2 \quad &: \quad \psi_2 = Q\psi_1 \\ \psi_0 \in \mathcal{H}_0 \quad &: \quad Q\psi_0 = 0, \psi_0 \notin \mathcal{H}_2 \end{aligned}$$

  - $\{|phys>\} = \mathcal{H}_0$

R_ξ-**Eichung im Abelschen Higgs-Modell,** $D = 2$

$$\phi(x) \;=\; \frac{1}{\sqrt{2}}\left(v \;+\; \phi_1(x) \;+\; i\phi_2(x)\right) \;\; \text{mit: } \phi_1, \phi_2 \;\in\; \mathbb{R}$$

Eichtransformationen, infinitesimal

$$\begin{array}{ccccl}
\phi_1(x) & \longrightarrow & \phi_1'(x) & = & \phi_1(x) \;+\; \alpha\phi_2(x) \;+\; \mathcal{O}(\alpha^2) \\
\phi_2(x) & \longrightarrow & \phi_2'(x) & = & \phi_2(x) \;-\; \alpha\left(\phi_1(x) \;+\; v\right) \;+\; \mathcal{O}(\alpha^2) \\
A_\mu(x) & \longrightarrow & A_\mu - \frac{1}{e}\partial_\mu\alpha & &
\end{array}$$

$$
\begin{aligned}
\mathscr{L} \;=\; & \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{2}(\partial_\mu\phi_1)^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 + \frac{m_{0,v}^2}{2}A_\mu^2 - evA_\mu(\partial_\mu\phi_2) + \frac{1}{2}e^2A_\mu^2\phi_1^2 \\
& + \frac{1}{2}e^2A_\mu^2\phi_2^2 + eA_\mu\phi_2\partial_\mu\phi_1 - eA_\mu\phi_1(\partial_\mu\phi_2) + e^2vA_\mu^2\phi_1 + \frac{m_0^2}{2}\phi_1^2 \\
& + \frac{g}{24}\left(\phi_1^2 + \phi_2^2\right)^2 + \frac{gv}{6}\phi_1^3 + \frac{gv}{6}\phi_1\phi_2^2 + \underbrace{\frac{m^2v^2}{2} + \frac{gv^4}{24}}_{\text{const.}}
\end{aligned}
$$

Eichfixierung:

$$F = \left[\partial_\mu A_\mu - 2i\frac{e}{\xi}(v, L\phi)\right]$$

mit Integrationsfaktor $-\frac{\xi}{2}$

$$\begin{aligned}
\phi &= \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + v \\ \phi_2 \end{pmatrix} \\
v &= \frac{1}{\sqrt{2}}\begin{pmatrix} v \\ 0 \end{pmatrix} \\
L &= i\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \\
\phi &\longrightarrow \phi' = \Big(1 + i\alpha L + \mathcal{O}(\alpha^2)\Big)\,\phi.
\end{aligned}$$

$$(v, L\phi) = -\frac{1}{2}\,iv\phi_2$$

$$F \;=\; \left[\partial_\mu A_\mu \;-\; \frac{ev}{\xi}\,\phi_2\right]$$

$$\int d^2x\; (\partial_\mu A_\mu)\phi_2 \;=\; \underbrace{\partial_\mu \int d^2x\; A_\mu \phi_2}_{=0} - \int d^2x\; A_\mu(\partial_\mu \phi_2)$$

$$\mathscr{L}_{GF} \;=\; \frac{\xi}{2}F^2 \;=\; \frac{\xi}{2}(\partial_\mu A_\mu)^2 \;+\; \frac{m_{0,v}^2}{2\xi}\phi_2^2 \;+\; evA_\mu\partial_\mu\phi_2$$

$$
\begin{aligned}
M_F &= \left.\frac{\delta F(A'_\mu, \phi'_1, \phi'_2)}{\delta\alpha}\right|_{\alpha=0} \\
&= -\left[\frac{1}{e}\partial_\mu^2 - \frac{ev}{\xi}(\phi_1 + v)\right]
\end{aligned}
$$

$$
\mathscr{L}_{\text{ghost}} = \underbrace{-\frac{1}{e}\bar{c}\partial_\mu^2 c + \frac{ev^2}{\xi}\bar{c}c}_{\mathscr{L}_{\text{ghost, quad}}} + \underbrace{\frac{ev}{\xi}\phi_1\bar{c}c}_{\mathscr{L}_{\text{ghost, int}}}
$$

$$
\mathscr{L}_{\text{ghost}} = = \underbrace{-\bar{c}\partial_\mu^2 c + \frac{m_{0,v}^2}{\xi}\bar{c}c}_{\mathscr{L}_{\text{ghost, quad}}} + \underbrace{\frac{e^2 v}{\xi}\phi_1\bar{c}c}_{\mathscr{L}_{\text{ghost, int}}}
$$

Lagrangedichte in der R_ξ-Eichung

$$\mathscr{L} = \mathscr{L}_{\mathrm{G,\ quad}} + \mathscr{L}_{\phi_1,\mathrm{quad}} + \mathscr{L}_{\phi_2,\mathrm{quad}} + \mathscr{L}_{\mathrm{ghost,\ quad}} + \mathscr{L}_{\mathrm{int}} + \mathscr{L}_{\mathrm{ghost,\ int}}$$

$$\mathscr{L}_{\mathrm{G,\ quad}} = \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{\xi}{2}(\partial_\mu A_\mu)^2 + \frac{m_{0,v}^2}{2}A_\mu^2$$

$$\mathscr{L}_{\phi_1,\mathrm{quad}} = \frac{1}{2}(\partial_\mu \phi_1)^2 + \frac{m_0^2}{2}\phi_1^2$$

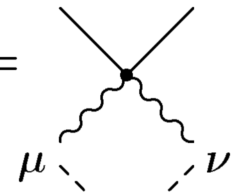
$$\mathscr{L}_{\phi_2,\mathrm{quad}} = \frac{1}{2}(\partial_\mu \phi_2)^2 + \frac{m_{0,v}^2}{2\xi}\phi_2^2$$

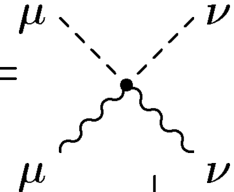
$$\mathscr{L}_{\mathrm{ghost,\ quad}} = -\bar{c}\partial_\mu^2 c + \frac{m_{0,v}^2}{\xi}\bar{c}c$$

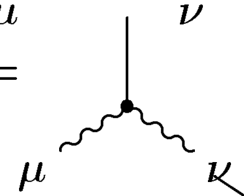
$$\begin{aligned}\mathscr{L}_{\mathrm{int}} + \mathscr{L}_{\mathrm{ghost,\ int}} = {} & \frac{1}{2}e^2A_\mu^2\phi_1^2 + \frac{1}{2}e^2A_\mu^2\phi_2^2 + e^2vA_\mu^2\phi_1 + eA_\mu(\partial_\mu\phi_1)\phi_2 \\ & - eA_\mu\phi_1(\partial_\mu\phi_2) + \frac{g}{24}\left(\phi_1^2+\phi_2^2\right)^2 + \frac{gv}{6}\left(\phi_1^3+\phi_1\phi_2^2\right) + \frac{e^2v}{\xi}\phi_1\bar{c}c\end{aligned}$$

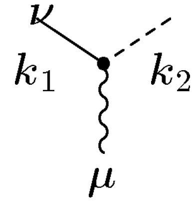
Feynman-Regeln, R_ξ-Eichung

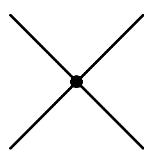
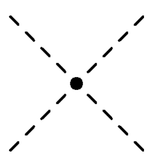
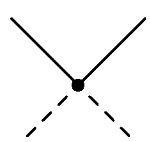
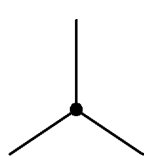
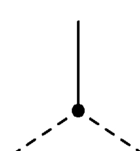
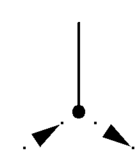
- Propagatoren
  - $\Delta_{\phi_1}(k) = \frac{1}{m_0^2 + k^2} =$
  - $\Delta_{\phi_2}(k) = \frac{1}{k^2 + \frac{m_{0,v}^2}{\xi}} =$
  - $\Delta_{\mu\nu,\xi}(k) = \frac{1}{(1+\xi)k^2 + m_{0,v}^2} \left(\delta_{\mu\nu} + \frac{1}{\xi} \frac{k_\mu k_\nu}{k^2 + \frac{m_{0,v}^2}{\xi}} \right) = \mu \quad \nu$
  - $\Delta_c(k) = \frac{1}{k^2 + \frac{m_{0,v}^2}{\xi}} =$

- Vertizes
  - $A_\mu^2\phi_1^2$-Vertex: $-2e^2\delta_{\mu\nu}\ =$ 

  - $A_\mu^2\phi_2^2$-Vertex: $-2e^2\delta_{\mu\nu}\ =$ 

  - $A_\mu^2\phi_1$-Vertex: $-2e^2 v\delta_{\mu\nu}\ =$ 

  - $A_\mu\phi_1\phi_2$-Vertex: $-ie(k_1 - k_2)_\mu\ =$ 

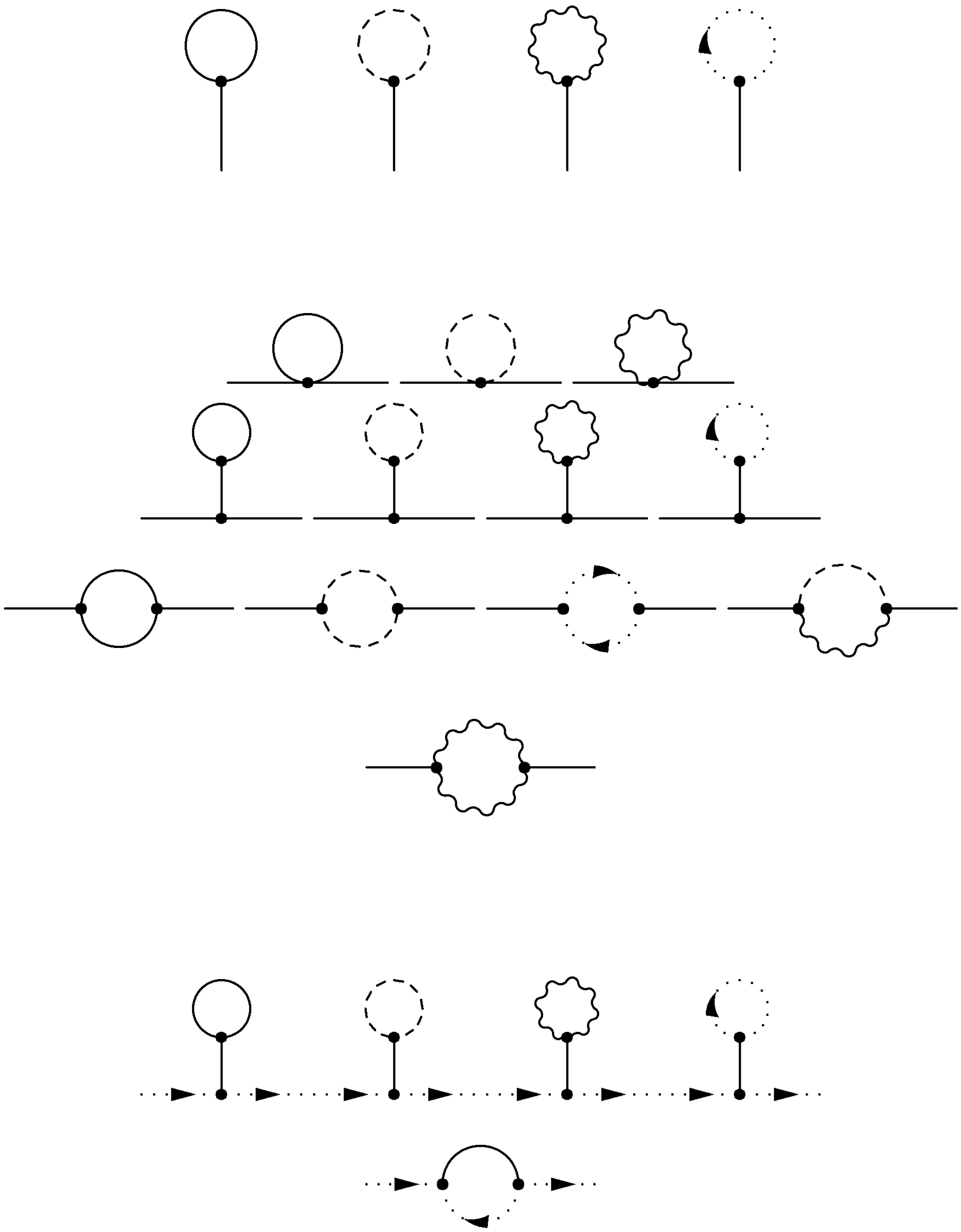
  - ϕ_1^4-Vertex: $-g\ =$ 
  - ϕ_2^4-Vertex: $-g\ =$ 
  - $\phi_1^2\phi_2^2$-Vertex: $-\frac{g}{3}\ =$ 
  - ϕ_1^3-Vertex: $-gv\ =$ 
  - $\phi_1\phi_2^2$-Vertex: $-\frac{gv}{3}\ =$ 
  - $\phi_1\bar{c}c$-Vertex: $\frac{e^2 v}{\xi}\ =$ 

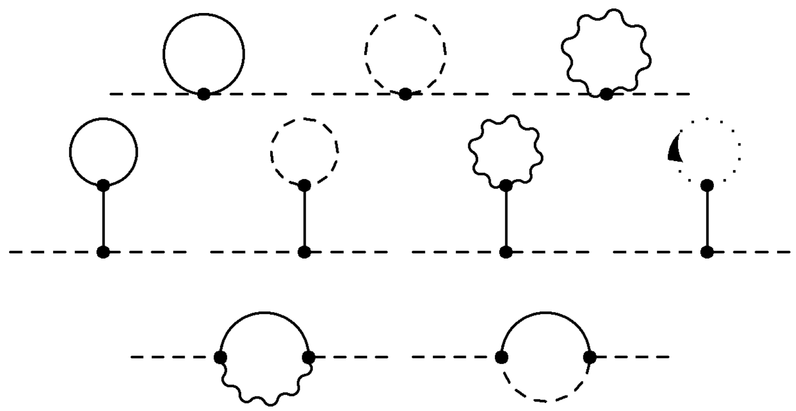
- Integration, Impulserhaltung
- $S \;=\; 2^l\,(-1)^L\,\left(\prod_i v_{i,\phi_1}!\right)\,\left(\prod_i v_{i,\phi_2}!\right)\,\left(\prod_i v_{i,A}!\right)\,\left(\prod_i v_{i,c}!\right)\,\left(\prod_i v_{i,\bar{c}}!\right)\,r$

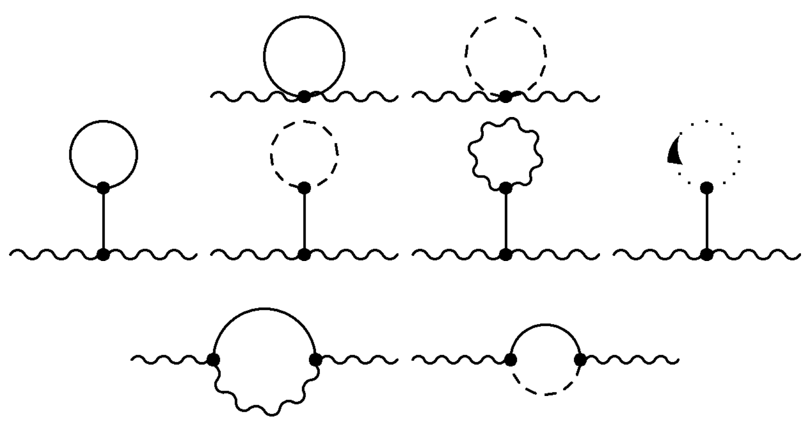
$$\frac{\delta^2}{\delta J_c(x)\delta J_c(y)} \;=\; -\frac{\delta^2}{\delta J_c(y)\delta J_c(x)}$$

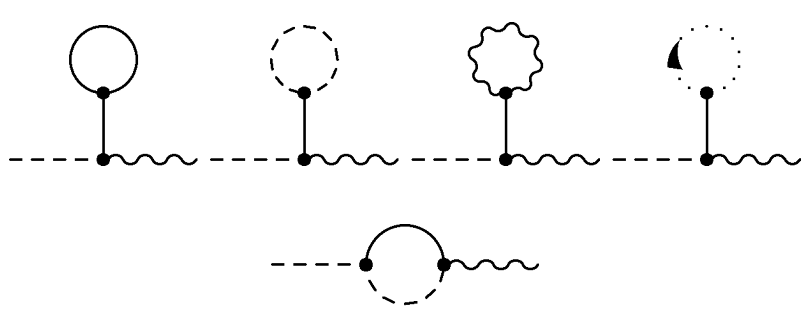
$$\begin{aligned}&\frac{\delta^2}{\delta \bar{J}_c(z_1)\delta J_c(z_1)}\frac{\delta^2}{\delta \bar{J}_c(z_2)\delta J_c(z_2)} \\ &\quad= \quad -\frac{\delta^2}{\delta \bar{J}_c(z_1)\delta J_c(z_2)}\frac{\delta^2}{\delta \bar{J}_c(z_2)\delta J_c(z_1)} \;\to\; -\Delta_c(z_1-z_2)\,\Delta_c(z_2-z_1)\end{aligned}$$

1-Loop Diagramme, R_ξ









Renormierung

work in progress...

-

$$\boldsymbol{D \;=\; 2 \;-\; 2\epsilon}$$

-

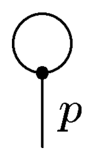
	D	$D = 2 - 2\epsilon$	$D = 2$
$[m_0^2]$	2	2	2
$[e^2]$	$4 - D$	$2 + 2\epsilon$	2
$[g]$	$4 - D$	$2 + 2\epsilon$	2
$[m_{v,0}^2] = [e^2 v^2]$	2	2	2
$[v^2] = [\frac{m_0^2}{g}]$	$D - 2$	-2ϵ	0

-

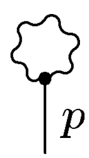
$$\begin{aligned} e &= \mu^{\epsilon}\, e' \\ g &= \mu^{2\epsilon}\, g' \\ v &= \mu^{-\epsilon}\, v' \end{aligned}$$

Vakuumgraphen

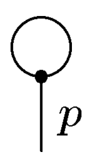
- Unitär



$$= \quad -\mu^{-\epsilon}\frac{gv}{8\pi m_0^2}\left(\frac{1}{\epsilon} - \ln\frac{m_0^2}{4\pi\mu^2} - \gamma + \mathcal{O}(\epsilon)\right)$$



$$= \quad -\mu^{-\epsilon}\frac{m_{0,v}^2}{4\pi v m_0^2}\left(\frac{1}{\epsilon} - \gamma - 2 - \ln\frac{m_{0,v}^2}{4\pi\mu^2} + \mathcal{O}(\epsilon)\right)$$



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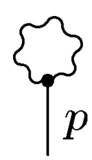
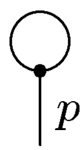


$=$

$$= \quad -\mu^{-\epsilon}\frac{3}{8\pi}\frac{e^2}{gv}\left[\left(2 + \frac{g}{e^2}\right)\left(\frac{1}{\epsilon} - \gamma - \ln\frac{m_0^2}{4\pi\mu^2}\right) - 2\ln\frac{3e^2}{g} - 4 + \mathcal{O}(\epsilon)\right]$$

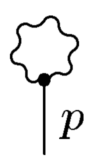
- R_ξ



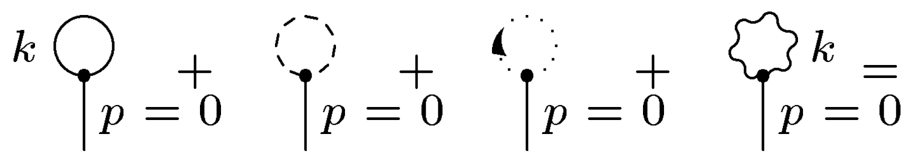
$$k \; \Big|_{p=0} = -\mu^{-\epsilon}\frac{gv}{8\pi m_0^2}\left[\frac{1}{\epsilon} - \gamma - \ln\frac{m_0^2}{4\pi\mu^2} + \mathcal{O}(\epsilon)\right]$$

$$\Big|_{p=0} = -\mu^{-\epsilon}\frac{gv}{24\pi m_0^2}\left[\frac{1}{\epsilon} - \gamma - \ln\frac{m_{0,v}^2}{4\pi\mu^2} + \ln\xi + \mathcal{O}(\epsilon)\right]$$

$$\Big|_{p=0} = \mu^{-\epsilon}\frac{e^2 v}{4\pi m_0^2}\,\frac{1}{\xi}\left[\frac{1}{\epsilon} - \gamma - \ln\frac{m_{0,v}^2}{4\pi\mu^2} + \ln\xi + \mathcal{O}(\epsilon)\right]$$

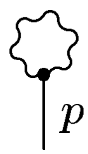
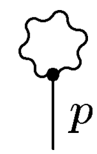


$$k \; \Big|_{p=0} = -\mu^{-\epsilon}\frac{e^2 v}{4\pi m_0^2}\left[\frac{2\xi+1}{\xi(\xi+1)}\left(\frac{1}{\epsilon} - \gamma - \ln\frac{m_{0,v}^2}{4\pi\mu^2}\right) - \frac{2}{1+\xi}\right.$$
$$\left. + \frac{1}{\xi+1}\ln(\xi+1) + \frac{1}{\xi}\ln\xi + \mathcal{O}(\epsilon)\right]$$

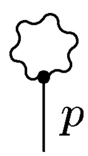
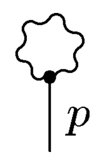


$$
= \quad -\mu^{-\epsilon}\frac{m_{0,v}^2}{8\pi v m_0^2}\Bigg[\left(\frac{1}{\epsilon} - \gamma - \ln\frac{m_0^2}{4\pi\mu^2}\right)\left(\frac{4}{3}\frac{g}{e^2} + \frac{2}{1+\xi}\right) \; - \; \left(\frac{1}{3} + \frac{2}{1+\xi}\right)\ln\frac{3e^2}{g}
$$

$$
- \frac{4}{1+\xi} \; + \; \frac{2}{1+\xi}\ln(1+\xi) \; + \; \frac{1}{3}\ln\xi \; + \; \mathcal{O}(\epsilon)\Bigg]
$$

 k, $p \underset{R_\xi}{=} 0$ $+$  $p \underset{R_\xi}{=} 0$ $= \dfrac{1}{1+\xi} \cdot \Bigg[$  k, $p \underset{\text{unitär}}{=} 0$ $+\; A \ln(1+\xi) \;+\; \mathcal{O}(\epsilon)\Bigg]$

$$A = -\mu^{-\epsilon}\frac{e^2 v}{8\pi m_0^2}$$

$\lim\limits_{\xi\to 0}$  k, $p \underset{R_\xi}{=} 0$ $+$  $p \underset{R_\xi}{=} 0$ $=$  k, $p \underset{\text{unitär}}{=} 0$