A mean field approach to self-organization in spatially extended perception-action and psychological systems

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Abstract

It is argued that perception-action systems should be considered as spatially extended systems on account of (i) the presence of spatially distributed synchronized brain activity during the performance of perceptual-motor tasks and (ii) the failure of conventional zero-dimensional theoretical approaches to deal with multistable perception-action systems and hysteresis in the presence of noise. It is shown that in spatially extended systems self-organization can arise due to the emergence of mean field attractors. This mean field approach is exemplified for a particular class of perception-action systems, namely, rhythmic movements. In addition, clinical implications of the mean field approach and the notion of spatially extended perception-action systems are briefly discussed in the context of psychotherapy and Parkinson's disease.

I. INTRODUCTION

Perception-action systems have been described with great success by means of synergetics [1–3] and dynamical system theory [4–7]. Recently, a plea has been made to incorporate structural elements in the descriptions of perception-action systems [8, 9]. In line with this suggestion, field theoretical models of movement-related brain activity have been developed [10–14] and the relevance of neural pathways [15, 16], musculo-skeletal constraints [17], and visual feedback [18] for movement coordination has been discussed. In this article, we address the issue of structural determinants of dynamics from a slightly different perspective. Perception-action systems are embodied in humans and animals and often include environmental aspects. Since humans, animals, and environments are spatially extended systems, from a structural perspective, perception-action systems are likely to be spatially extended too. Nevertheless, in many cases they have been modeled as zero-dimensional systems, that is, as systems without a spatial extension. Therefore, the question arises do we need to incorporate a spatial dimension in the descriptions of perception-action systems? And if so, what are the benefits of spatially extended models with respect to zero-dimensional models? We will answer the first question in the affirmative. As regards the second question, we will demonstrate that spatially extended models can explain the phenomena of multistability and hysteresis in systems subjected to noise. In contrast, conventional zero-dimensional models fail to account for these phenomena.

II. ZERO-DIMENSIONAL VERSUS SPATIALLY EXTENDED SYSTEMS

A Synchronization of task-related brain activity

An important concept in neuroscience is that the neocortex of humans and animals is organized as a map. Particular areas correspond to sensory and motoric organs and functional abilities (e.g., memory). In fact, encephalographic measurements corroborate this notion [19–21]. We may regard these areas as "black boxes", neural dipoles, or elementary neural units and describe their functioning by means of zero-dimensional models (just as we would describe the functioning of an Ohmic resistance). According to a new paradigm in neuroscience, however, neural computation involves the cooperative activity of many spatially distributed neurons. In particular, it is believed that perception and action involve spatially extended systems composed of many neurons that are bound to task-related entities by synchronizing their activity. Using cross-correlation analysis, in animal studies with cats, synchronized neural activity has been found in functional columns of the visual cortex [22, 23], among several unilateral areas of the visual cortex [24], and between left- and right-hemispheric visual areas [25]. Although synchronization typically occurs during the presentation of an object, it is assumed that the visual stimulus does not act as a driving force in order to establish a population of synchronized neurons. On the basis of theoretical reasoning and experimental evidence, we can assume that synchronization emerges due to mutual couplings between so-called neural oscillators. Accordingly, the synchronization of particular populations of neurons corresponds to particular perceptions [26–28]. Moreover, if in animal experiments the corticocortical connections between cortical hemispheres are removed, then synchronization between the hemispheres vanishes despite of the presence of a visual stimulus [25].

Synchronized oscillatory neural activity has also been found during the performance of manual tasks. There are significant cross-correlations between neural populations of different unilateral cortical regions when monkeys perform perceptual-motor tasks [29]. Such correlations are almost entirely absent before and after task performance. In similar animal studies, it has been found that arm movements involve both uni- and bilateral synchronized brain activity [30]. By means of analytical methods other than cross-correlation analysis, synchronized brain activity has also been revealed in man. Visual flicker signals can induce synchronized oscillatory neural activity over the whole cortex [31]. In addition, synchronization between different cortical areas occurs during tremor [32, 33], the production of isometric forces [34], and when humans listen to music [35]. The conflict between zero-dimensional and spatially distributed neural units is a conflict between localized and nonlocalized task-related neural activity. Probably the truth is to be found somewhere in between these extremes. In order to illustrate this, we may compare the situation with the theories of electromagnetism and gravity. Both theories deal with localized elements (charged particles and particles with mass, respectively) as well as nonlocalized elements (electromagnetic fields and gravitational fields, respectively). There are problems that can be treated solely by reference to the localized elements. Other problems only involve the field theoretical aspects of electromagnetism and gravity. Similarly, a comprehensive account of neural task-related activity may combine zero-dimensional and spatially distributed task-related units. Nevertheless, in special cases it may be sufficient to model perception-actions systems as either zero-dimensional or spatially extended systems.

B Multistability and hysteresis

If, under the same circumstances, perception-action systems exhibit multiple stable behavioral states, then they are said to be multistable. Examples of multistable isofrequency coordination patterns are found in animal locomotion [36], rhythmic single limb movements [37–39], and rhythmic multilimb movements [40–44]. Multistability has also been observed in polyrhythmic movements [45–47]. In line with the dynamical systems approach, multistable perception-action systems are described by potentials with multiple minima. Each minimum corresponds to one stable behavioral state. This is illustrated in Fig. 1 for a bistable system. According to the dynamical systems approach, the double-well potential $V(x)$ can be regarded as an energy measure or a measure of effort, where x represents a suitably chosen state variable. Stable perception-action systems occupy states of minimal effort or energy, that is, they occupy a minimum of the potential V. If a perception-action system is initially located outside a potential minimum, then it performs an overdamped energy-decreasing motion and finally converges to a minimum. Consequently, in the deterministic case, we find a bistable perception-action system in either one the two minima depicted in Fig. 1.

Insert Figure 1 about here

In order to take fluctuations into account, we may follow the conventional approach and regard the bistable perception-action system as a zero-dimensional unit. In this case, we can describe the system in terms of an ordinary linear Fokker-Planck equation with additive noise involving the potential $V(x)$ [12, 44, 48–50]. The stationary state of the perception-action system is described by a stationary probability density. For the linear Fokker-Planck equation the stationary solution is given by the Boltzmann distribution [51] of $V(x)$ as depicted in Fig. 1. Obviously, this conventional approach predicts a unique solution and, therefore, fails to account for the phenomenon of multistability. Alternatively, we may view the bistable perception-action system as a spatially extended system composed of many interacting and almost identical components (e.g., neural oscillators). According to the mean field theory, we can describe the spatially extended bistable perception-action system by a nonlinear Fokker-Planck equation involving the potential $V(x)$ and a mean field force arising from the interactions among the components. For example, we may consider the Desai-Zwanzig model [52] or extensions of it [53, 54]. Then, there are two stationary probability densities, as in Fig. 1. Consequently, the spatially extended model can describe a bistable perception-action system in the presence of noise.

Another fascinating phenomenon that is closely related to multistability is hysteresis. Hysteresis means that under the same environmental conditions a system shows two different behaviors. The behavior that is actually realized depends on the way how the environmental conditions were established, that is, the history of the system. For example, let us consider half a glass of beer. If we have a full glass of beer and drink half of it, we usually see the glass *half empty.* If we have an empty glass and we fill it up halfway, then we usually see the glass *half full.* Although, strictly speaking, hysteresis requires bistability, it is sometimes easier from an experimental point of view to prove hysteresis than to prove bistability [55]. Hysteresis has been observed, for example, in isofrequency interlimb coordination [56], polyrhythmic movements [57], and discrete movements such as hitting a ball with a table-tennis bat [58]. In addition, perceived apparent motion is known to show hysteresis [59].

Let us consider a simple type of hysteresis, namely, a three-stage hysteresis emerging in a system with a single control parameter α and two parameter regimes: a bistable and a monostable regime. We change the control parameter such that the system evolves from the bistable regime with the stable states A and B to the monostable regime with the stable state A and back to the bistable regime with the stable states A and B. Note that in this case the environmental conditions (described by the control parameter α) for the first and last stage are identical. Nevertheless, if we initially prepare the system in the B-state, then it will abandon the B-state and occupy the A-state in the second stage and remain there during the third and final stage. For a deterministic system this behavior is illustrated in the upper panels of Fig. 2. Again, the system in described in terms of a potential $V(x;\alpha)$. For $\alpha > \alpha_c$ the potential has two minima. For $\alpha < \alpha_c$ there is a single potential minimum. An explicit example of such a potential is the one proposed by Haken, Kelso, and Bunz in the context of coordinated finger movements [1].

Insert Figure 2 about here

In the presence of fluctuations, we may supplement the deterministic model with an additive fluctuation force. Thus, we obtain a zero-dimensional stochastic model described by a linear Fokker-Planck equation [51]. The stationary probability density $P(x)$ predicted by this stochastic model is the Boltzmann distribution of $V(x,\alpha)$ depicted in Fig. 2. We realize that in the stochastic zero-dimensional description there is no hysteresis. Alternatively, we may use a spatially extended stochastic model, for example, the mean field model introduced above. Then, there are multiple stationary probability densities that show hysteresis [60, 61] and [62] (with $\nu = 2$). The stationary probability densities of the mean field model are depicted in the lower panels of Fig. 2.

C The mean field approach to self-organization in spatially distributed systems

So far, we illustrated the benefits of spatially extended models. They are in line with a new paradigm in neuroscience and can describe multistability and hysteresis of noisy perception-action systems. The objective now is to elucidate how the structural properties of spatially extended systems relate to their behavioral properties. To this end, we concentrate on the aforementioned stochastic systems that can be described in terms of mean field models. As pointed out by *Haken* [63–65] and others [52] systems of this kind are of particular interest because they are self-organizing.

Let us assume we deal with a many particle system composed of spatially distributed, interacting, and almost identical particles. Note that we here use the notion of particles in a broad sense. In general, the particles represent subsystems such as neural oscillators, neurons involved in memory and association processes, muscular cross-bridges, or humans involved in social interactions. A detail of such a many particle system is shown in the upper panel of Fig. 3. Hexagons denote particles, connecting lines represent interactions, and black dots are so-called heat bath particles that permanently impinge on the system's particles and thus produce fluctuations [65]. F_0 is an external force that acts on each particle. Next, we summerize the effects of all particles but one in a particle-particle interaction force F_{pp} which acts on the remaining particle but is affected by that particle as well (see middle panel). In many cases, we can assume that the states of the particles differ only by statistical fluctuations. In particular, in large systems with many almost identical particles, we can assume that individual properties can be described approximately as fluctuations. In line with this notion, we pick out an arbitrary particle, call it a representative particle, and describe all other particles in terms of the representative particle and fluctuations of its state variable. Then, we can approximate F_{pp} by a so-called mean field force F_{MF} that depends on the stochastic properties of the representative particle, see lower panel of Fig. 3. Thus, we obtain a self-organizing system characterized by a circular causality structure (cf. also [3]).

The mean field force F_{MF} can distort the potentials shown in Figs. 1 and 2. For example, in the lower left panel of Fig. 1, when assuming attractive particle-particle interactions, the particles in the vicinity of the right potential minimum produce an attractive mean field force F_{MF} which attracts all particles such that they remain preferably in the vicinity of the right potential minimum. The lower right panel depicts the opposite situation. The particles are situated close to the left potential minimum of $V(x)$ and due to their interactions produce an additional force, the mean field force F_{MF}, that renders the left potential minimum more attractive than the right one.

III. APPLICATION OF MEAN FIELD MODELS

A Paced rhythmic finger movements

In a series of experiments subjects were asked to tap with their right index fingers along with the beat of a metronome [60, 66, 67]. They were requested to tap either on the beat (on-beat condition) or between two consecutive beats (off-beat condition). During the experiments the frequency of the beats was increased from 1 Hz to about 3 Hz. At low pacing frequencies ($\approx$ 1 Hz) both off-beat and on-beat tapping could be stably performed, whereas at high pacing frequencies ($\approx$ 3 Hz) only on-beat tapping could be performed in a stable fashion. If a subject was asked to tap off-beat with the frequency of the metronome, then often an involuntary switch occurred from the required off-beat tapping to on-beat tapping when the metronome frequency exceeded a critical (subject-dependent) value. This transition has been studied extensively in the literature [37, 68]. In our experiments, during the performance of on-beat and off-beat tapping, brain activity was measured by means of magnetoencephalography (MEG) [69] over the whole cortex. A detailed data analysis yielded the following results. The power spectra of single-site recordings were dominated by a peak corresponding to the movement frequency (which coincided with the pacing frequency). Irrespective of the task conditions (on-beat vs. off-beat) we could distinguish two polarity regimes of brain activity. Roughly speaking, these regimes represent cortical regions of in-flowing and out-flowing magnetic fluxes. The observed bipolarity of cortical activity is in line with other experimental studies on finger movements [20, 70, 71]. We found that MEG signals of the same polarity region were statistically phase-locked. That is, synchronization of neural activity was observed [60, 72]. In what follows, we will focus on the synchronization within a polarity region. A detailed discussion of the interplay of both polarity regions can be found in [60]. From the MEG signals of a polarity region we computed a phase distribution. In the low-frequency regime (≈ 1 Hz), we observed two phase distributions. One distribution was related to off-beat tapping, the other to on-beat tapping. In the high-frequency regime (≈ 3 Hz) there was a unique phase distribution which was related to on-beat tapping. From this observation, we concluded that paced tapping involves a stochastic perception-action system that is bistable in a particular parameter regime and monostable in another.

In order to model a stochastic, multistable perception-action system, we proposed a spatially extended model and evaluated it with the help of mean field theory [60, 73]. According to this model, the cortex is conceived of as a population of N interacting neural oscillators described by $s_i(t) = A\cos(\Omega t + \phi_i(t))$, where A denotes a time-independent amplitude and Ω corresponds to the pacing and tapping frequency. The phases ϕ_i with $i = 1, \ldots, N$ are assumed to perform a stochastic overdamped motion given by [60, 73]

$$\frac{\mathrm{d}}{\mathrm{d}t}\phi_i(t) = \underbrace{-\frac{\mathrm{d}}{\mathrm{d}\phi_i}V_{HKB}(\phi_i; a, b)}_{F_0} \underbrace{-\frac{K}{N}\sum_{k=1}^{N}\sin(\phi_i - \phi_k)}_{F_{pp}} + \underbrace{\sqrt{Q}\,\Gamma_i(t)}_{\text{noise}}$$

with $K > 0$, $Q > 0$, $\langle \Gamma_i(t)\Gamma_k(t')\rangle = \delta_{ik}\delta(t-t')$ [51]. Here, the external force F_0 is described by the HKB potential $V_{HKB}(z; a, b) = -a\cos(z) - b\cos(2z)$ which depends on two parameters $a \geq 0$ and $b \geq 0$ [1, 2]. The HKB potential has two potential minima for $b/a > 1/4$ and then looks like the potentials in the left panels of Fig. 2. For $b/a < 1/4$ it has a unique minimum and resembles the potentials shown in the middle panels of Fig. 2. The interactions between the neural oscillators is described by a coupling function F_{pp} proposed by *Kuramoto* [74, 75]. The strength of the couplings is measured by the parameter K. The action of the heat bath particles are described by fluctuation forces $\Gamma_i(t)$, which represent statistically independent white noise forces (for details see [51, 65]). The overall fluctuation strength is given by Q. Using the mean field approximation, we can derive the evolution equations for the probability density $P(\phi, t)$ of a representative neural oscillator [60, 74], which reads

$$\frac{\partial}{\partial t}P(\phi,t) = \frac{\partial}{\partial \phi}\left[\frac{\mathrm{d}V_{HKB}(\phi;a,b)}{\mathrm{d}\phi} - F_{MF}(\phi;K)\right]P(\phi,t) + Q\frac{\partial^2}{\partial \phi^2}P(\phi,t)$$

with the mean field force defined by

$$F_{MF}(\phi;K) = K\int_0^{2\pi} \sin(\phi-\phi')P(\phi',t)\,\mathrm{d}\phi' \ .$$

The mean field model can be evaluated using different techniques such as transcendent equation analysis, linear stability analysis, and stability analysis by Lyapunov's direct method [60, 73]. Thus, we can identify bistable and monostable regimes of the neural oscillator model. These regimes can be conveniently illustrated in the parameter space defined by the rescaled parameters $K' = K/Q$, $a' = a/Q$, and $\epsilon = b/a$, see Fig. 4 (upper panel). In addition, a simulation of the model for spatially extended neural oscillators agrees with the experimental findings, see Fig. 4 (lower panel).

B Parkinson's disease

Several authors have suggested that Parkinsonian tremor is caused by particular populations of neurons that synchronize their oscillatory activity [76]. In a series of papers, *Tass* [77–79] proposed a model of the synchronized neural oscillators involved in Parkinsonian tremor (see also [80]). This model is quiet similar to the model described in Sec. IIIA. According to the model proposed by *Tass*, the couplings between the neural oscillators in healthy people are so weak that they do not lead to the degree of synchronization necessary to create Parkinsonian tremor. Therefore, in healthy people neural systems for the control of finger posture are well described by zero-dimensional approaches. Then, the phase ϕ of each oscillator can assume any value between 0 and 2π and — on account of the lack of sufficiently strong couplings between the oscillators — the distribution of phases is uniform, see Fig. 5 (panels a) and b)). The oscillators are said to be de-synchronized. In contrast, in patients with Parkinson's disease the oscillators are assumed to be connected with each other. Consequently, we deal with a spatially extended neural system. According to the *Tass*'s model, a mean field force emerges in this system and results in a self-organized synchronization of the oscillator population. For this reason, the phase distribution becomes non-uniform and single-peaked, see Fig. 5 (panel c)). This synchronization, however, can be destroyed by stimulating the synchronized neurons with an appropriate external signal F_0. Put differently, the external signal F_0 can be used to effectively de-couple the neural oscillators. Consequently, the oscillator's phase distribution becomes uniform again and synchronization along with Parkinsonian tremor vanishes, see Fig. 5 (panel d)).

C Patterned versus diffusive selves

Dynamical systems theory and synergetics have not only been applied to perception-action systems, but also to psychological processes, see, for example, [81, 82]. Since psychological conditions such as anxiety can affect perception-action systems (e.g., [83]), in general, we deal with systems in which psychological, perceptual, and motoric processes are integrated. In this paragraph the focus is on psychological processes. We will discuss a rather specific issue. How can a patterned self emerge under the impact of two contrasting potential self-states? Here, we mean by 'a patterned self' a personality with articulated properties. For example, charismatic and authoritarian people exhibit clearly articulated characteristics. Drug-addicts, work-aholics, and fanatic people are extreme examples of people with patterned selves. In contrast, 'a diffusive self' is a personality with a lack of characteristic features. For example, there are school leavers without definite future plans because they do not feel a preference for a particular job or higher education. Indifferent people, people who switch from TV channel to TV channel, and people who have the experience of being pushed throughout their lives instead of making their own decisions, show aspects of a diffusive self.

Let us now concentrate on two opposing self-states, say, a selfish self and an altruistic self. We consider the case in which both self-states are available for a single person. Furthermore, we describe the person by a scalar time-dependent psychological variable $p(t)$ and assign $p = 1$ to a selfish self and $p = -1$ to an altruistic self. Then, we can model the psychological process of creating a self by a potential dynamics

$$\frac{\mathrm{d}}{\mathrm{d}t}p(t) = -\frac{\mathrm{d}}{\mathrm{d}p}V(p) \ ,$$

where $V(p)$ corresponds to a double-well potential with minima at ± 1 as shown schematically in Fig. 1. In the deterministic case the person eventually becomes either altruistic (minimum at -1) or selfish (minimum at $+1$). In both cases, we meet with a patterned self. Aiming at a more comprehensive model for psychological processes we may take fluctuations into account. Assuming that the stochastic process $p(t)$ evolves in a zero-dimensional system, we can rewrite its evolution equation as

$$\frac{\mathrm{d}}{\mathrm{d}t}p(t) = -\frac{\mathrm{d}}{\mathrm{d}p}V(p) + \sqrt{Q}\,\Gamma(t) \ ,$$

where $\Gamma(t)$ describes a fluctuation force [51] and $Q > 0$ is a measure for the fluctuation strength. According to this model, in the stationary state the person will permanently switch between altruistic and selfish behavior because the stationary distribution of p has peaks at both potential minima, see Fig. 1 (middle panels). Consequently, in such a stochastic zero-dimensional psychological system a diffusive self emerges. Finally, we consider a spatially extended psychological system. Then, the psychological variable reads $p(x,t)$, where x denotes a continuous spatial coordinate. We will defer the discussion about the meaning of a spatial coordinate to the end of this paragraph. For a so-called diffusive spatial coupling, the evolution equation reads

$$\frac{\partial}{\partial t}p(x,t) = -\frac{\mathrm{d}}{\mathrm{d}p}V(p) + K\triangle_x p(x,t) + \sqrt{Q}\,\Gamma(t) \ , \quad K \geq 0$$

where $\triangle_x$ denotes the Laplace operator. This stochastic partial differential equation is a special case of the general evolution equation for $p(x,t)$ as proposed in [84]. Alternatively, we may consider a discrete spatial coordinate x_i. Then, we deal with a psychological system composed of many similar subsystems $p_i(t) = p(x_i, t)$ with $i = 1, \ldots, N \gg 1$. In the case of a so-called ferromagnetic coupling [52] between the subsystem the evolution equations for $p_i(t)$'s read

$$\frac{\partial}{\partial t} p_i(t) = -\frac{\mathrm{d}}{\mathrm{d}p_i} V(p_i) - \frac{K}{N} \sum_{k=1}^{N} (p_i - p_k) + \sqrt{Q}\,\Gamma_i(t) \ .$$

The stochastic behavior of the spatially extended models can be read off from Fig. 1 (lower panels). Two possible patterned selves can emerge. The person may be selfish on the average (probability density centered at $+1$) or altruistic on the average (probability density centered at -1). Since p is distributed according to a probability density it is possible that there are brief episodes with large deviations from the averaged behavior. That is, an altruistic person can sometimes behave in a selfish fashion and a selfish person can now and then perform a selfless deed.

There are at least two fundamental interpretations of the spatial dimension of the psychological system of a person: a structural and a social one. Psychological processes take place in human bodies and involve neural computations. Therefore, the $p_i(t)$'s may represent almost identical structural subsystems of the neural system involved in a particular psychological process. Alternatively, we may regard the index i as a label for different people. Then, the diffusive coupling $\triangle_x$ and the ferromagnetic coupling $\sum_{k=1}^{N} (p_i - p_k)$ describe social interactions between people. In particular, the relevance of social interactions for the development of the self was recently pointed out [85]. According to this latter interpretation of the spatial dimension, people with 'backbones' or with a patterned self emerge due to social interactions, whereas people without social interactions tend to develop diffusive selves.

The dynamical model described here and suggested in [84] has also clinical implications. For example, people may suffer from a diffuse self (e.g., schizophrenics). According to the current model, a diffusive self may arises from a lack of connecting elements. Psychotherapy and drug treatment may aim at an increase of coupling constants such as K or an increase of the number N of connected subsystems. In a second example, we may consider people who suffer from a self that is trapped in a particular self-state (e.g., drug addicts). Although there are in principle other self-states available, these people cannot occupy them due to the couplings with other people (e.g., other drug addicts) or due to spatial couplings between structural parts of their psychological systems. Put differently, they cannot change their way of living because of the impacts of mean field forces that tend to re-establish their actual personal situations as soon as deviations from these situations occur. In this case a therapy may act as an external force F_0 leading to

$$\frac{\partial}{\partial t} p_i(t) = -\frac{\mathrm{d}}{\mathrm{d}p_i} V(p_i) - \frac{K}{N} \sum_{k=1}^{N} (p_i - p_k) + F_0(p_i, t) + \sqrt{Q}\, \Gamma_i(t) \ .$$

In doing so, the psychological system can be driven out of the actual self-state and can be put close to a desired self-state.

By comparing the proposed use of F_0 in this section and in Sec. IIIB, we realize that there are two possible effects of F_0. On the one hand, F_0 may interact with the potential V in order to drive a many particle system out of stable stochastic state and to put it in the vicinity of another stable state. On the other hand, F_0 may interact with the coupling force F_{pp} in order to reduce the impact of F_{pp}, to destroy particle-particle interactions, and to force the spatially extended many particle system into a state in which it can be regarded as a collection of many independent zero-dimensional systems (see Sec. IIIB).

IV. CONCLUSIONS

We presented experimental and theoretical evidence for the relevance of spatially extended perception-action systems. First, we showed that there are several instances in which action and perception involve spatially extended neural circuitries. Then, we illustrated the failure of conventional stochastic zero-dimensional models for multistability and hysteresis. In contrast, spatially extended models, such as mean field models, can describe these phenomena. In this context, we discussed the coordination of rhythmic finger movements, Parkinsonian tremor, and psychological processes involved in the creating of selves. We would like to emphasize that at present these three research fields are studied extensively. That is, many scientists are nowadays interested in exploring the link between neural activity and movement coordination, in constructing dynamical models for psychological processes, and in developing novel treatments for Parkinsonian tremor on the basis of spatially extended stochastic models. From a phenomenological perspective, when we observe a noisy multistable system or hysteresis in the presence of noise, we can take these observations as a strong hint that we should consider the system under consideration as a spatially extended one. In addition to bistability and hysteresis, further indicators for spatially extended perception-action systems have been documented such as cutoff-distributions, anomalous diffusion, and noise-induced shifts of bifurcation points (or instability points) [86] (Sec. 6.2).

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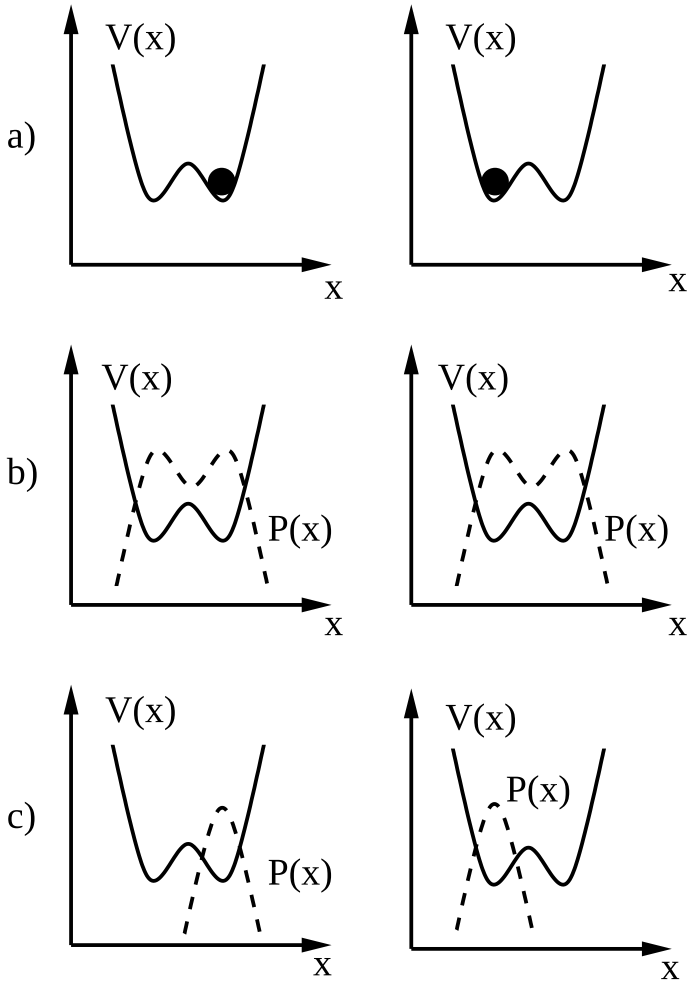

FIG. 1: Examples of models involving a typical double-well potential $V(x)$: a) deterministic model, b) stochastic zero-dimensional model, c) stochastic spatially extended model. The black balls describe the stationary states of the model a). $P(x)$ corresponds to the stationary probability densities of the models b) and c).

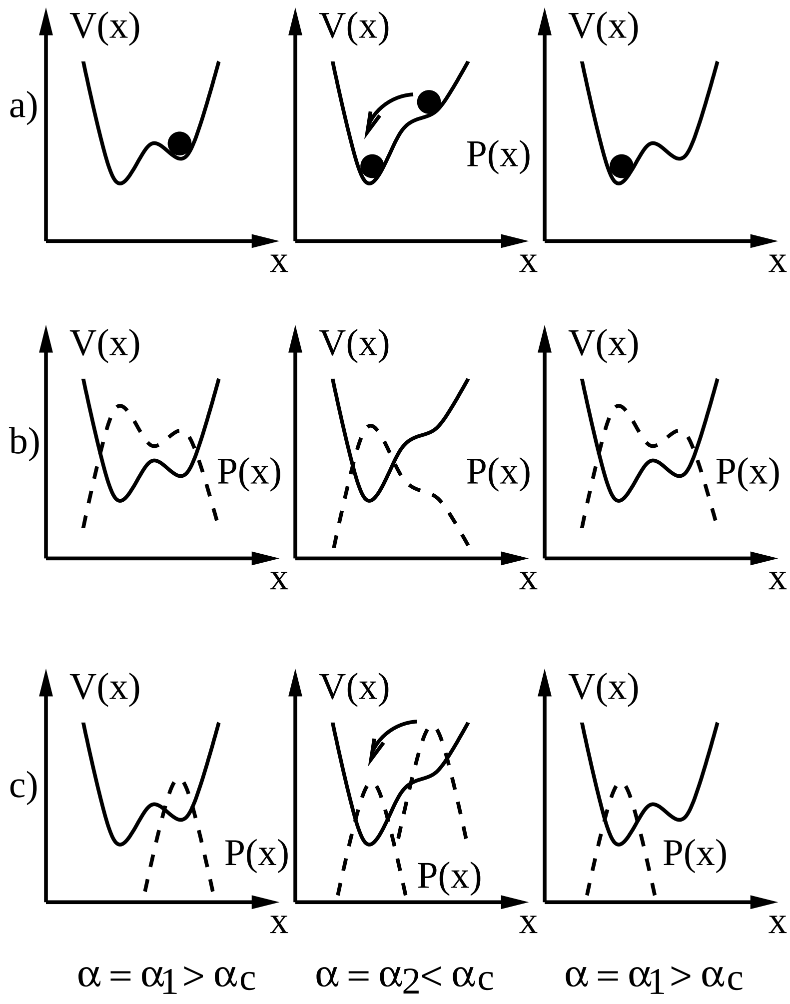

FIG. 2: Examples of models involving a three-stage hysteresis: a) deterministic model, b) stochastic zero-dimensional model, c) stochastic spatially extended model. The potential $V(x;\alpha)$ is shown for two values of α (α_1 and α_2). The control parameter α is varied according to the sequence $\alpha_1, \alpha_2, \alpha_1$. The black balls describe the stationary states of the model a). $P(x)$ corresponds to the stationary probability densities of the models b) and c).

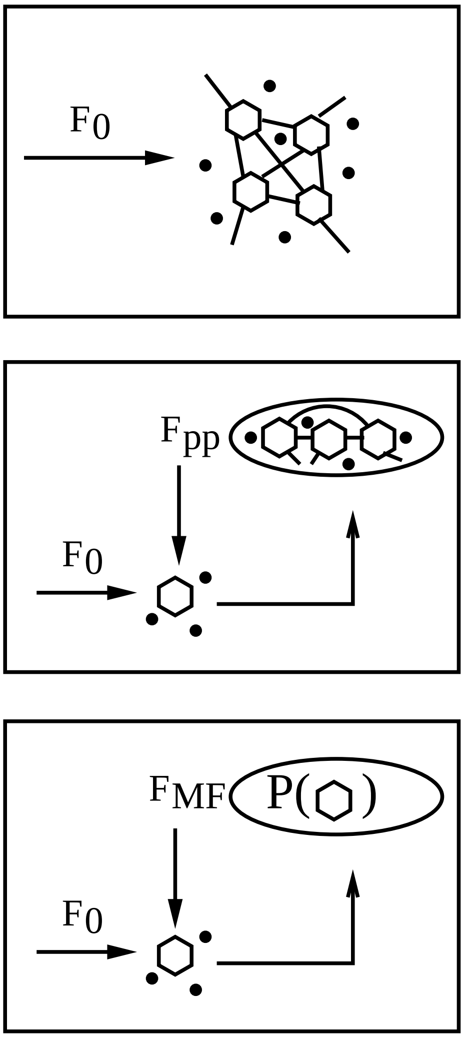

FIG. 3: Self-organization of a spatially distributed system. Upper panel: Many interacting and almost identical particles subjected to an external force F_0 and fluctuation forces (black dots). Middle panel: The symbolic re-arrangement of particles and forces results in the introduction of a particle-particle interaction force F_{pp} that acts on a representative particle and is affected by that particle. Lower panel: The behavior of the particles contributing to F_{pp} is approximately described by the stochastic properties of the representative particle (mean field approximation); here P stands for the distribution function.

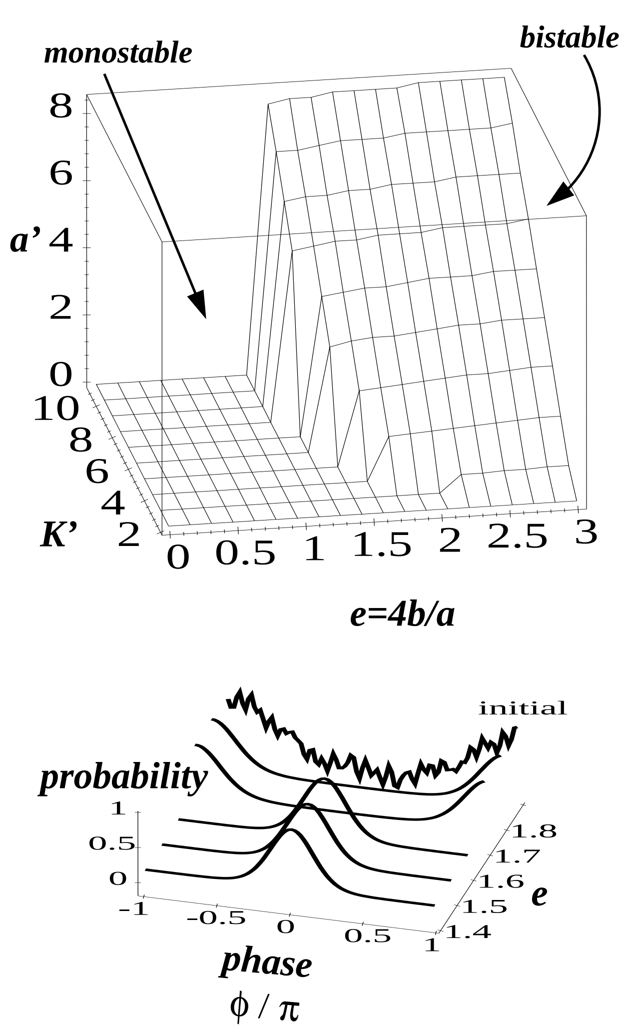

FIG. 4: Upper panel: the three-dimensional parameter space of the mean field HKB model is composed of two subspaces describing a regime with a single (monostable) and two (bistable) stationary phase distributions. Lower panel: a simulation of the model is shown that mimics the experimentally observed transition between two characteristic phase distributions of MEG signals.

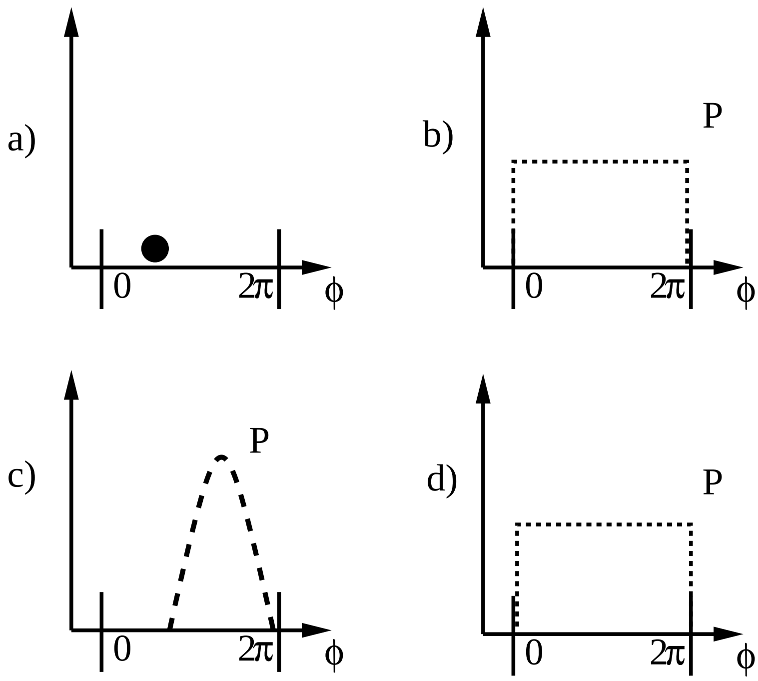

FIG. 5: Neural oscillator models of synchronized brain activity related to Parkinsonian tremor and treatment via electric stimulation. Panel a): deterministic case for a healthy person; the ball represents the phase of a free oscillator. Panel b): stochastic zero-dimensional model for a healthy person; phases are uniformly distributed. Panel c): stochastic spatially extended model for a patient with Parkinson's disease; oscillator phases are attracted to each other due to a mean field force. Panel d): stochastic spatially extended model under the impact of an external stimulus; the impact of couplings between neural oscillators is reduced by the external stimulus. P represent the phase distributions of models b), c), and d).