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Drift bifurcation detection for dissipative solitons

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Abstract. We report on the experimental detection of a drift bifurcation for dissipative solitons, which we observe in the form of current filaments in a planar semiconductor–gas-discharge system. By introducing a new stochastic data analysis technique we find that due to a change of system parameters the dissipative solitons undergo a transition from purely noise-driven objects with Brownian motion to particles with a dynamically stabilized finite velocity.

Contents

1. Introduction

Brownian motion of particles is a well known phenomenon in physical, chemical and biological systems. The motion of these particles is overdamped in the absence of external forces, such that they would remain stationary without the driving force of fluctuations. On the other hand there are synergetic objects which, under the conditions given above and depending on the system parameters, may stay at rest or propagate with a dynamically stabilized velocity. Well known examples of these objects are non-equilibrium Ising–Bloch fronts in the one-dimensional case or, in one- and higher-dimensional systems, localized dissipative structures, so-called dissipative solitons [1, 2]. These particle-like structures are commonly observed in biological systems as nerve pulses [3], in chemical systems as concentration drops [4] and in physical systems as current filaments [5]. In this context it can be shown for a system with continuous symmetries that the transition from stationary to travelling objects or patterns occurs due to a drift bifurcation, which breaks the symmetry of the structures [6]–[15]. Experimentally this effect has been observed in the case of periodic structures, e.g. for the Faraday instability [16], Rayleigh–Bénard convection [17], the printer's instability [18]–[20] and cellular flame patterns [21]. However, in the case of dissipative solitons, this effect has been theoretically predicted [13, 14, 22]–[26], but has not been experimentally verified.

In this paper we report on an experiment in which we observe the theoretically predicted drift bifurcation [14, 22]–[26] for dissipative solitons in the form of localized current filaments in a semiconductor–gas-discharge system. In particular, we detect the transition from noise-driven dissipative solitons behaving like Brownian particles to dissipative solitons moving with an intrinsic velocity, so-called active Brownian particles [27]. The basis of this investigation is a new stochastic data analysis technique for the separation of deterministic and stochastic dynamics which is applicable under quite general assumptions and therefore can be used for a large class of systems. This enables the investigation of the dynamics of localized objects in experimental systems, in which quantitative studies were not possible due to large fluctuations. In the case of the discussed experiment the presented results give the first experimental evidence for the existence of the theoretically predicted drift bifurcation of dissipative solitons and show that the bifurcation can be hidden by noise.

2. Experimental observations

The investigated dc gas-discharge system consists of a high ohmic planar semiconductor cathode (GaAs⟨Cr⟩) and a transparent, low ohmic planar anode consisting of a glass substrate coated with indium tin oxide (ITO). Both electrodes are separated by a narrow gas-discharge space of width $d = 0.25$–0.75 mm filled with nitrogen (figure 1). The active discharge area has a diameter of $D \approx 17$ mm. The specific resistivity of the semiconductor is controlled by an external light source via the internal photo effect. During the preparation of the experimental set-up, great care is taken to ensure the spatial homogeneity of the system. For appropriate parameters, well localized current filaments, or dissipative solitons, can be observed through the transparent anode as bright spots of high luminance being emitted from the discharge gap [28, 29]. While in general several filaments can be generated in the active area of the system due to a Turing destabilization of the homogeneous discharge [29, 30], we have chosen the experimental parameters for the following investigations such that only one filament exists at a given time.

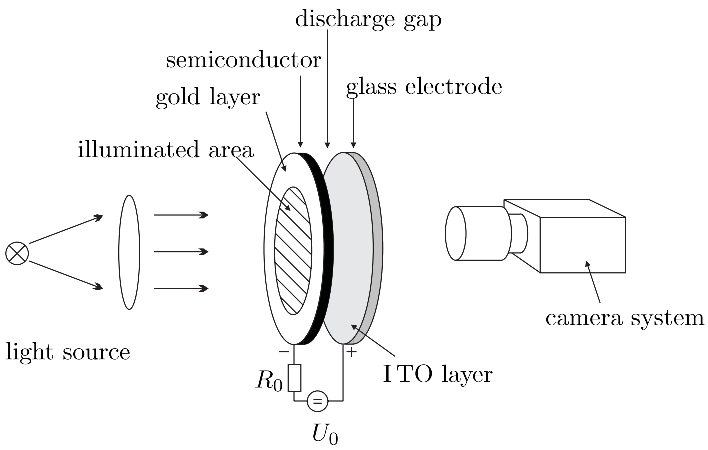

Figure 1. Schematic plot of the experimental set-up. It mainly consists of a high ohmic semiconductor electrode, contacted with a gold layer, a discharge gap filled with nitrogen and a transparent ITO electrode. The resistivity of the semiconductor is controlled by homogeneous illumination with visible light. The maximum global current of the discharge is restricted due to a series resistor R_0. The discharge can be observed with a CCD camera system through the transparent ITO electrode.

In this case we observe that the single filament moves on an irregular path, indicating a strong influence of noise, which might be related to noise in the semiconductor (generation–recombination, $1/f$ noise), thermal fluctuations in the gas and noisy processes of charge transportation through the semiconductor–gas-discharge interface [31]. Depending on the parameters of the system, the motion of the filaments exhibits qualitative differences. In order to illustrate these differences, we choose a threshold value ϕ_T for the luminance distribution $\phi(\vec{x}, t)$ in order to distinguish between the dissipative soliton and the background discharge. Starting from this definition the trajectory $\vec{p}(t) \in \mathbb{R}^2$ of the dissipative soliton can be defined and calculated as

$$\vec{p}(t) = \frac{\int_{\vec{x}:\phi(\vec{x},t)\geq\phi_T} \vec{x}\, \phi(\vec{x}, t)\, \mathrm{d}\vec{x}}{\int_{\vec{x}:\phi(\vec{x},t)\geq\phi_T} \phi(\vec{x}, t)\, \mathrm{d}\vec{x}}. \tag{1}$$

Two typical trajectories $\vec{p}(t)$ for different system parameters are presented in figures 2(a) and 3(a), in which the differences in the dynamics of the dissipative solitons become clearly visible. Both figures show a snapshot of the luminance distribution $\phi(\vec{x}, t)$ emitted from the gas discharge and the corresponding trajectories $\vec{p}(t)$. While the trajectory depicted in figure 2(a) shows that the current filament moves with frequent random changes of its direction of motion, resembling the typical motion of a Brownian particle, the trajectory of the current filament in figure 3(a) looks much smoother. These observations give rise to the question of whether the different dynamics is related to a qualitative change of an intrinsic property of the dissipative solitons.

3. Stochastic data analysis

In order to separate the stochastic part of the dynamics from the deterministic part, we assume, using a particle approach, that the dynamics of the current filament located at $\vec{p}(t) \in \mathbb{R}^2$ can be

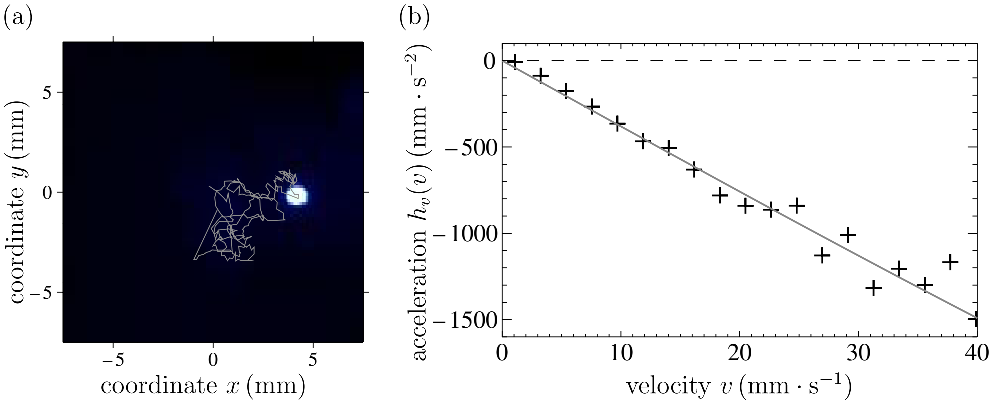

Figure 2. The dynamics of a dissipative soliton exhibiting purely Brownian motion. (a) Trajectory of a current filament and snapshot of the luminance distribution in false colour representation. The overall recording time was 72 s, from which a section of 31 s is depicted. The aspect ratio between the diameter of the dissipative soliton and the diameter of the active area is approximately 20. (b) Result of the stochastic data analysis technique for the trajectory. Crosses mark the deterministic part $h_v(v)$ of the dynamics for the respective velocity intervals of width $\Delta v = 2$ mm s^{-1}. A linear fit to $h_v(v)$ is shown as a grey curve. Parameters: supply voltage $U_0 = 3600$ V, specific resistivity of the semiconductor $\rho_{SC} = 2.02 \times 10^6$ Ω cm, series resistor $R_0 = 10$ MΩ, pressure $p = 282$ mbar, temperature $T = 105$ K, width of gas gap $d = 550$ μm, exposure time $t_{exp} = 0.02$ s, global current $I = 116$ μA.

described by a Langevin equation

$$\ddot{\vec{p}}(t) = \dot{\vec{v}}(t) = \vec{h}(\vec{v}(t)) + R(\vec{v})\vec{\Gamma}(t), \tag{2}$$

which is interpreted according to the Itô calculus. Here the terms $\vec{h}(\vec{v}(t))$ and $R(\vec{v})\vec{\Gamma}(t)$ denote the deterministic and stochastic parts of the dynamics, as $R(\vec{v})$ is a velocity-dependent noise amplitude and $\vec{\Gamma}(t)$ is a vector of noise forces with vanishing mean. Concerning the noise force, we assume that its autocorrelation decays on a smaller timescale than the characteristic timescale of the dynamics of the filaments. Furthermore, we use the fact that $\vec{h}(\vec{v})$ possesses rotational symmetry (i.e. $\vec{h}(\vec{v}) = h_v(v)\vec{v}/v$ with $v = |\vec{v}|$) due to the O(2) symmetry of the experimental system, if the finite size of the system is neglected. Note that no further details about the function $h_v(v)$ have to be known. From this, we deduce the following projection technique for the analysis of two-dimensional particle trajectories:

$$h_v(v) \approx \frac{1}{\Delta t}\left\langle \frac{(\vec{v}(t+\Delta t) - \vec{v}(t)) \cdot \vec{v}(t)}{v(t)} \right\rangle\Bigg|_{v(t)\approx v}, \tag{3}$$

whereby the velocity $\vec{v}(t)$ is calculated in first-order approximation from two successive filament positions $\vec{p}(t)$ and $\vec{p}(t + \Delta t)$. The projection technique (3) holds as long as the time interval Δt is small enough to resolve the investigated dynamics [32] and large compared to the correlation time of the fluctuations [34]. In principle, it is also possible to estimate the fluctuation strength in a similar way [32], which is not the topic of this investigation. Note that an interpretation of equation (2) according to the Stratonovich calculus would lead to an additional spurious drift

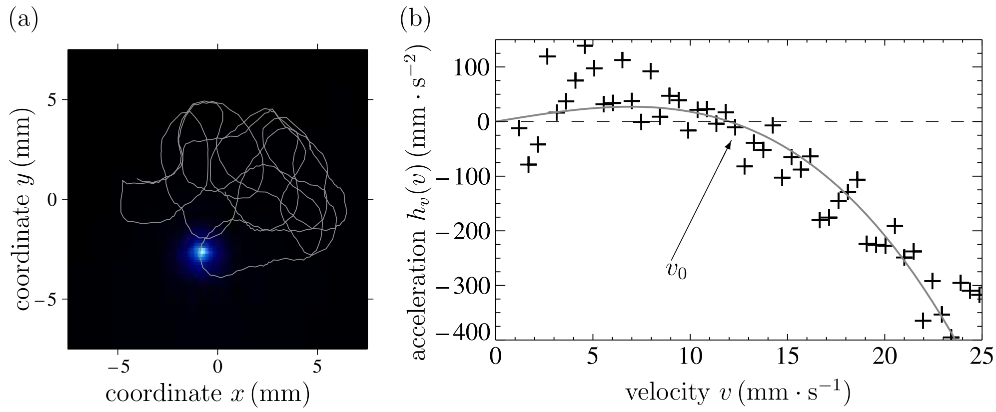

Figure 3. The dynamics of a dissipative soliton with a finite intrinsic velocity. (a) Trajectory of a current filament and snapshot of the luminance distribution in false colour representation. The overall recording time was 62 s, from which a section of 39 s is depicted. (b) Result of the stochastic data analysis technique for the trajectory. Crosses mark the deterministic part $h_v(v)$ of the dynamics for the respective velocity intervals of width $\Delta v = 0.5$ mm s^{-1}. A cubic fitting function referring to the deterministic dynamics of equation (4) is shown as a grey curve. Parameters: $U_0 = 2740$ V, $\rho_{SC} = 4.95 \times 10^7\ \Omega$ cm, $R_0 = 20$ MΩ, $p = 280$ mbar, $T = 105$ K, $d = 250\ \mu$m, $t_{exp} = 0.02$ s, with $I = 46\ \mu$A.

term on the left-hand side of equation (3), which vanishes for additive noise. The same holds if a generalized Stratonovich integral is used [33].

In order to test the presented technique, we use an equation taken from a particle model for dissipative solitons in a related model system [35], which we heuristically extend to a Langevin equation with appropriate additive noise terms

$$\dot{\vec{v}}(t) = a_1\vec{v}(t) - a_3|\vec{v}(t)|^2\vec{v}(t) + R\vec{\Gamma}(t), \tag{4}$$

where a constant noise amplitude R has been chosen for the purpose of simplification. Concerning the deterministic dynamics of (4) two generic cases are known, which depend on the sign of a_1, since a_3 is always positive, so that the cubic term can be interpreted as a velocity dependent friction. Therefore, the case $a_1 > 0$ describes the dynamics of a dissipative soliton which would propagate with a finite dynamically stabilized intrinsic velocity $v_0 = \sqrt{a_1/a_3}$ if no noise was present in the system. On the other hand the case $a_1 \leq 0$ describes the dynamics of an overdamped and purely noise-driven filament. On the basis of model equation (4), data series have been numerically generated using a stochastic integration method [36, 37] and have afterwards been analysed using (3). It turned out that the data analysis technique reproduces the deterministic dynamics of the model equation (4) very well for sufficiently long time series. In detail this means that, for the deterministic dynamics considered here, the average in equation (3) should be computed from at least 20 events $v(t) \approx v$ in order to get a useful result for the corresponding value $h_v(v)$.

After validating the efficiency of the presented analysis technique, it can be applied to experimentally recorded trajectories. For this task all data points of the trajectories are taken into account for the analysis, since it has been tested that the improvement of the statistics due to the gain on data points rules out possible deviations at the boundary.

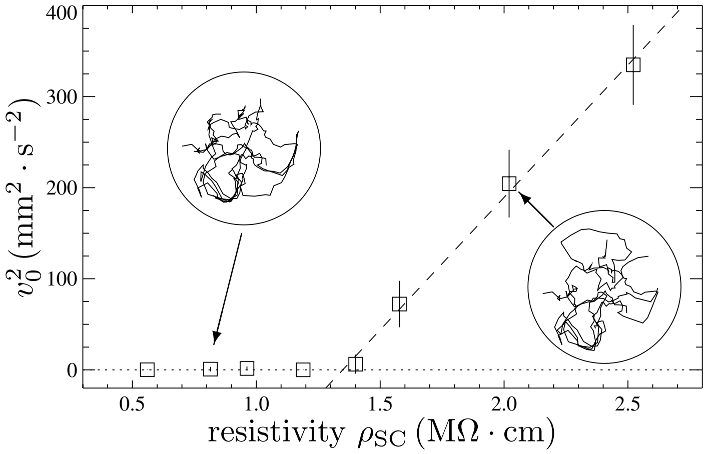

Figure 4. Experimental results for the square of the intrinsic speed v_0 as a function of the bifurcation parameter ρ_{SC} and exemplary trajectories for parameters below and above the bifurcation point with circles denoting the active discharge area. Parameters: $U_0 = 3700$ V, $R_0 = 10$ MΩ, $p = 286$ mbar, $T = 105$ K, $d = 750$ μm, $t_{exp} = 0.02$ s, with $I = 107$ μA.

The analysis reveals the existence of two qualitatively different dynamical states which are discussed on the basis of the trajectories shown in figures 2(a) and 3(a) for which the respective results of the analysis are depicted in figures 2(b) and 3(b). In the example presented in figure 2, the motion of the current filament is purely damped with a damping constant of (37.6 ± 1.5) s^{-1}, which can be computed from a linear fit (grey curve) to the acceleration (crosses) resulting from the data analysis (figure 2(b)). In this case the filament behaves like a classical Brownian particle as the fluctuations are the driving force of the movement. In contrast to this finding is the example presented in figure 3, where the data analysis reveals the dynamics of a particle with finite, dynamically stabilized velocity v_0 (figure 3(b)). This velocity can be estimated to be 11 mm s^{-1} using a cubic fitting function (grey curve), which has been chosen corresponding to the deterministic dynamics of the model equation (4). Here, the internal drive, causing an intrinsic velocity, dominates the dynamics and the fluctuations only play the role of a disturbance. This case is referred to as *active Brownian motion* [27].

4. Drift bifurcation

Motivated by these findings, systematic measurements have been made which show that a transition between the two different dynamical states can occur due to a change of the specific resistivity ρ_{SC} of the semiconductor. Plotting the square of the intrinsic velocity v_0 as function of the control parameter ρ_{SC} reveals the course of a supercritical pitchfork bifurcation (figure 4) with a bifurcation point at $\rho_c = 1.35$ MΩ cm. We identify this transition as a drift bifurcation, which is theoretically predicted for a large class of different dissipative systems with continuous symmetries where dissipative solitons are observed [11, 14, 22, 23, 26]. The importance of the stochastic data analysis technique becomes clear by comparing the trajectories of the dissipative solitons (plotted within figure 4) with the naked eye. For the chosen set of parameters the qualitative difference in the dynamical behaviour of the dissipative solitons is not obvious as in figures 2(a) and 3(a). In the case of figure 4 the transition between the different dynamical states can only be uncovered by using the discussed data analysis technique.

In the context of modelling dissipative solitons with systems of reaction–diffusion type the drift bifurcation typically occurs due to an increase of a time constant over a critical threshold [14, 22]–[26]. These theoretical predictions can be related to the presented experimental findings by referring to an early phenomenological equivalent circuit model for the investigated experiment [38]. This approach shows that an increase of the specific resistivity of the semiconductor leads among other things to an increasing time constant and therefore enables the drift bifurcation of dissipative solitons. We would also like to note that the deterministic dynamics of equation (4) is characteristic for parity breaking bifurcations and therefore has theoretically been discussed for a huge class of systems.

5. Conclusion and outlook

The transition from stationary to travelling objects due to a change of system parameters is a well known phenomenon for non-equilibrium systems with continuous symmetries. Although this drift bifurcation has been theoretically predicted for dissipative solitons in systems of reaction–diffusion type, it has not been experimentally verified. In the case of planar semiconductor–gas-discharge systems this discrepancy is related to the strong influence of noise. By describing the dynamics of the observed dissipative solitons on the basis of a particle approach we are able to introduce a new stochastic data analysis technique for the separation of the deterministic and the stochastic parts of the dynamics. The application of this analysis technique to experimentally recorded trajectories of dissipative solitons reveals that the dissipative solitons are either purely noise driven or move with a stabilized finite velocity. Systematic investigations show that the transition between these different dynamic states can occur due to a change of the specific resistivity of the semiconductor. Identifying this transition as a supercritical drift bifurcation, we give for the first time experimental evidence for the theoretically predicted transition.

Furthermore, the presented results demonstrate that the observed dissipative solitons can be effectively described by a particle approach naturally motivating an extension of the presented technique towards the investigation of interaction phenomena of dissipative solitons. This is a topic of current research. Due to the practicability of the introduced data analysis technique and due to the underlying general assumptions the presented technique can be applied to a large class of systems for which the deterministic dynamics of localized structures has not yet been experimentally investigated.

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