Dissipative Solitons in Physical Systems

Talk

given by

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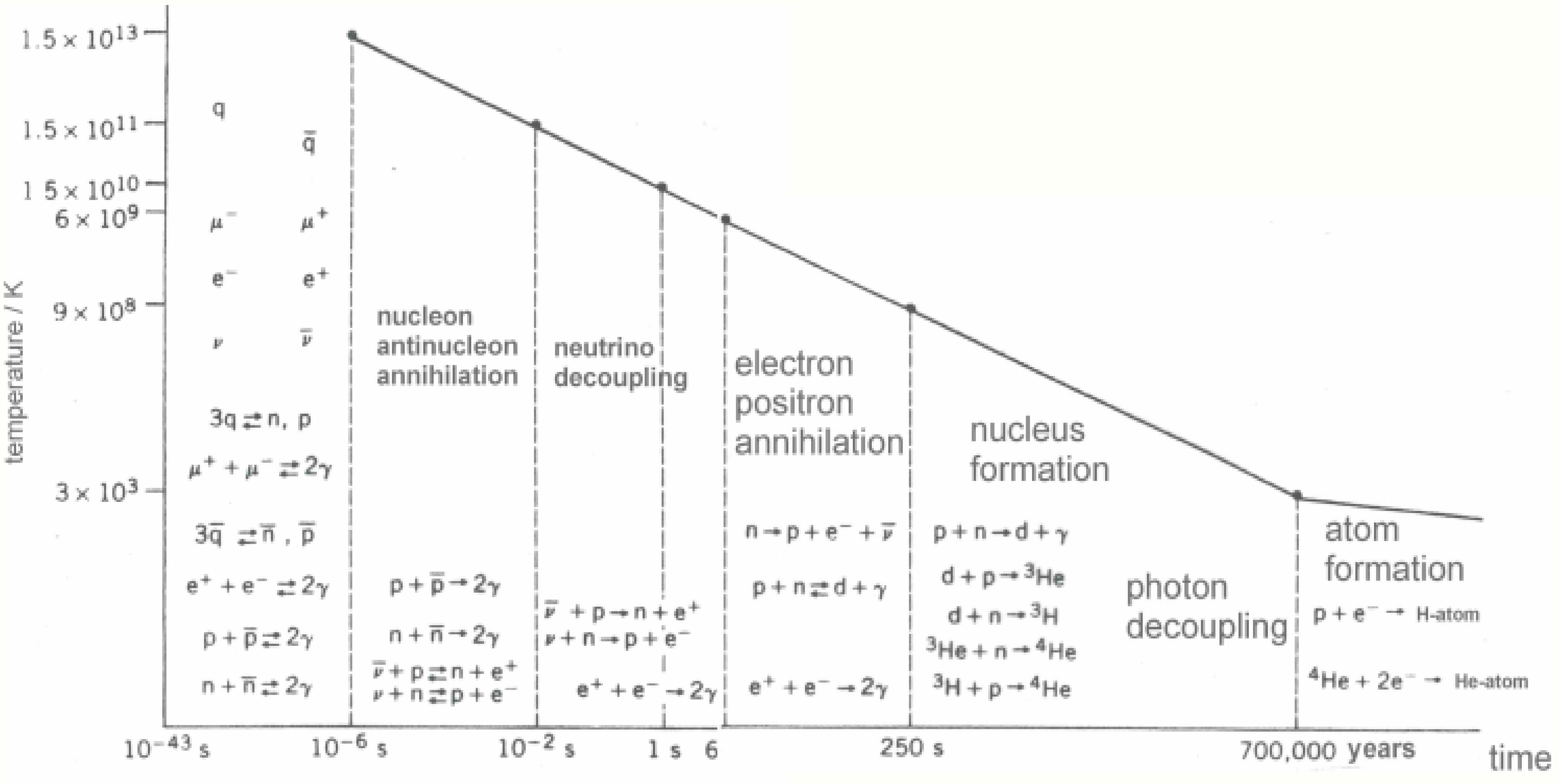
Introduction

Cosmology and Pattern Formation



Complex Behaviour of Pattern Forming Nonlinear Dissipative Systems

- **Patterns in space and time**
- **Attractors**
- **Complex dependence on**
  - **initial conditions**
  - **boundary conditions**
  - **parameters**
- **Dependence on initial conditions $\Rightarrow$ multistability**
- **Dependence on parameters and boundary conditions $\Rightarrow$ bifurcation**
- **Lack of reproducibility in the presence of noise**
- **Chaos**
- **Understanding of self-organized patterns is one of the most important problems of modern science**

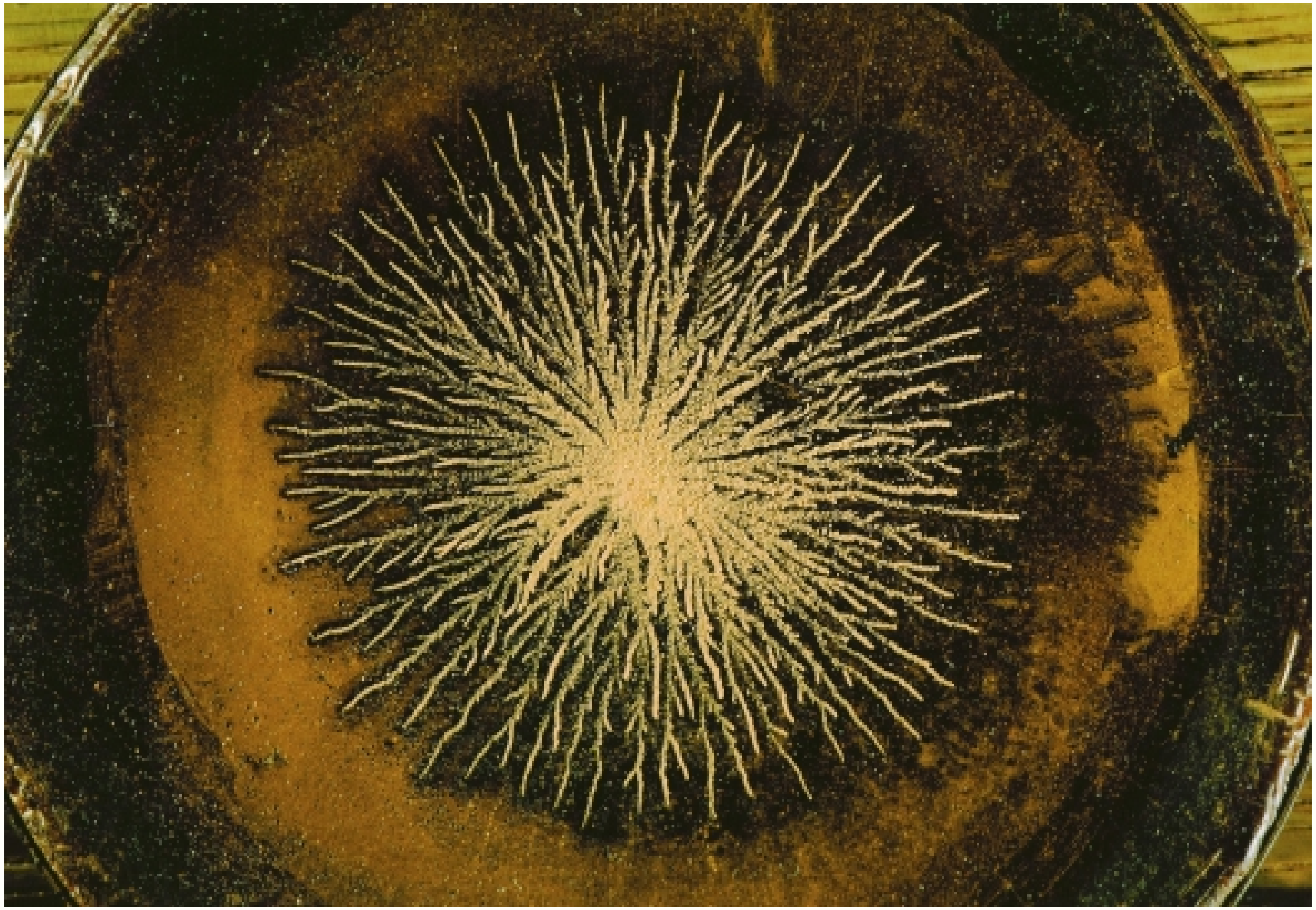
Chapter 2

Gas-Discharge Systems

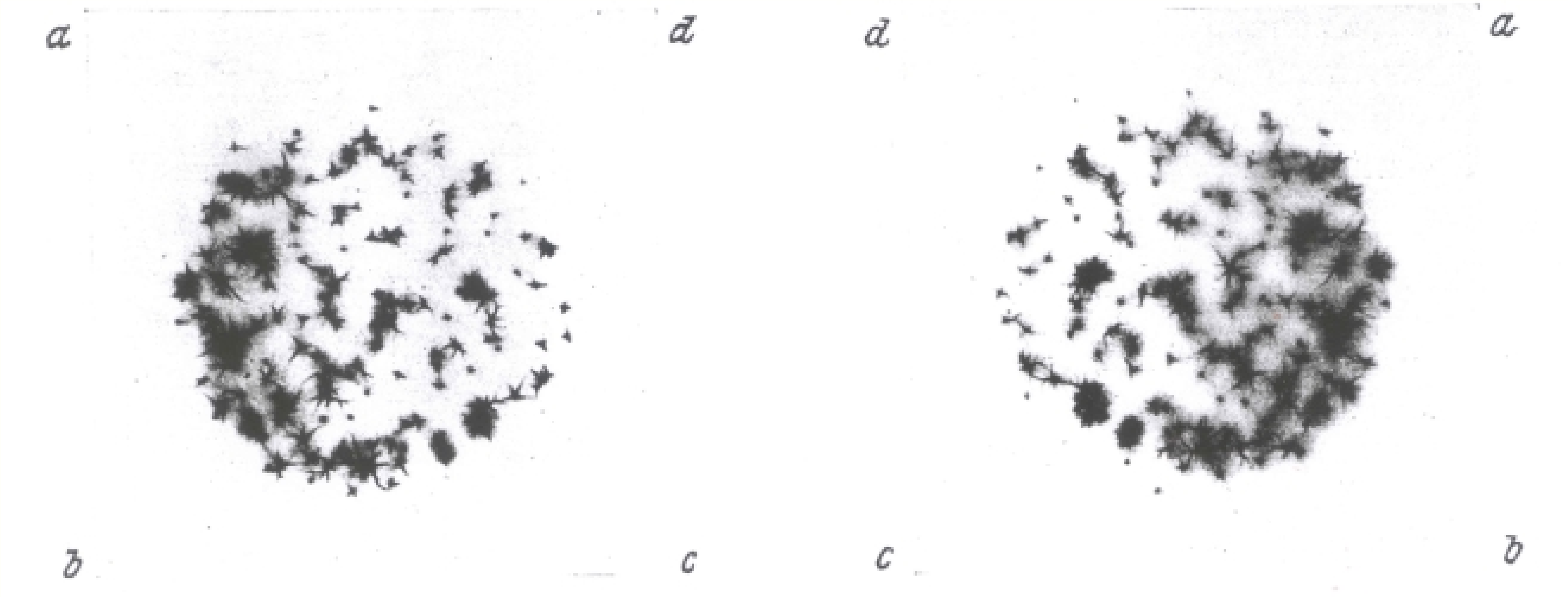
Lichtenberg Pattern I: Experimental Set-Up

elektrophorus

Lichtenberg Pattern II: Reproduction of the Original Pattern



Current Filaments in a Pulse Driven Planar Gas-Discharge System with Dielectric Barrier



photograph of the luminescence radiation from the discharge space measured on one electrode

Chapter 3

Experimental Results

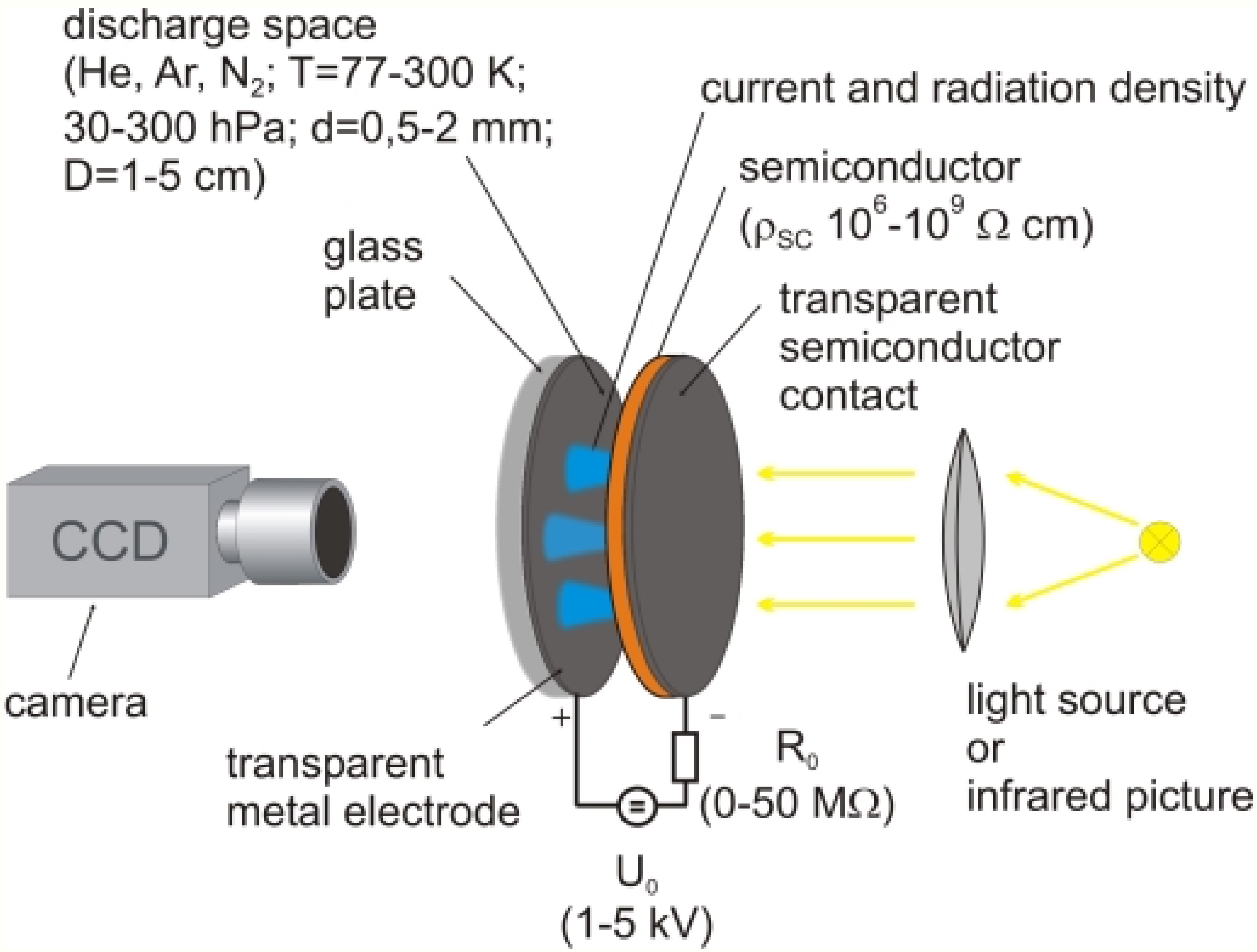
from

Gas-Discharge Systems

with

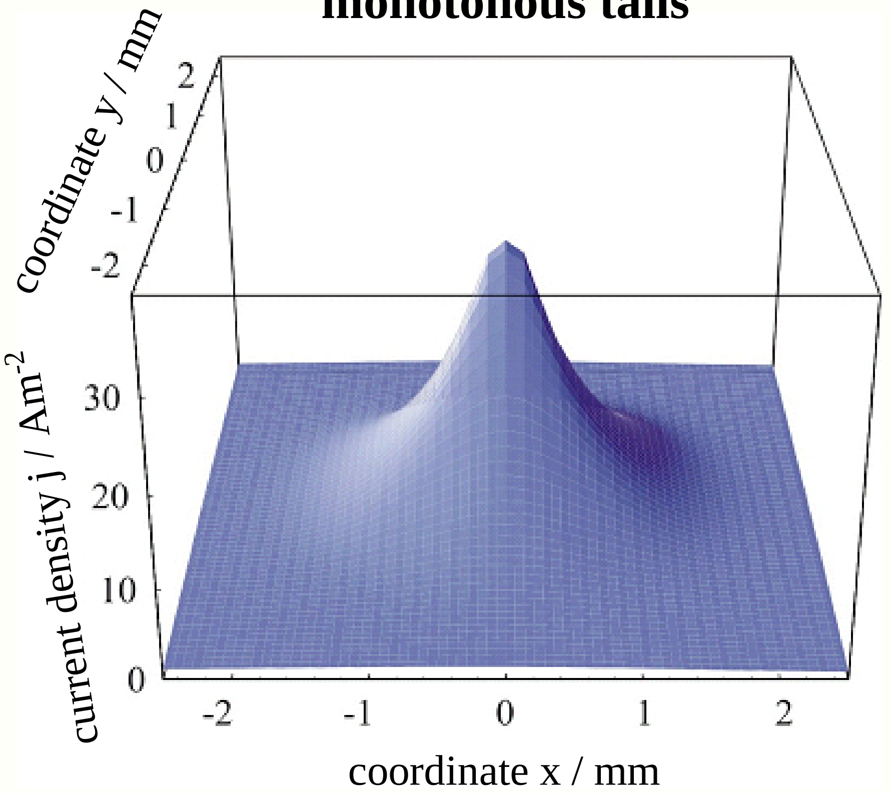
High Ohmic Barrier

Experimental Set-Up for Measuring Self-Organized Patterns in Planar DC Gas-Discharge Systems



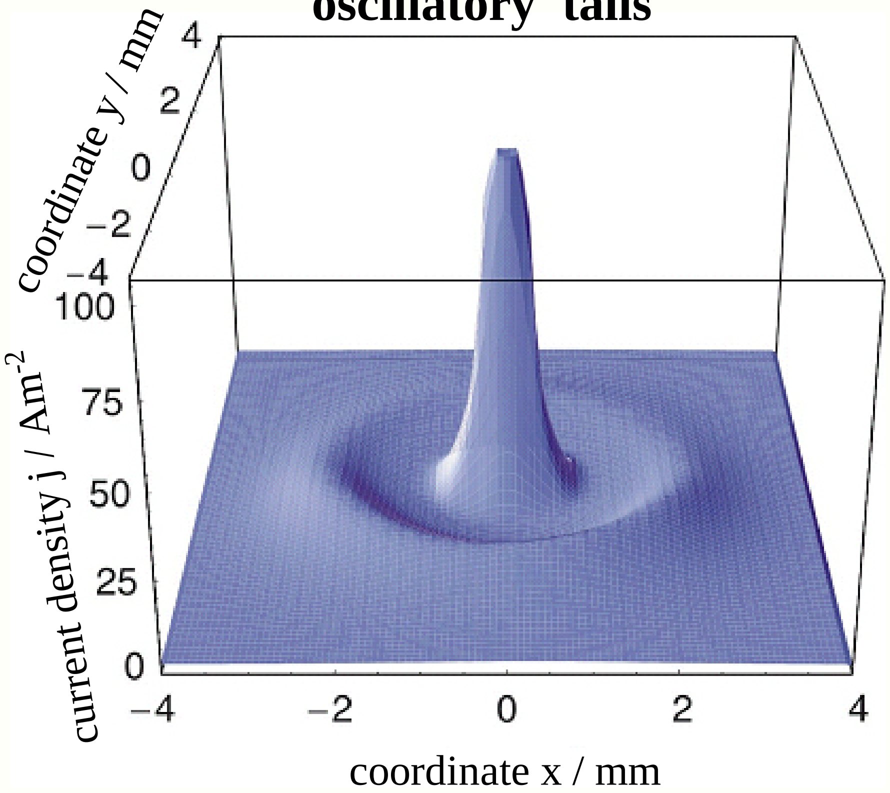
Superposition of 1000 Subsequent Experimental Frames of a Single Propagating Current Filament

monotonous tails



parameters: U_0 = 2,74 kV, ρ_{SC} = 4,95 MΩ cm, R_0 = 20 MΩ, Gas: N_2, T=100 K, p = 280 hPa, D=30 mm, d = 250 µm, a_{SC}=1 mm, I = 46 µA, t_{exp}=20 ms

oscillatory tails



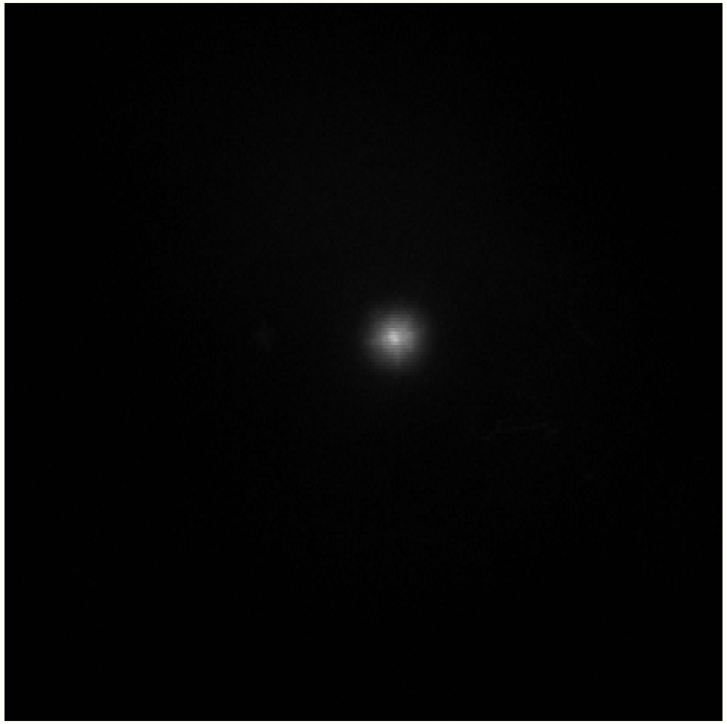
parameters: U_0 = 3,6 kV,ρ_{SC} = 3,05 MΩ cm, R_0 = 4,4 MΩ, Gas: N_2, T=100 K, p = 279 hPa, D=30 mm, d = 500 µm, a_{SC}=1 mm, I = 200 µA, t_{exp}=20 ms

Bödeker et al. 2003

Typical Dynamic Behaviour of a Single Experimental Current Filament in the Discharge Plane

1 cm

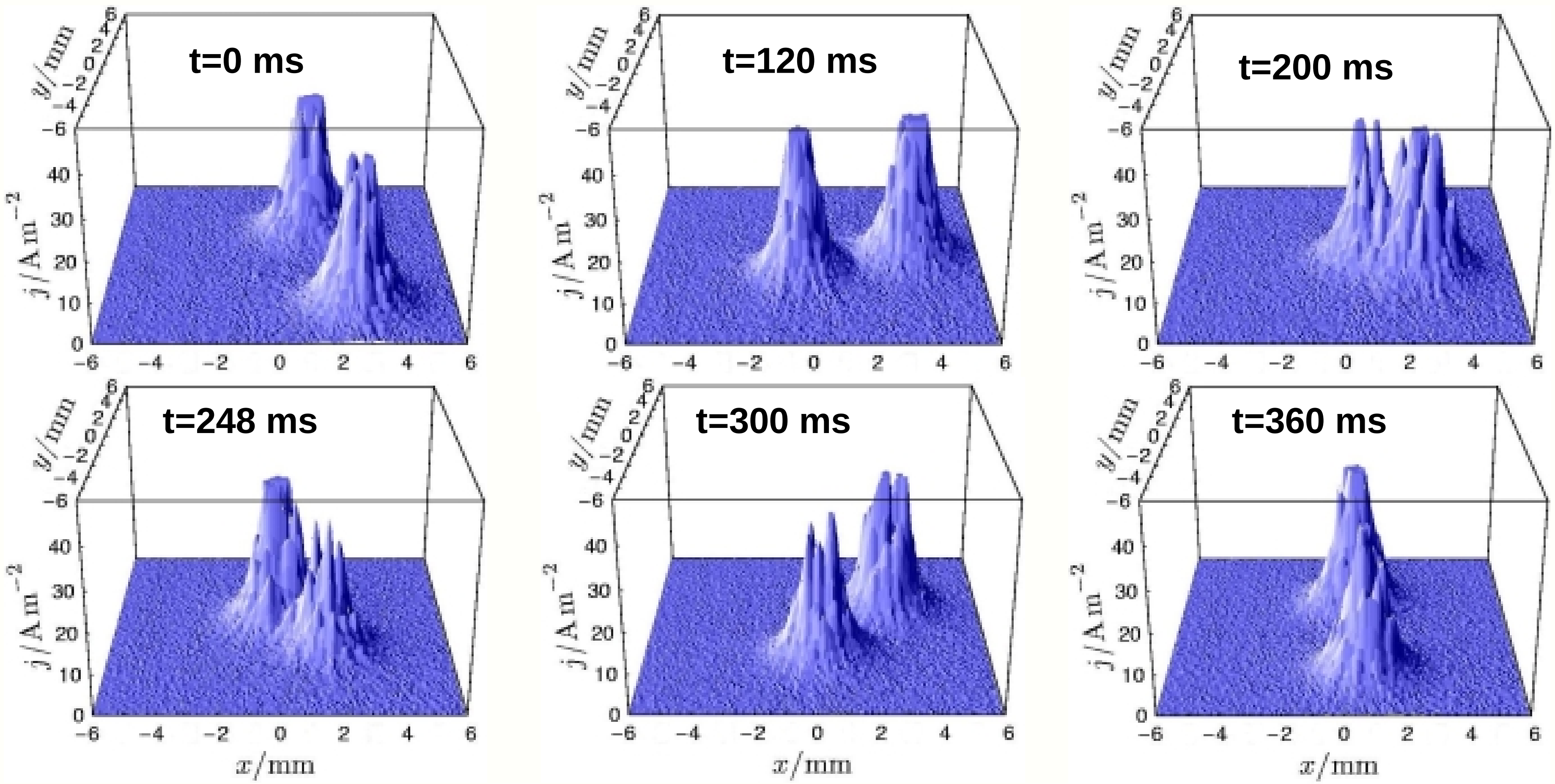
real time



parameters: U_0=2,7 kV, ρ_{SC}=4,95 MΩ cm, R_0=20 MΩ, Gas: N_2, T=100 K, p=280 hPa, D=30 mm, d=250 µm, a_{SC}=1 mm, I=46 µA

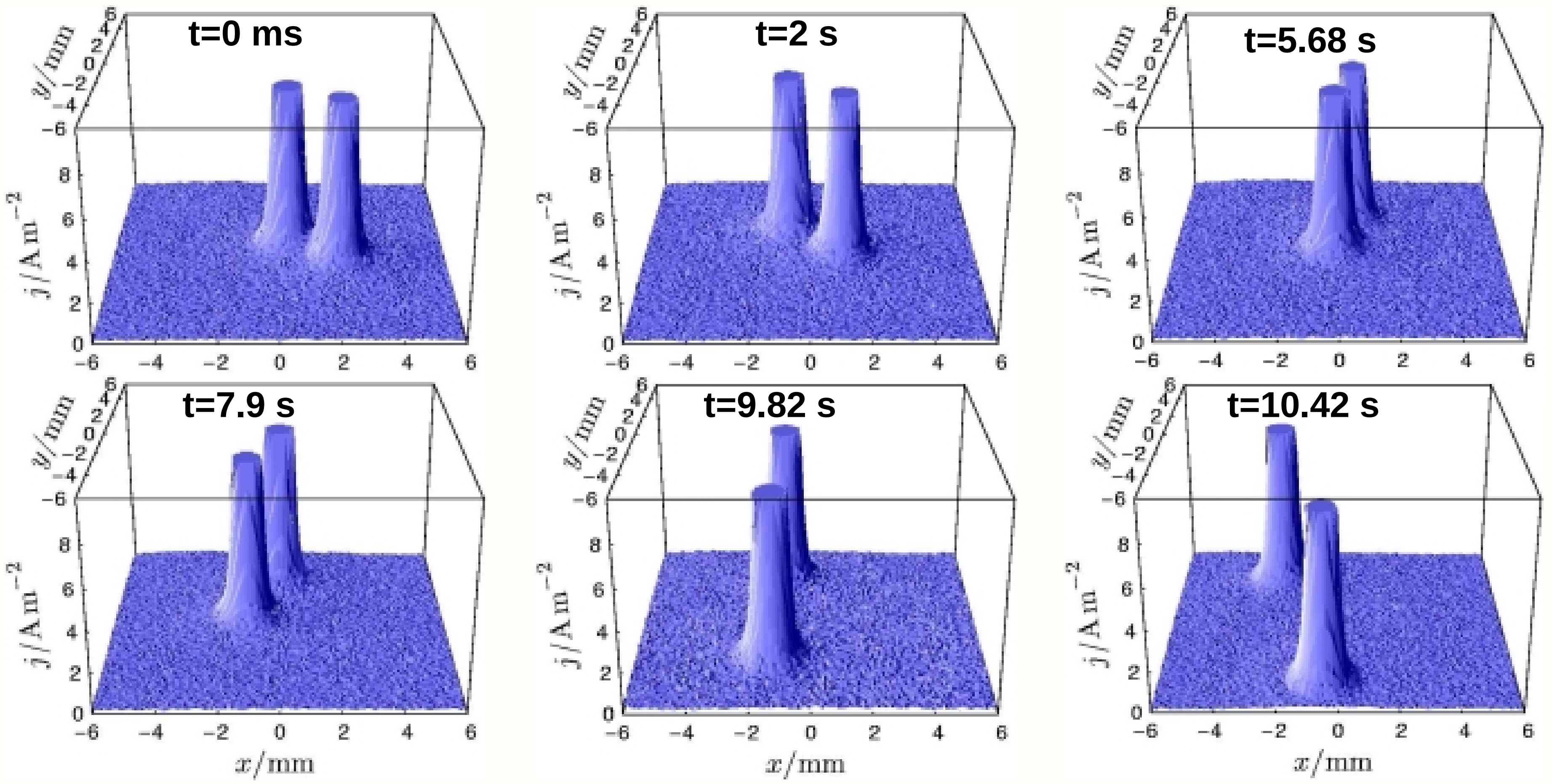
Bödeker et al. 2003

Typical Scattering Process for two Interacting Filaments in the Discharge Plane



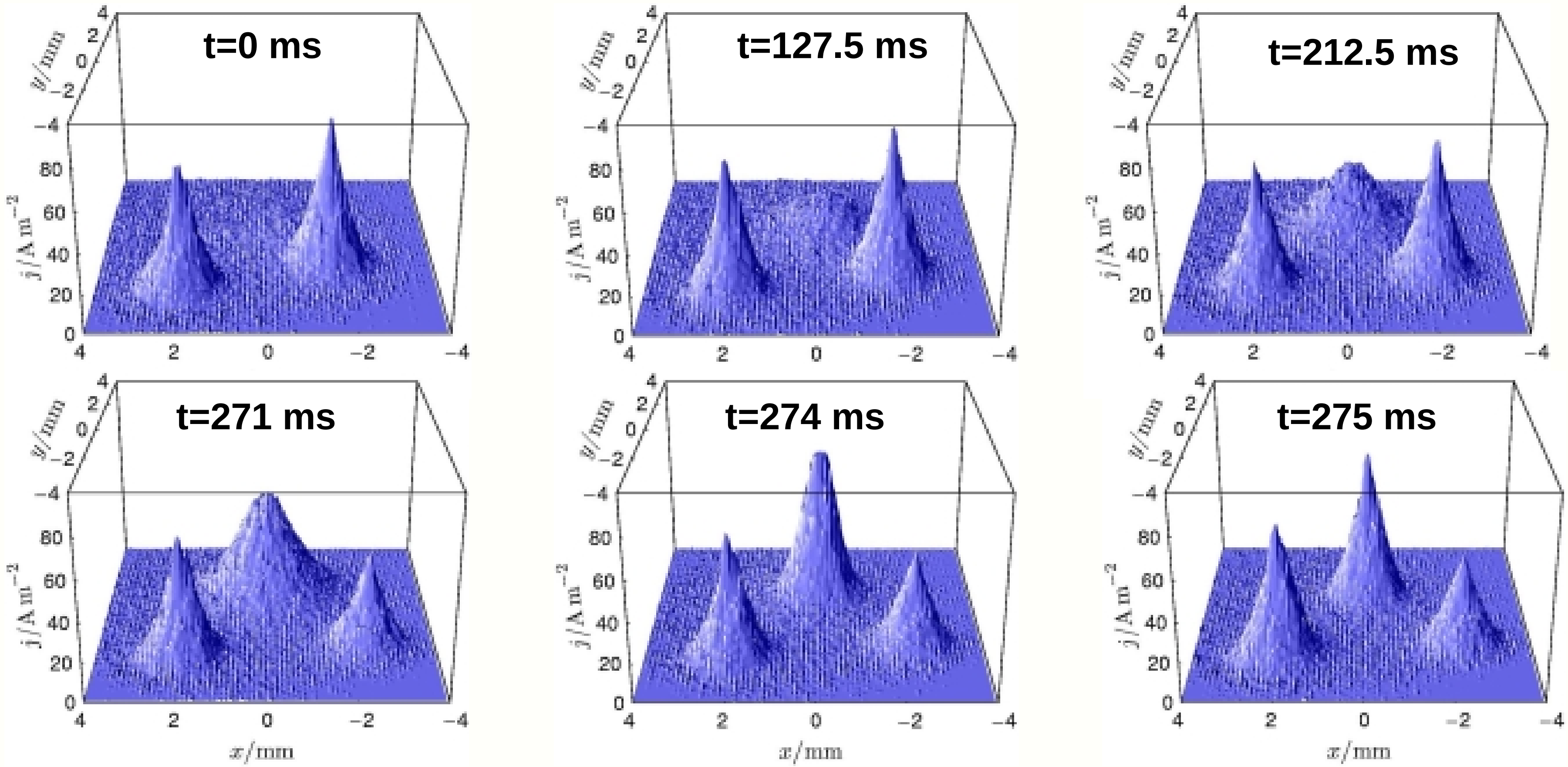
parameters: U_0=3 kV, ρ_{SC}=1.98 MΩ cm, R_0=4.4 MΩ, Gas: N_2, T=100 K, p=244 hPa, D=30 mm, d=500 µm, a_{SC}=1 mm, I=140 µA, t_{exp}=20 ms, f_{rep}=50 Hz

Typical Process for Molecule Formation of two Interacting Filaments in the Discharge Plane



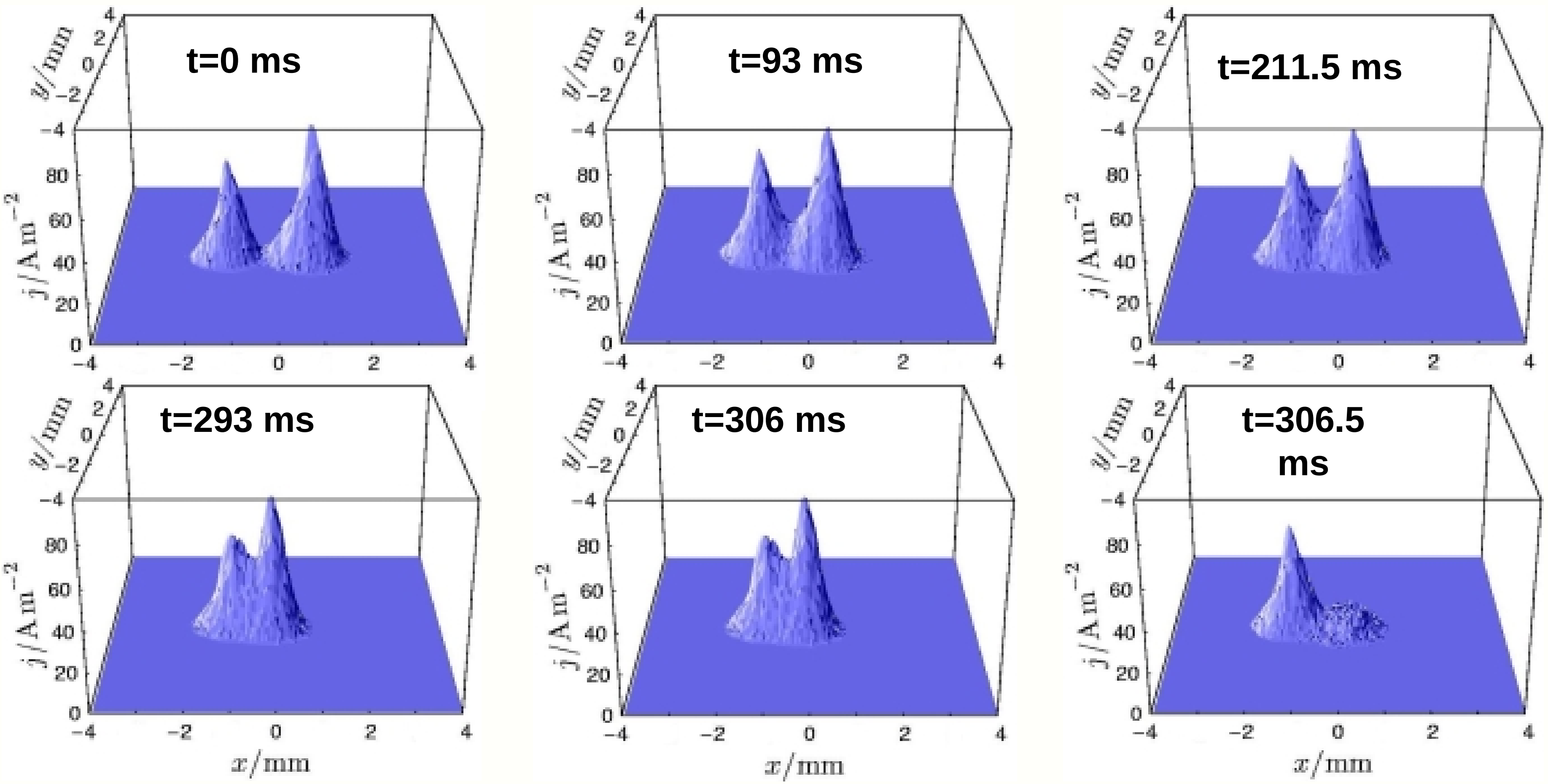
parameters: U_0=3.1 kV, ρ_{SC}=4.19 MΩ cm, R_0=4.4 MΩ, Gas: N_2, T=100 K, p=290 hPa, D=30 mm, d=500 µm, a_{SC}=1 mm, I=170 µA, t_{exp}=20 ms, f_{rep}=50 Hz

Typical Process for the Generation of a Current Filament in the Discharge Plane



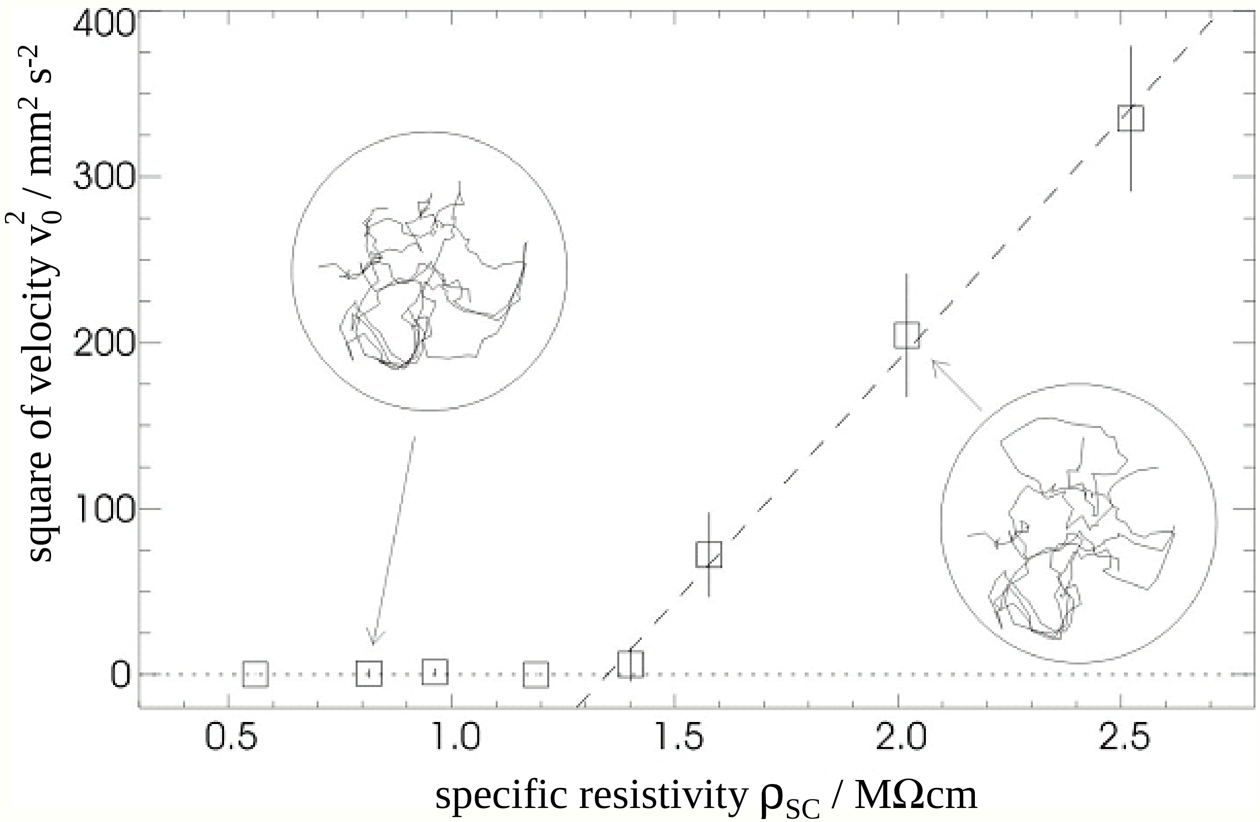
parameters: U_0=3,8 kV, ρ_{SC}=4,14 MΩ cm, R_0=20 MΩ, Gas: N_2, T=100 K, p=290 hPa, D=30 mm, d=500 µm, a_{SC}=1 mm, I=100-250 µA, t_{exp}=0,2 ms, f_{rep}=2 kHz

Typical Process for the Annihilation of a Current Filament in the Discharge Plane



parameters: U_0=3,8 kV, ρ_{SC}=4,14 MΩ cm, R_0=20 MΩ, Gas: N_2, T=100 K, p=290 hPa, D=30 mm, d=500 µm, a_{SC}=1 mm, I=100-250 µA, t_{exp}=0,2 ms, f_{rep}=2 kHz

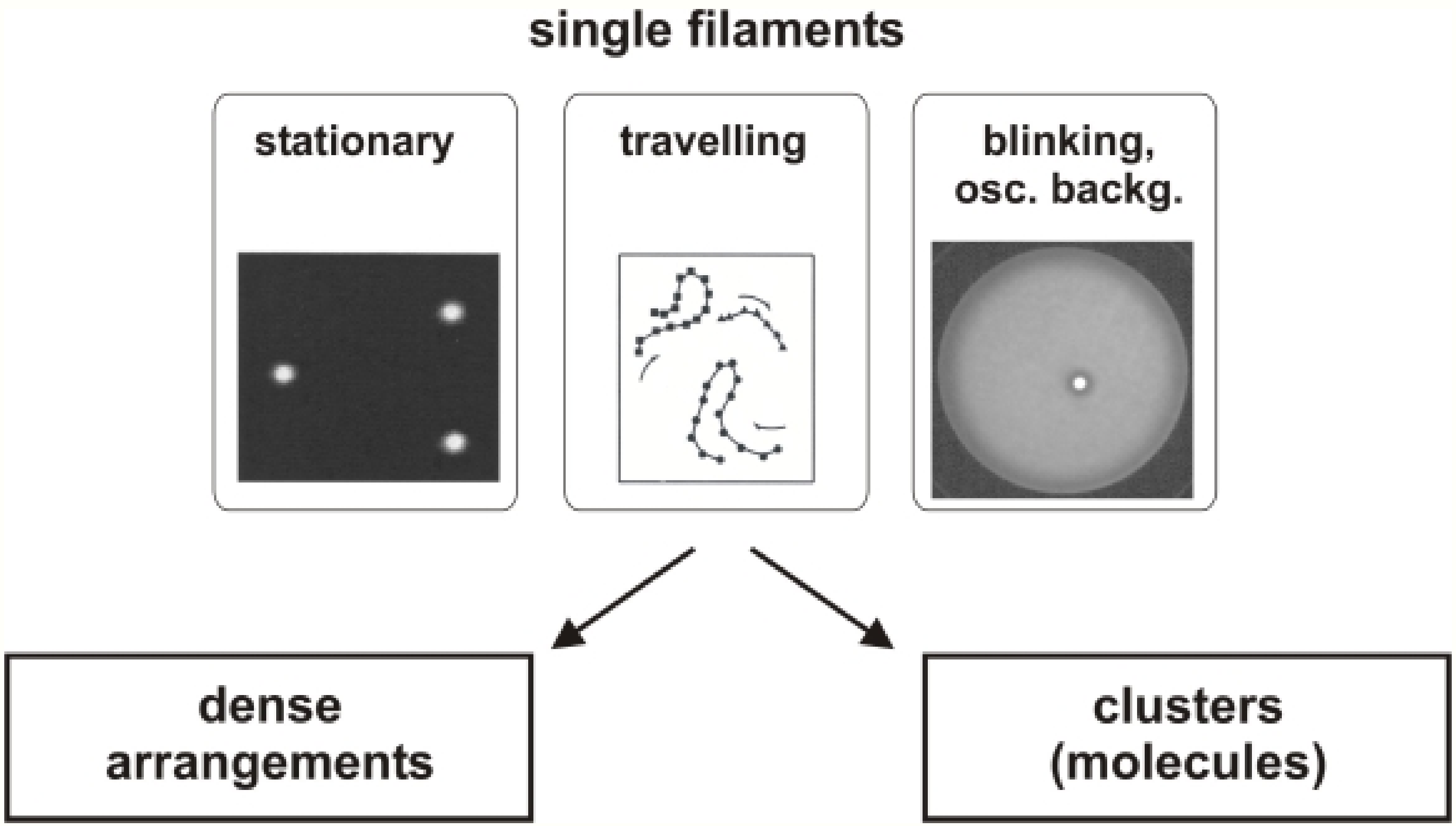
Drift Bifurcation of an Experimental Filament in the Discharge Plane Obtained from Stochastic Data Analysis of Trajectories



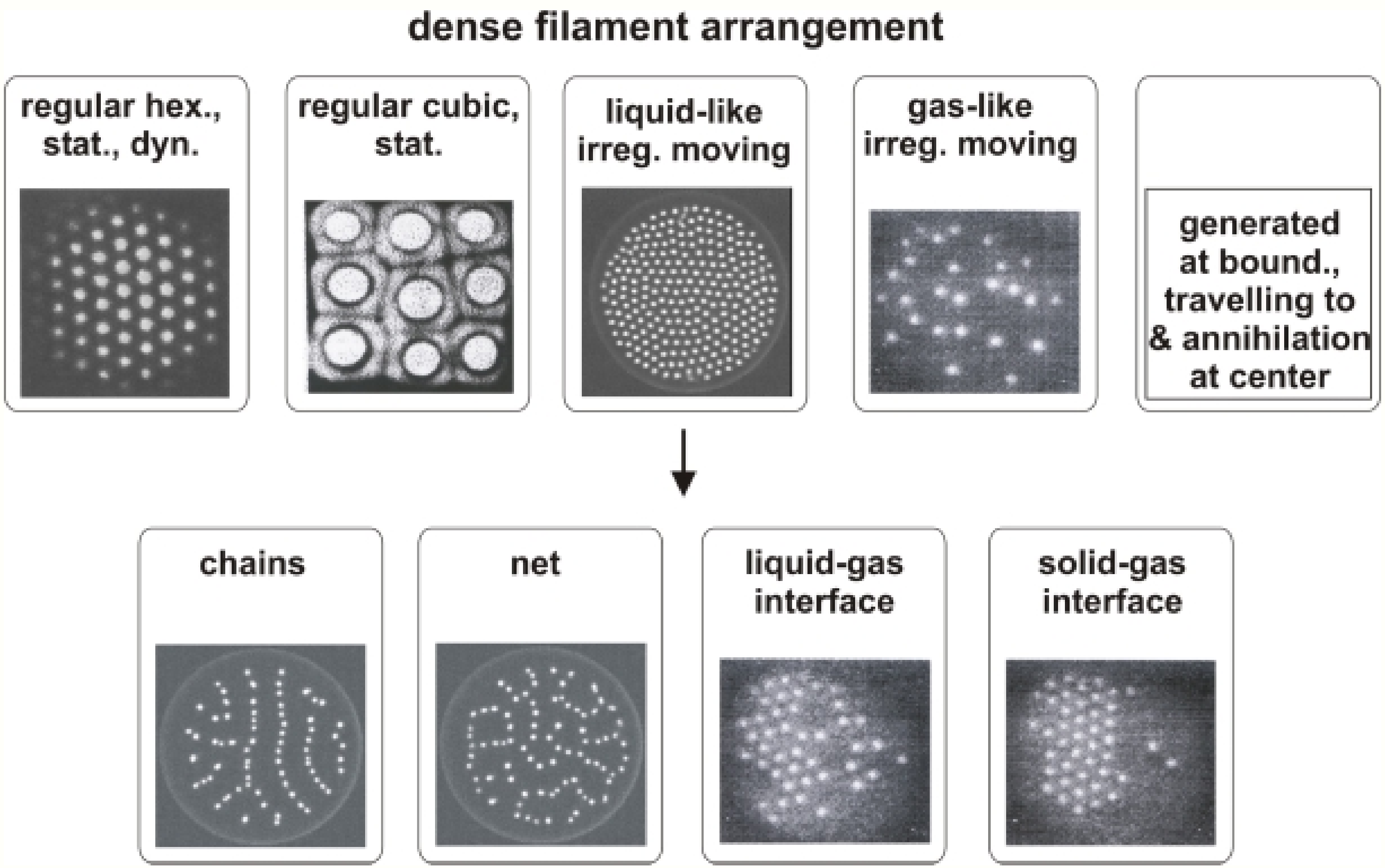
parameters:
U_0=3,7 kV, R_0=10 MΩ,
Gas: N_2, T=100 K,
p=286 hPa, D=30 mm,
d=750 μm, a_{SC}=1 mm,
I=107 μA, t_{exp}=20 ms,
f_{rep}=50 Hz

square of the intrinsic velocity as a function of the specific resistivity of the semiconductor wafer and typical experimental trajectories

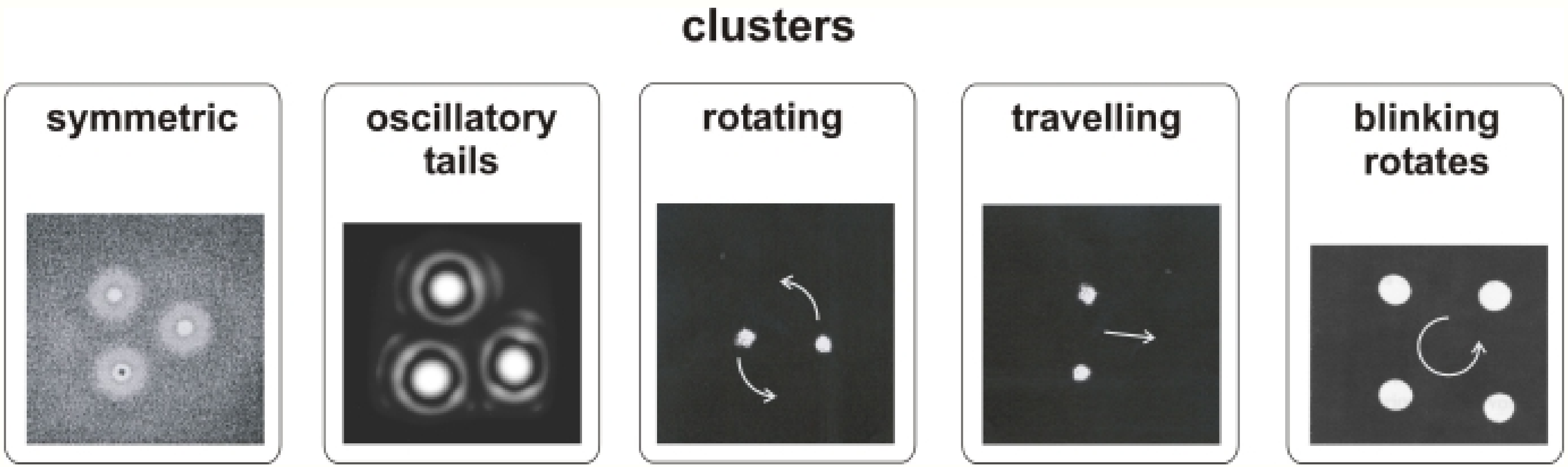
Experimentally Observed Hierarchy of Filamentary Patterns in the DC Gas-Discharge System I



Experimentally Observed Hierarchy of Filamentary Patterns in the DC Gas-Discharge System II



Experimentally Observed Hierarchy of Filamentary Patterns in the DC Gas-Discharge System III



Chapter 4

Qualitative Model

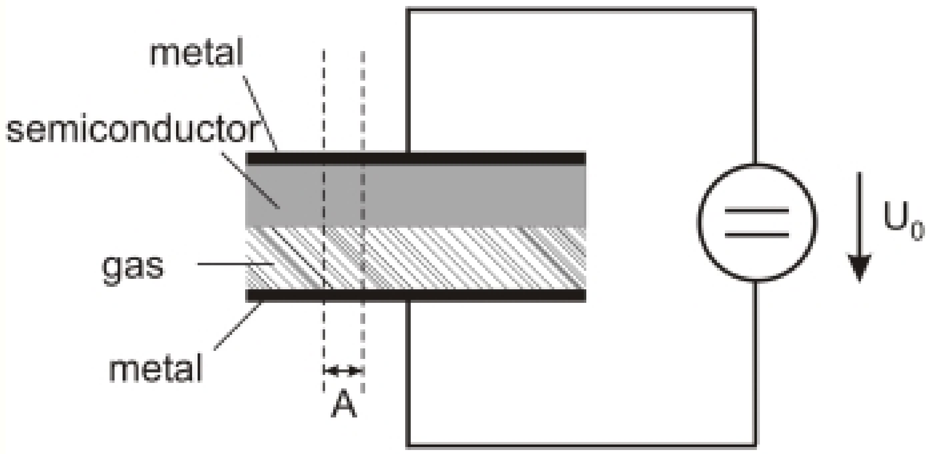
for

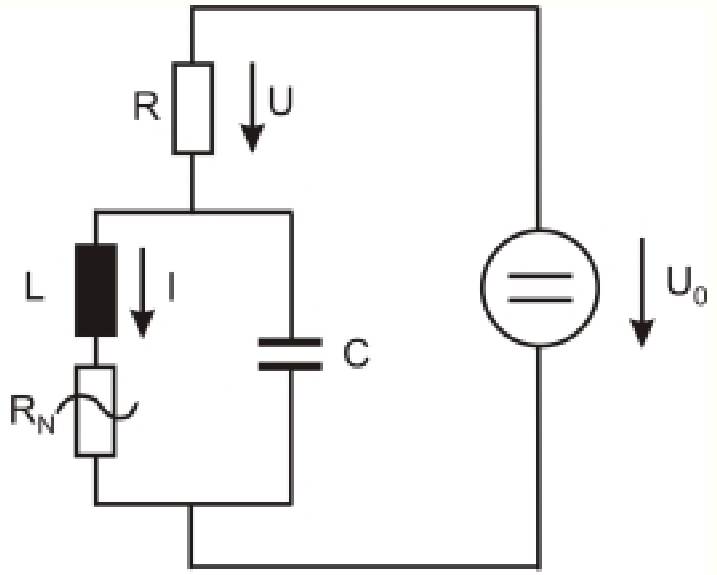
Planar Gas-Discharge Systems

with

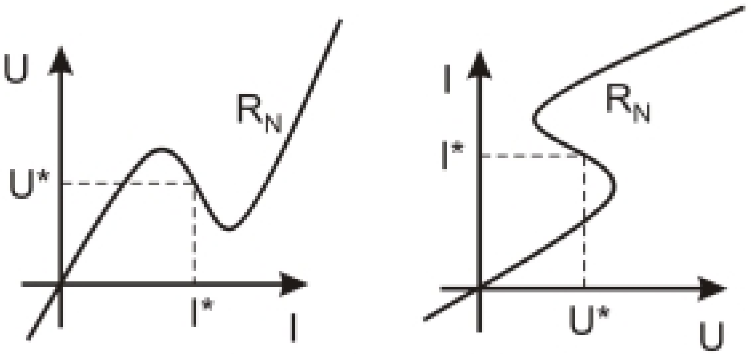
High Ohmic Barrier

The Local DC Gas-Discharge System as Activator-Inhibitor System





equivalent circuit for the area A without spacial coupling



characteristic of R_N

behaviour of the circuit

- $\Delta I > 0, \Delta U = 0 \rightarrow \frac{dI}{dt} > 0, \frac{dU}{dt} > 0$
- $\Delta I = 0, \Delta U > 0 \rightarrow \frac{dI}{dt} < 0, \frac{dU}{dt} < 0$

The 3-Component Reaction-Diffusion-Equation (3-k-RD-System)

$$u_t = D_u \Delta u + f(u) - \kappa_3 v - \kappa_4 w + \kappa_1 - \frac{\kappa_2}{\|\Omega\|} \int_\Omega u \, d\Omega,$$

$$\tau v_t = D_v \Delta v + u - v,$$

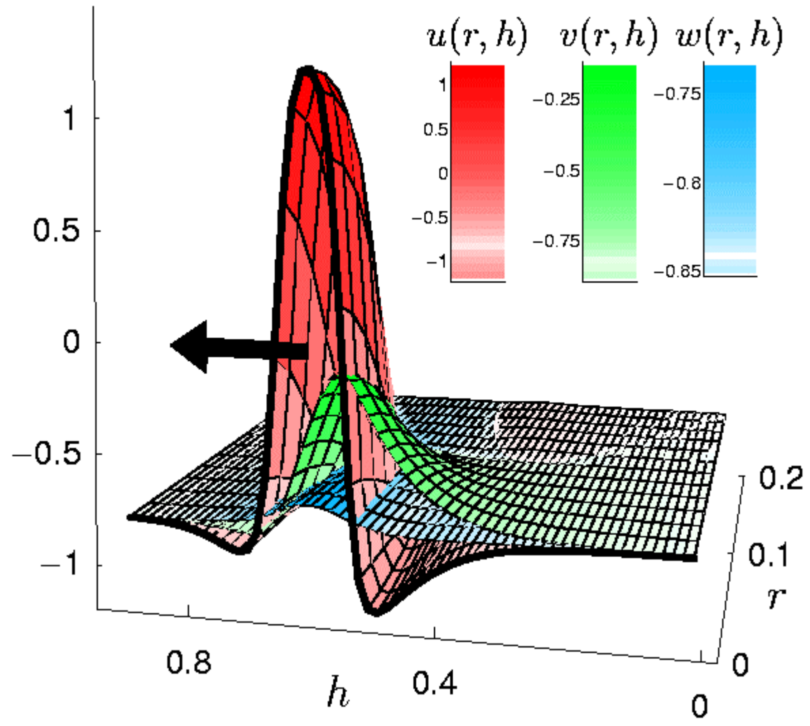
$$\theta w_t = D_w \Delta w + u - w,$$

$$f(u) \approx \lambda u - u^3,$$

$$u = u(\vec{x};t),\ v = v(\vec{x};t),\ w = w(\vec{x};t),\ \vec{x} \in \Omega \subset \mathbb{R}^1, \mathbb{R}^2, \mathbb{R}^3,$$

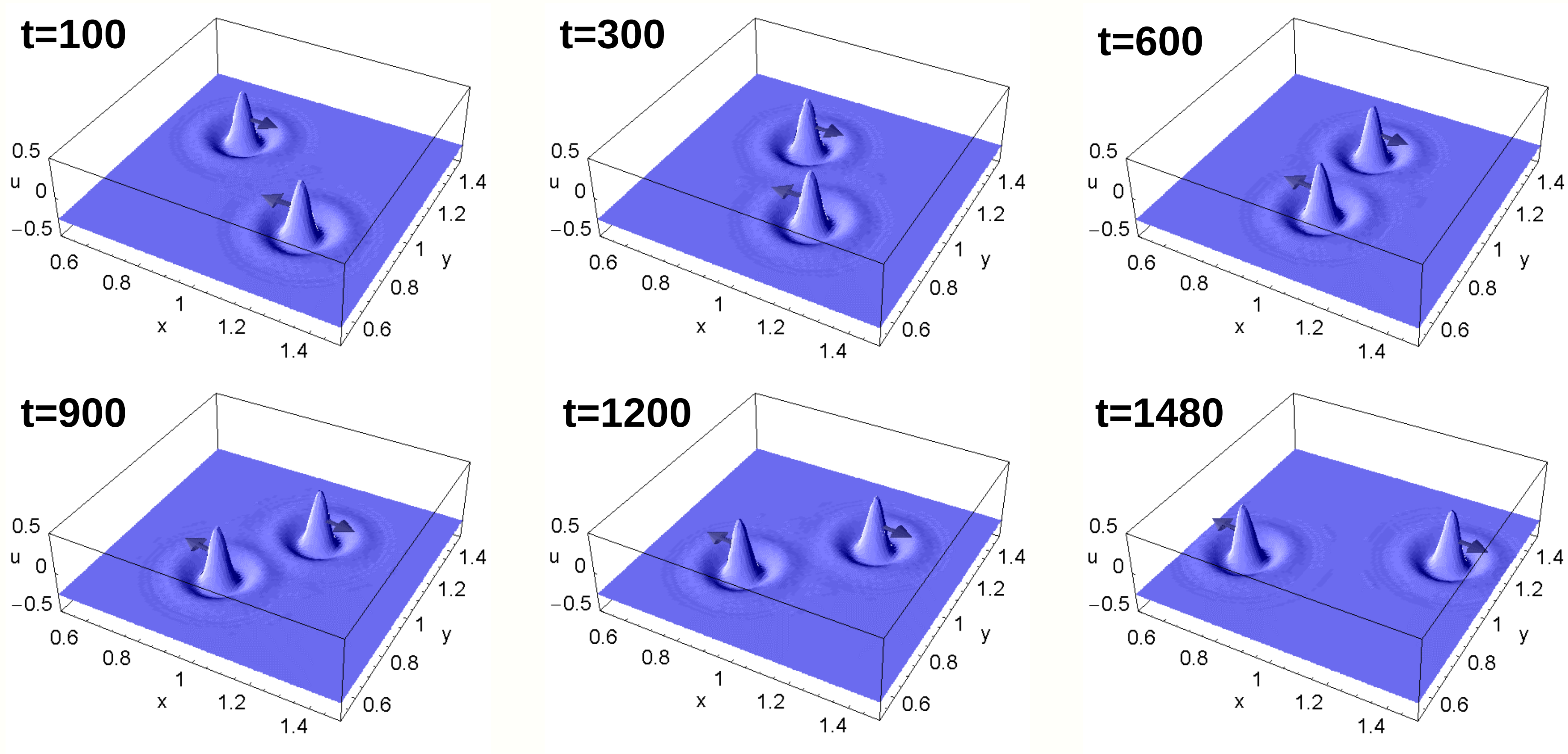
$$D_u, D_v, D_w, \tau, \theta, \lambda, \kappa_2, \kappa_3, \kappa_4 \geq 0.$$

Numerical Solution of the 3-k-RD-System: Dissipative Soliton as Localized Travelling Solitary Structure in $\mathbb{R}^3$



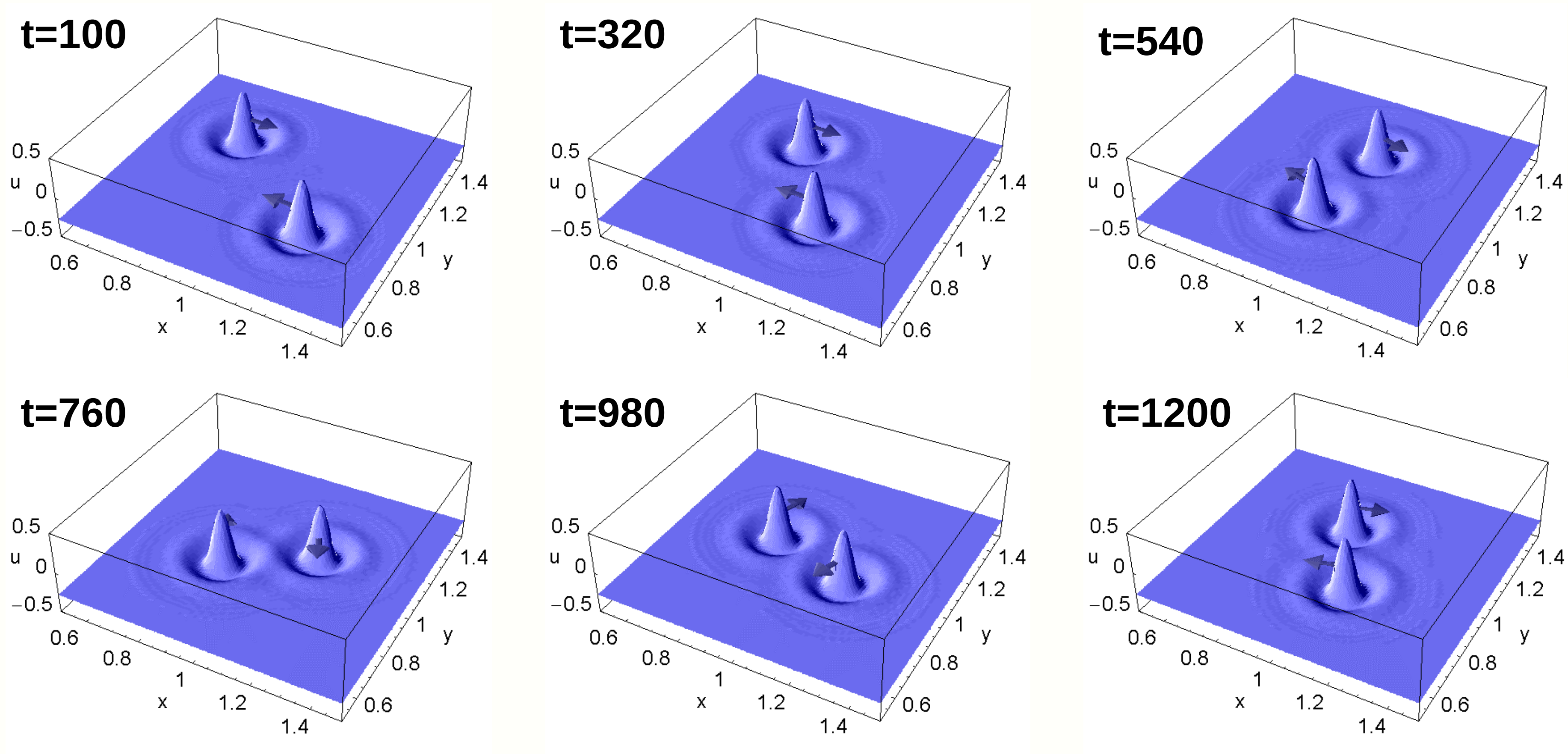
τ=48.0, θ=0.5, D_u=1.5*10^{-4}, D_v=1.86*10^{-4}, D_w=9.6*10^{-3}, λ=2.0, κ_1=-6.92, κ_2=0, κ_3=8.5, κ_4=1.0, Ω=[0,0.466]x[0,0.932], Δx=0.0155, Δt=0.01 .

Numerical Solutions of the 3-k-RD-System: Scattering of Interacting Dissipative Solitons in $\mathbb{R}^2$



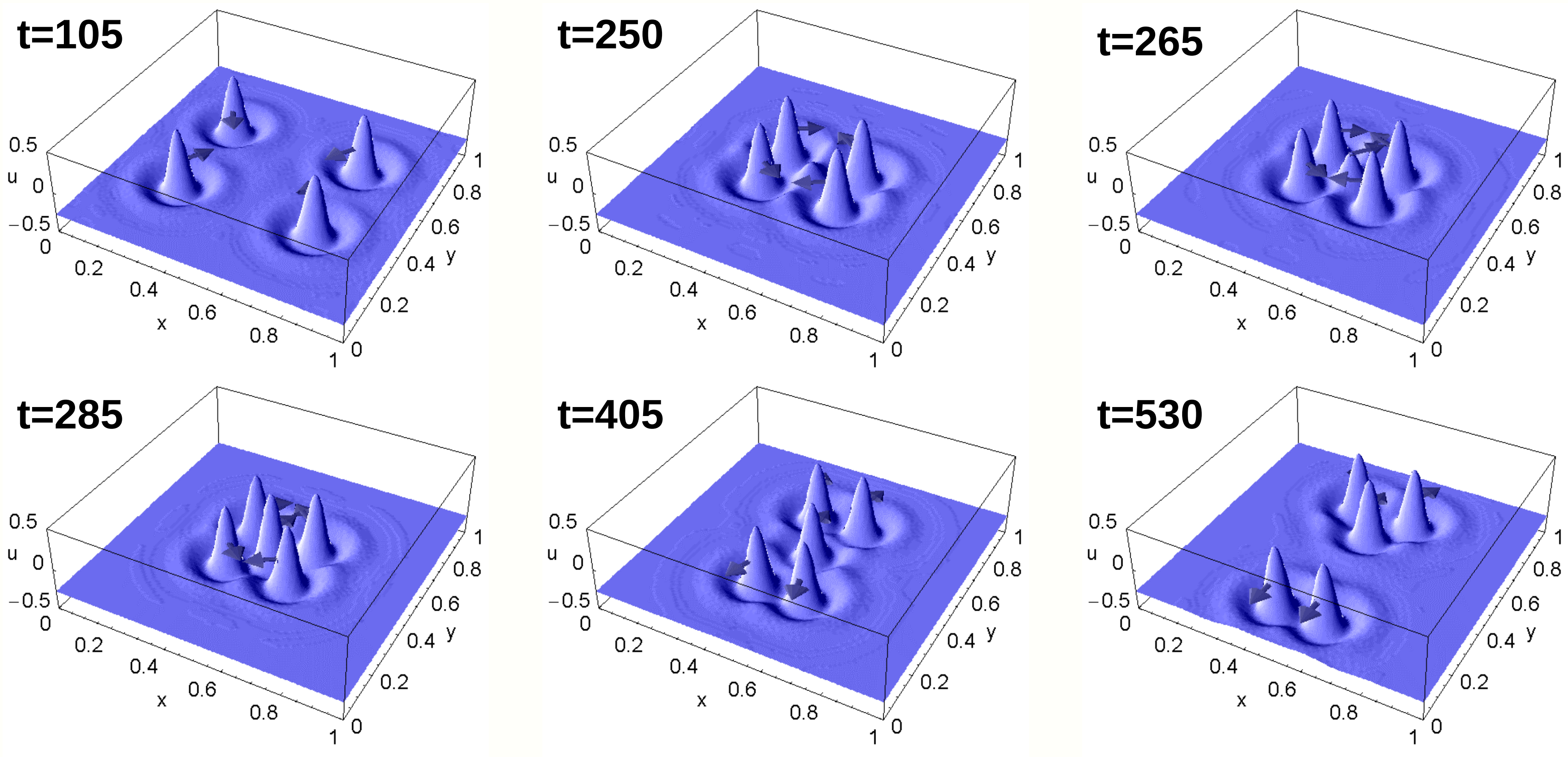
τ=3.35, θ=0, D_u=1.1*10^{-4}, D_v=0, D_w=9.64*10^{-4}, λ=1.01, κ_1=-0.1, κ_3=0.3, κ_4=1.0 Ω=[0,1]x[0,1], Δx=5*10^{-3}, Δt=0.1 .

Numerical Solutions of the 3-k-RD-System: Formation of Rotating Molecules Due to Collision of Dissipative Solitons in $\mathbb{R}^2$



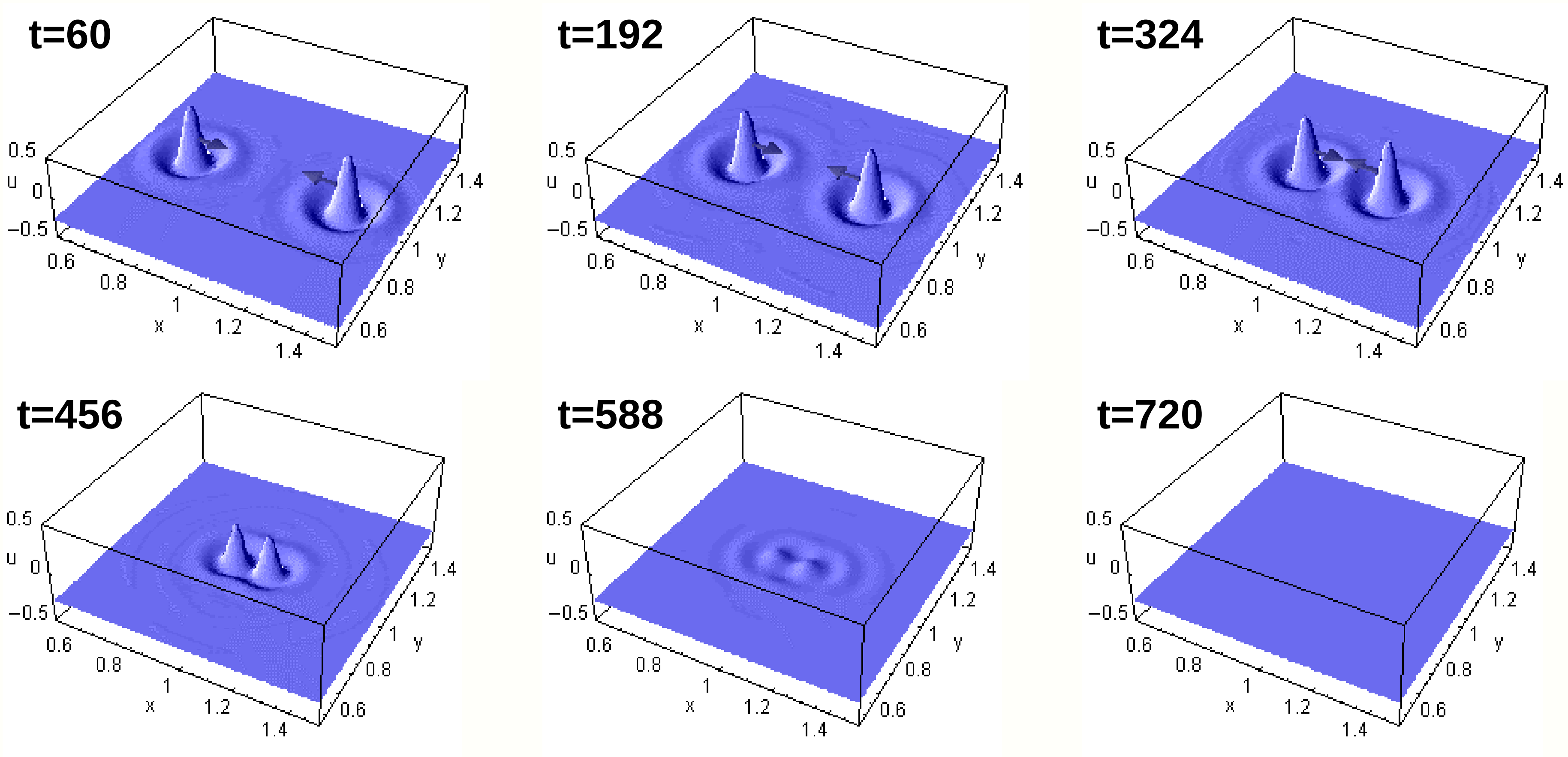
τ=3.35, θ=0, D_u=1.1*10^{-4}, D_v=0, D_w=9.64*10^{-4}, λ=1.01, κ_1=-0.1, κ_3=0.3, κ_4=1.0, Ω=[0,1]x[0,1], Δx=5*10^{-3}, Δt=0.1 .

Numerical Solutions of the 3-k-RD-System: Generation of a Dissipative Soliton Due to Collision in $\mathbb{R}^2$



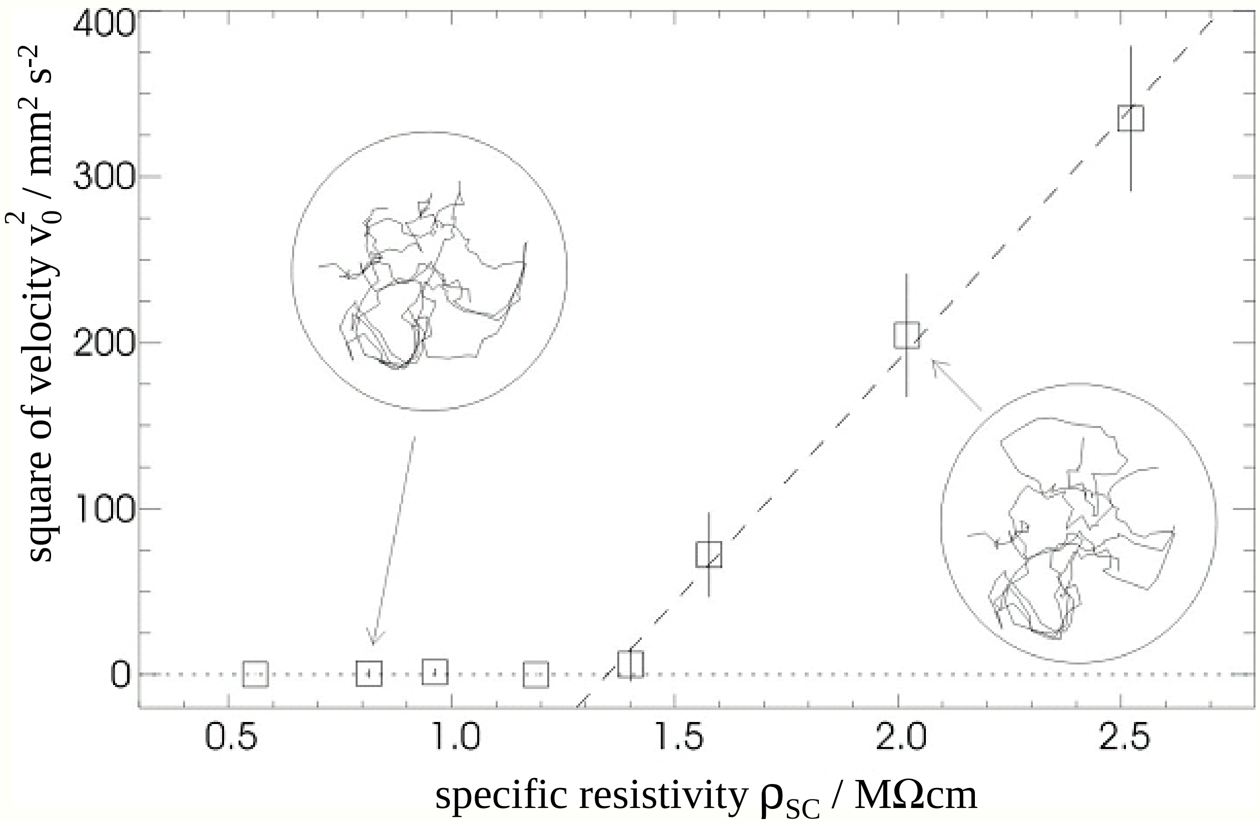
τ=3.47, θ=0, D_u=1.1*10^{-4}, D_v=0, D_w=9.64*10^{-4}, λ=1.01, κ_1=-0.1, κ_3=0.3, κ_4=1.0, Ω=[0,1]x[0,1], Δx=5*10^{-3}, Δt=0.1 .

Numerical Solutions of the 3-k-RD-System: Annihilation of a Dissipative Soliton Due to Collision in $\mathbb{R}^2$



$\tau=3.59$, $\theta=0$, $D_u=1.1*10^{-4}$, $D_v=0$, $D_w=9.64*10^{-4}$, $\lambda=1.01$, $\kappa_1=-0.1$, $\kappa_3=0.3$, $\kappa_4=1.0$, $\Omega=[0,1]x[0,1]$, $\Delta x=5*10^{-3}$, $\Delta t=0.1$.

Drift Bifurcation of an Experimental Filament in the Discharge Plane Obtained from Stochastic Data Analysis of Trajectories



parameters:
U_0=3,7 kV, R_0=10 MΩ,
Gas: N_2, T=100 K,
p=286 hPa, D=30 mm,
d=750 µm, a_{SC}=1 mm,
I=107 µA, t_{exp}=20 ms,
f_{rep}=50 Hz

square of the intrinsic velocity as a function of the specific resistivity of the semiconductor wafer and typical experimental trajectories

Relevance of the 3-k-RD-System

exemplaric theoretical investigation

- **3-component nonlinear partial differential equation**
- **simple structure**
- **solutions reflect a large variety of particle properties**
- **strong relation to electrical transport systems**
- **theoretical predictions could be manifested**
- **experimentally observed phenomena could be found in the solutions of the equations**
- **expectation: deep insight into the formation of self-organized patterns**
- **deep insight into the mechanisms of pattern formation of nonlinear dissipative systems in general**

The Reduced Equation

The 3-Component Reaction-Diffusion-Equation (3-k-RD-System)

$$u_t = D_u \Delta u + f(u) - \kappa_3 v - \kappa_4 w + \kappa_1 - \frac{\kappa_2}{\|\Omega\|} \int_\Omega u \, d\Omega,$$

$$\tau v_t = D_v \Delta v + u - v,$$

$$\theta w_t = D_w \Delta w + u - w,$$

$$f(u) \approx \lambda u - u^3,$$

$$u = u(\vec{x};t),\ v = v(\vec{x};t),\ w = w(\vec{x};t),\ \vec{x} \in \Omega \subset \mathbb{R}^1, \mathbb{R}^2, \mathbb{R}^3,$$

$$D_u, D_v, D_w, \tau, \theta, \lambda, \kappa_2, \kappa_3, \kappa_4 \geq 0.$$

Reduction of the Field Equation to a Dynamical Equation for Dissipative Solitons I

ansatz:

normalization of the homogeneous stationary background to 0,

$$\mathbf{u}(\vec{x},T_1,T_2,T_3)=\bar{u}(\vec{x}-\vec{p}_1)+\bar{u}(\vec{x}-\vec{p}_2)+\varepsilon^2 r_u-\varepsilon^3 R_u,$$

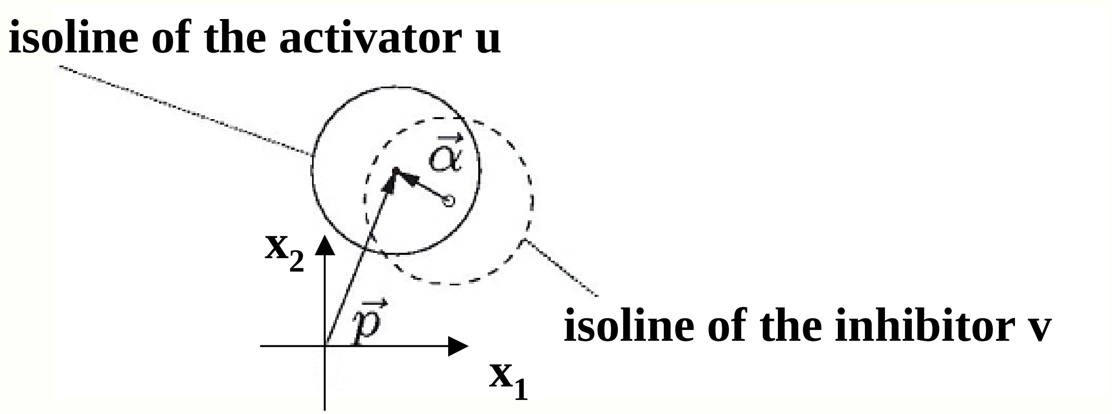
$$\mathbf{v}(\vec{x},T_1,T_2,T_3)=\bar{u}(\vec{x}-\vec{p}_1)+\bar{u}(\vec{x}-\vec{p}_2)+\varepsilon\vec{\alpha}_1\nabla\bar{u}(\vec{x}-\vec{p}_1)+\varepsilon\vec{\alpha}_2\nabla\bar{u}(\vec{x}-\vec{p}_2)+\varepsilon^2 r_v+\varepsilon^3 R_v.$$

time scale separation:

$$\vec{p}_i=\vec{p}_i(T_1,T_2,T_3),\ i=1,2, \qquad r_{u,v}=r_{u,v}(\vec{x},T_1), \qquad T_j=\varepsilon^j t,\ \ j=1,2,3,$$

$$\varepsilon\vec{\alpha}_i=\varepsilon\vec{\alpha}_i(T_1,T_2), \qquad R_{u,v}=R_{u,v}(\vec{x}).$$

visualization of $\vec{p}$ and $\vec{\alpha}$ in $\mathbb{R}^2$ for a dissipative Soliton



Reduction of the Field Equation to a Dynamical Equation for Dissipative Solitons II

equation of motion for N dissipative solitons

$$\dot{\vec{p}}_i = \kappa_3 \vec{\alpha}_i - \vec{W}_i(\vec{p}_1, \ldots, \vec{p}_N), \qquad i = 1, 2, \ldots, N$$

$$\dot{\vec{\alpha}}_i = \kappa_3^2 \left(\tau - \frac{1}{\kappa_3} \right) \vec{\alpha}_i - \kappa_3 Q \alpha_i^2 \vec{\alpha}_i - \vec{W}_i(\vec{p}_1, \ldots, \vec{p}_N)$$

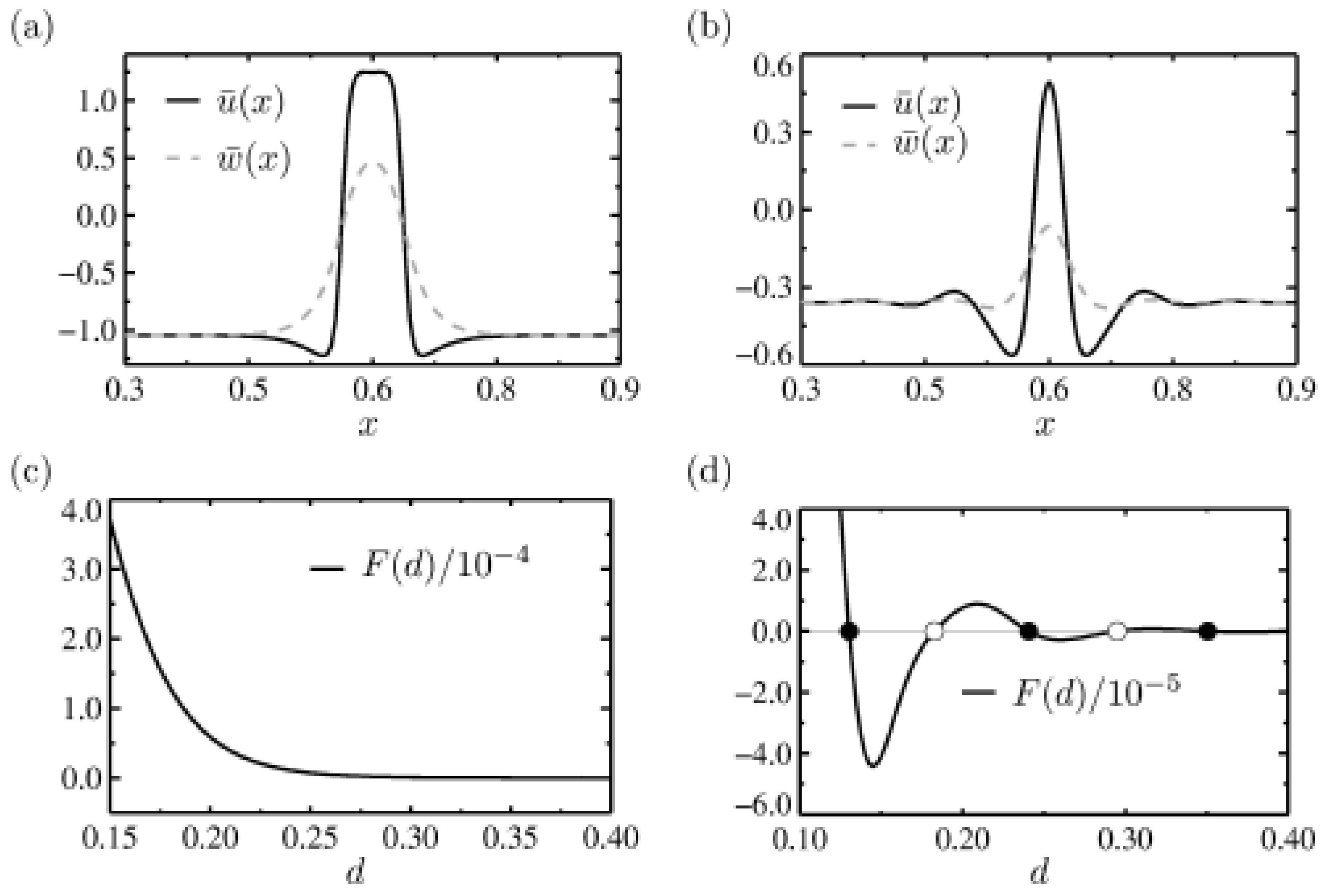
interaction function

$$\vec{W}_i(\vec{p}_1, \ldots, \vec{p}_N) = \sum_{\substack{j=1 \\ j \neq i}}^{N} \vec{W}_{DS}(\vec{p}_i, \vec{p}_j) = \sum_{\substack{j=1 \\ j \neq i}}^{N} F\left(\left| \vec{p}_j - \vec{p}_i \right| \right) \frac{\vec{p}_j - \vec{p}_i}{\left| \vec{p}_j - \vec{p}_i \right|}$$

Interaction Function of the Equation of Motion for Dissipative Solitons Derived from the Field Equation

$\bar{u}(x) = \bar{v}(x)$

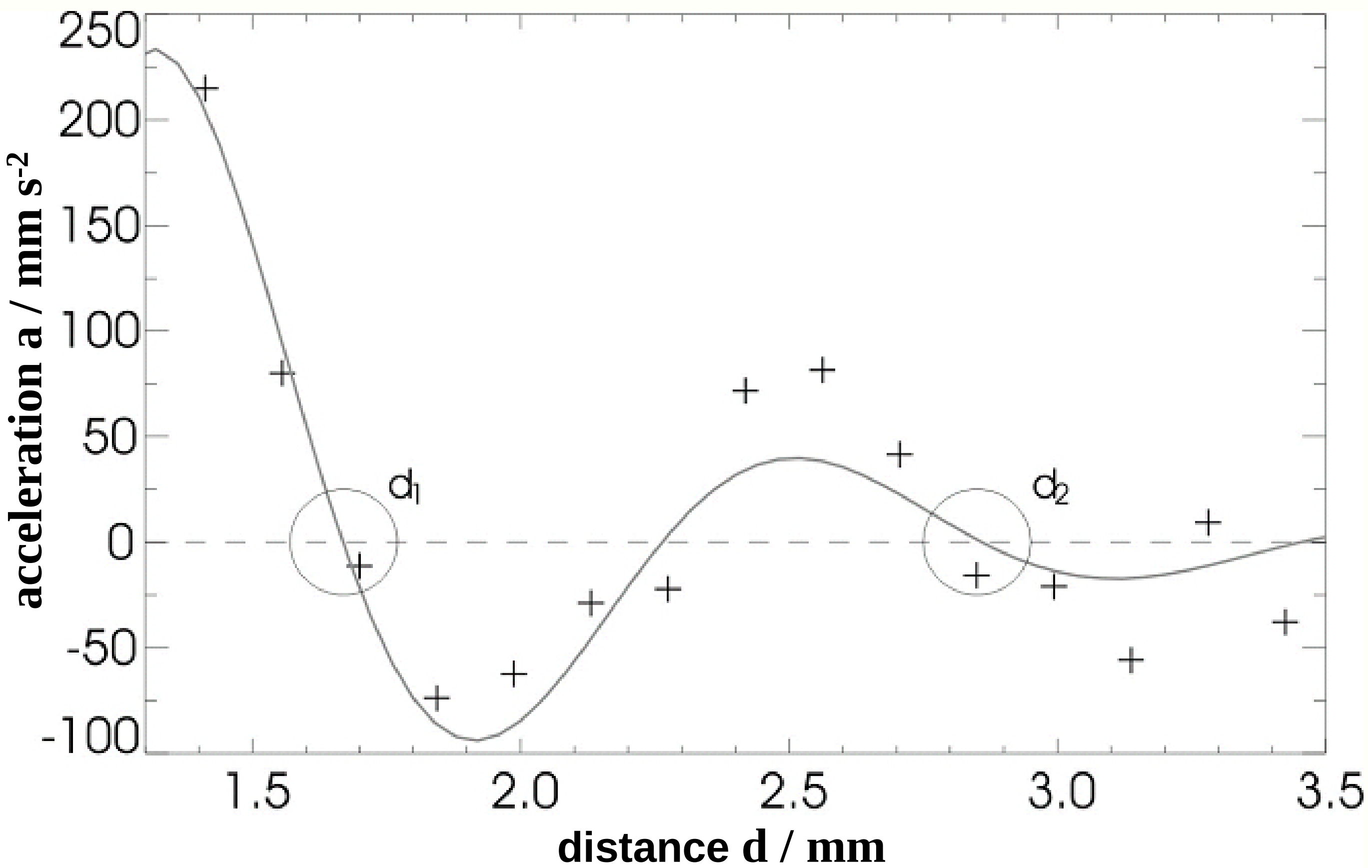
$F(d) = F\left(\left|\vec{p}_2 - \vec{p}_1\right|\right)$



a+c: $\tau=0$, $\theta=0$, $D_u=0.5*10^{-4}$, $D_v=0$, $D_w=10^{-3}$, $\lambda=3.0$, $\kappa_1=-0.1$, $\kappa_3=1.0$, $\kappa_4=1.0$, $\Omega=[0,1.2]$, $\Delta x=5*10^{-3}$,

b+d: $D_u=1.1*10^{-4}$, $D_w=9.64*10^{-4}$, $\lambda=1.71$, $\kappa_1=-0.15$, $\Delta x=2.5*10^{-3}$, others as in a+c.

Determination of the Interaction Function from Experimental Trajectories Using New Stochastic Data Analysis

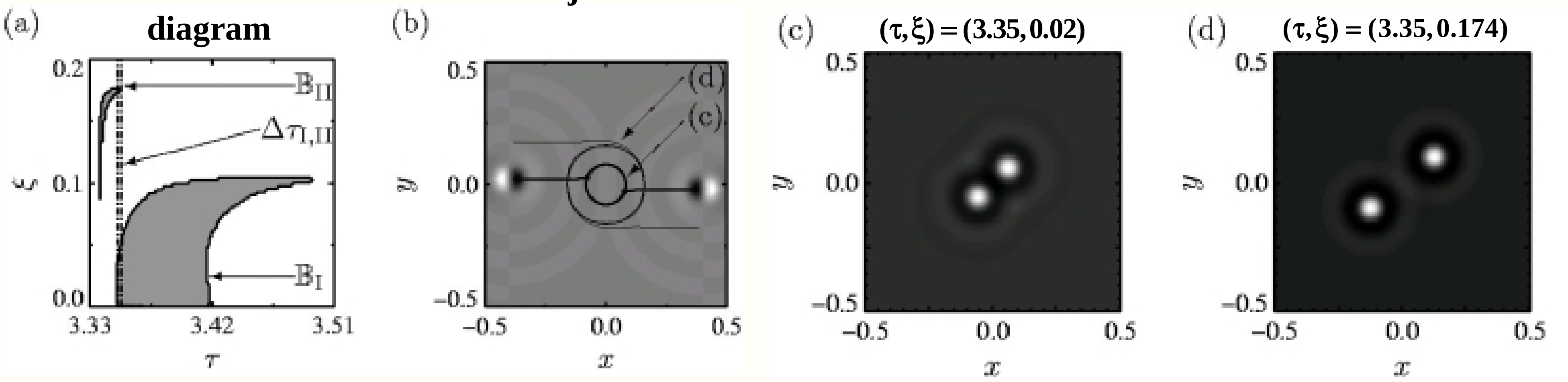


parameters: U_0=4600 V, ρ_{SC}=3,55 MΩ cm, R_0=4,4 MΩ, Gas: N_2, T=100 K, p=283 hPa, D=30 mm, d=550 µm, a_{SC}=1 mm, I=233µA, t_{exp}=20 ms, f_{rep}=50 Hz

Bödeker et al. 2003

Examples for the Interaction Behaviour of Dissipative Solitons in $\mathbb{R}^2$ Obtained from the Model Equations

reduced equation (a), field equation (b)-(d)



white area: scattering

B_I **: rotating molecules I**

B_{II} **: rotating molecules II**

ξ **: parallel offset**

τ **: relaxation time**

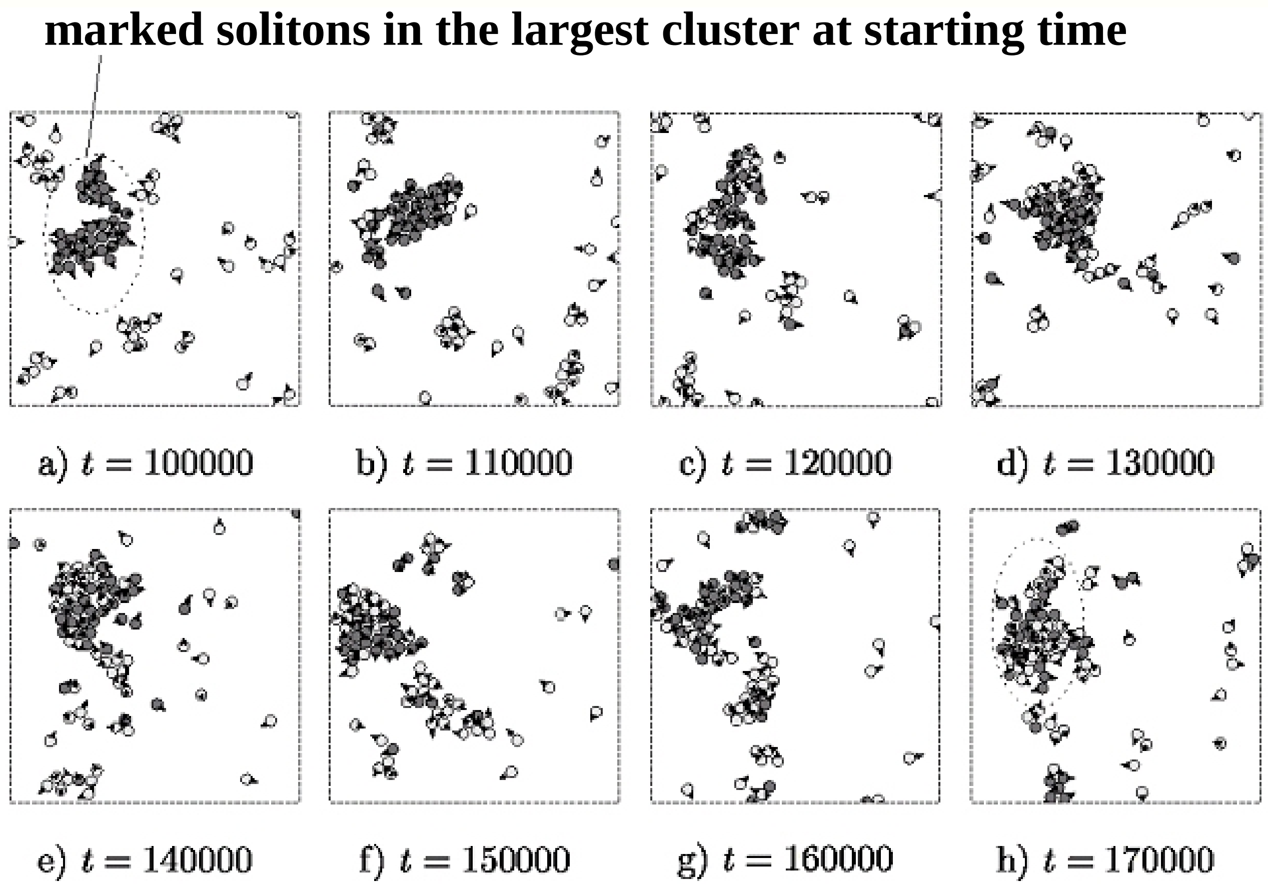
$$F(d) = -\frac{6.87\cdot 10^{-4}}{\sqrt{d}} e^{-15.7d} \cos\left(43.15\left(d-0.199\right)\right), \quad Q=1950$$

θ=0, D_u=1.1*10^{-4}, D_v=0, D_w=9.64*10^{-4}, λ=1.01, κ_1=-0.1, κ_3=0.3, κ_4=1.0, Ω=[-0.5,0.5]x[-0.5,0.5], Δx=5*10^{-3}, Δt=0.1 .

Liehr et al. 2003

Phase Separation in the Solutions of the Reduced Equation for many Dissipative Solitons

N=81



τ=3.34, θ=0, D_u=1.1*10^{-4}, D_v=0, D_w=9.64*10^{-4}, λ=1.01, κ_1=-0.1, κ_3=0.3, κ_4=1.0, Ω=[-2,2]x[-2,2], Δx=5*10^{-3}, Δt=0.1; F(d), Q = 1950.

Quantitative Model

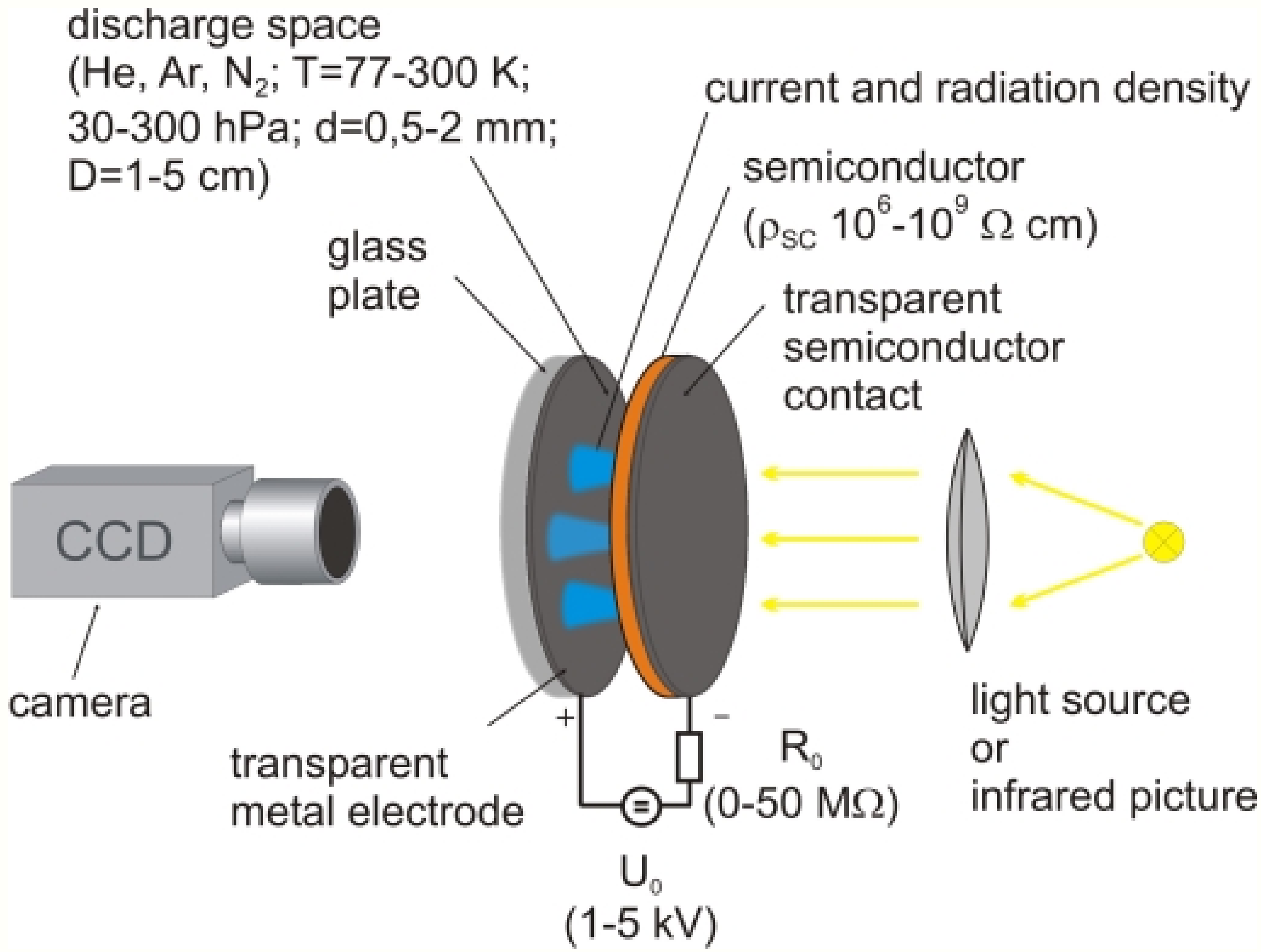
for

Planar Gas-Discharge Systems

with

High Ohmic Barrier

Experimental Set-Up for Measuring Self-Organized Patterns in Planar DC Gas-Discharge Systems



Model Equation
for the DC Gas-Discharge System I:
Gas-Discharge Space (-d < z < 0)

$$\partial_t n_e + \mathrm{div}(\vec{\Gamma}_e) = S_e,$$
$$\partial_t n_p + \mathrm{div}(\vec{\Gamma}_p) = S_p,$$
$$\Delta\varphi = -\frac{1}{\varepsilon_0} e(n_e - n_p),$$

$$\vec{\Gamma}_e = -\mu_e n_e \vec{E} - D_e \vec{\nabla} n_e,$$
$$\vec{\Gamma}_p = \mu_p n_p \vec{E} - D_p \vec{\nabla} n_p,$$
$$S_e = S_p = \nu n_e - \beta n_e n_p,$$
$$\vec{E} = -\vec{\nabla}\varphi,$$

$$\mu_{e,p} = \mu_{e,p}(E),\ \nu = \nu(E),$$
$$D_{e,p} = k T_{e,p} \mu_{e,p}(E)/e,$$
$$\vec{j}_g = e(\vec{\Gamma}_p - \vec{\Gamma}_e).$$

n_e, n_p	electron/ion density
S_e, S_p	electron/ion source term
$\vec{\Gamma}_e$, $\vec{\Gamma}_p$	electron/ion particle current density
μ_e, μ_p	electron/ion mobility
$\vec{E}$	electrical field
D_e, D_p	electron/ion diffusion constant
ν	ionisation rate
β	recombination rate
φ	electrical potential
T_e, T_p	electron/ion temperature
$\vec{j}_g$	global electrical current density

Model Equation for the DC Gas-Discharge System II: Semiconductor Wafer ($0 < z < d_{SC}$)

$$\vec{j}_{SC} = \lambda \vec{E},$$
$$\vec{E} = -\vec{\nabla} \varphi,$$
$$\mathrm{div}(\varepsilon \vec{\nabla} \varphi) = 0.$$

$\vec{j}_{SC}$	global electrical current density
λ	specific electrical conductivity
$\vec{E}$	electrical field
φ	electrical potential
ε	dielectric constant of the semiconductor

Model Equation for the DC Gas-Discharge System III: Boundary Conditions Gas – Semiconductor at z=0

$$\partial_t \sigma - D_S \Delta\sigma = \vec{e}_z (\vec{j}_g - \vec{j}_{SC})_{z=0},$$

$$\frac{1}{\varepsilon_0}\sigma = (\varepsilon \vec{e}_z \vec{E})_{z=+0} - (\vec{e}_z \vec{E})_{z=-0},$$

$$(\vec{\Gamma}_p \vec{e}_z)_{z=-0} = (\mu_p n_p \vec{E}\vec{e}_z + \frac{1}{4} n_p \langle v_p \rangle)_{z=-0},$$

$$(\vec{\Gamma}_e \vec{e}_z)_{z=-0} = (-\mu_e n_e \vec{E}\vec{e}_z + \frac{1}{4} n_e \langle v_e \rangle - \gamma \vec{\Gamma}_p \vec{e}_z)_{z=-0},$$

$$\langle v_{e,p} \rangle = \sqrt{8k T_{e,p} / \pi m_{e,p}}.$$

$$(\varphi)_{z=-d} - (\varphi)_{z=d_{SC}} = U.$$

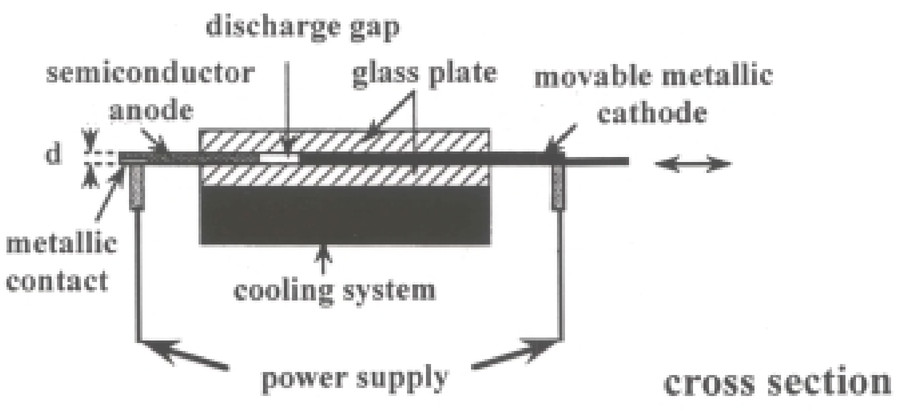
σ	surface charge
D_S	diffusion constand of surface charge
$\vec{e}_z$	unity vector in z-direction
ε	dielectric constant of the semiconductor
v_e, v_p	thermal electron/ion speed
γ	γ -Townsend-coefficient
k	Boltzmann-constant
m_e, m_p	electron/ion mass
U	voltage drop at the component

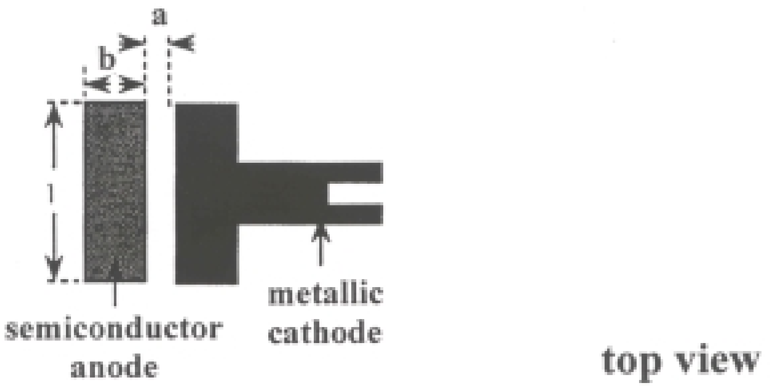
Dissipative Solitons

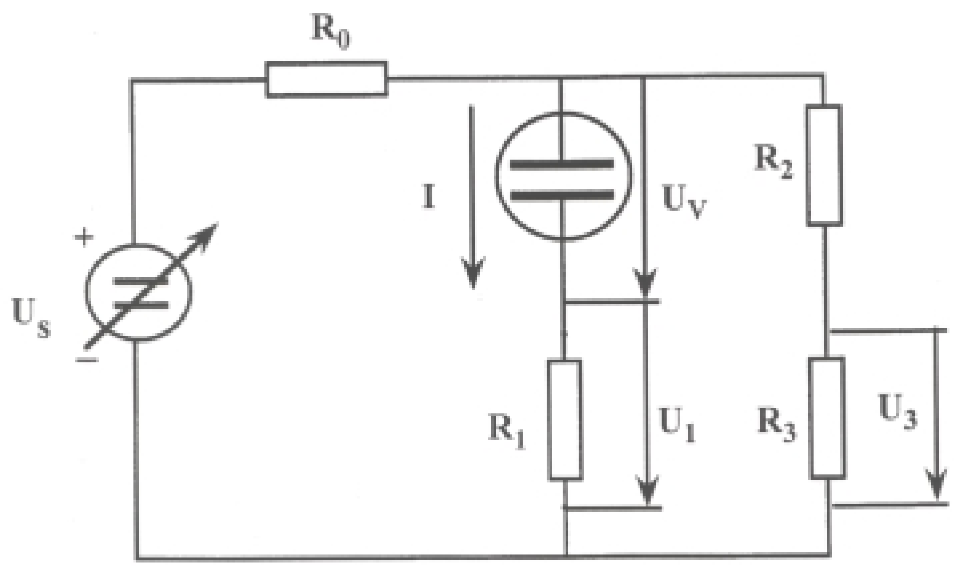
in

Various Systems

Experimental Set-Up for Measuring Self-Organized Patterns in Quasi-1-Dimensional DC Gas-Discharge Systems



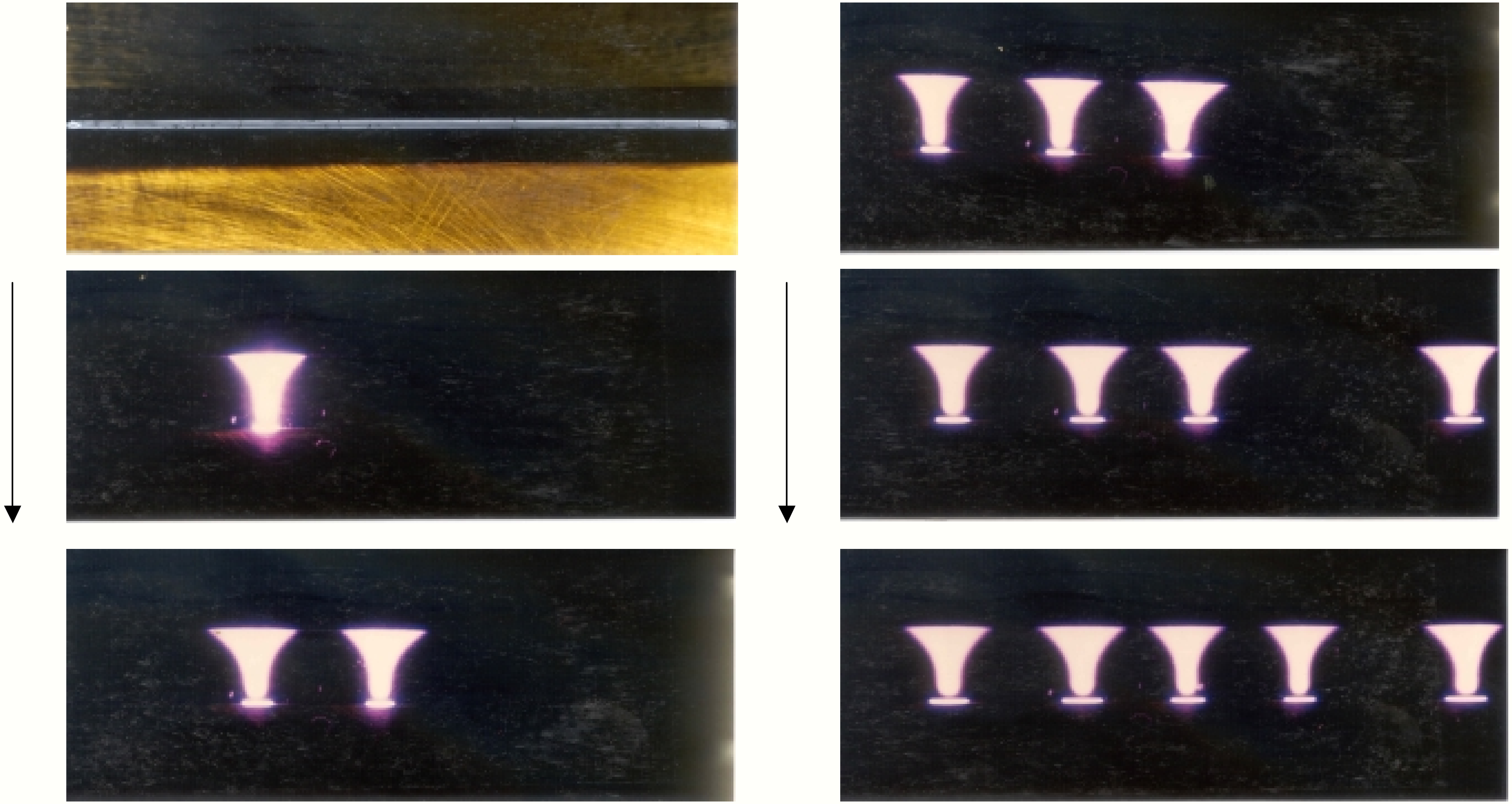




electric circuit

Radehaus et al. 1987

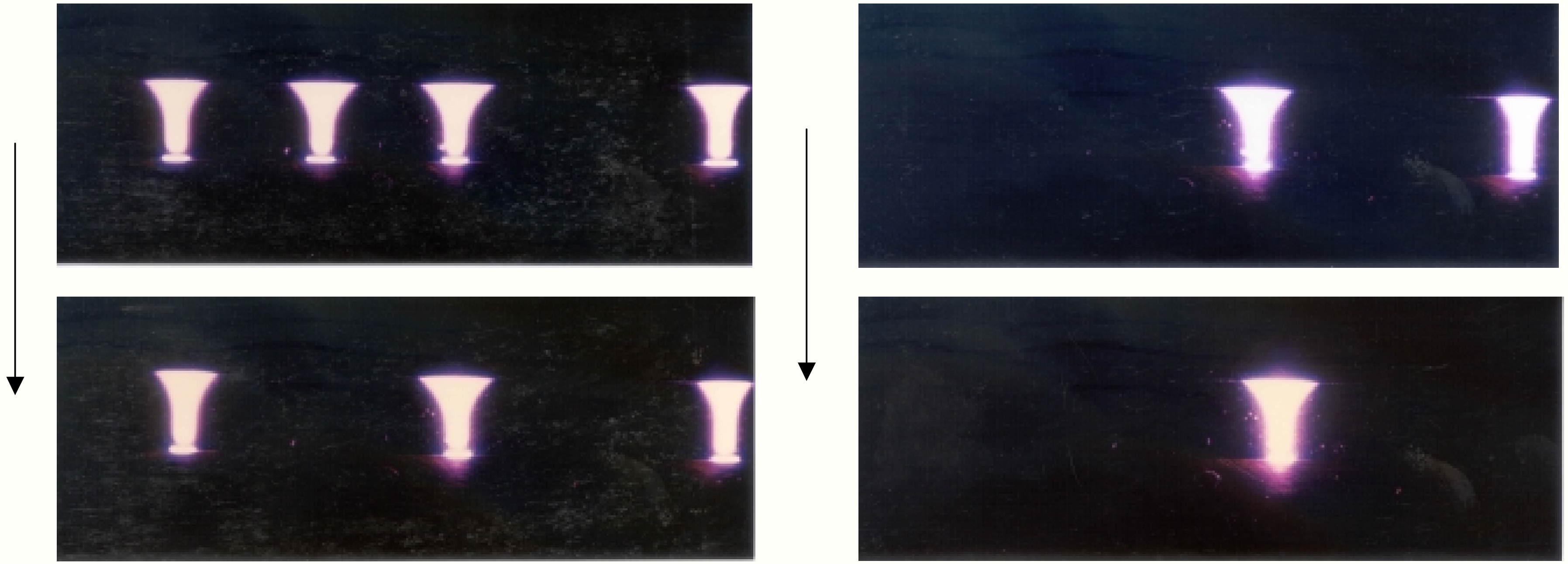
Cascade of Current Filaments in a Quasi-1-Dimensional DC Gas-Discharge System I



increasing driving voltage U_S

Radehaus et al. 1987, Willebrand et al. 1992

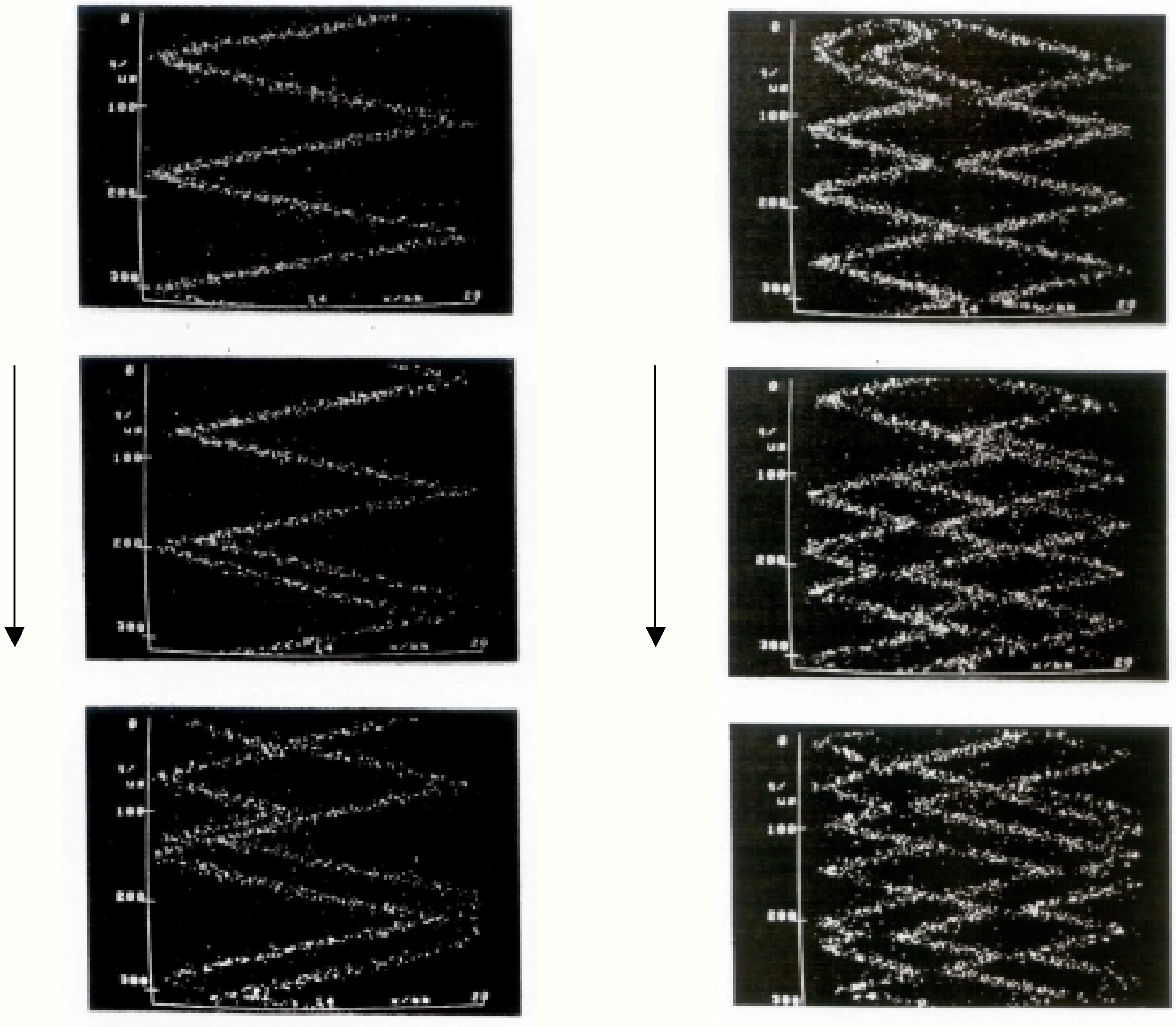
Cascade of Current Filaments in a Quasi-1-Dimensional DC Gas-Discharge System II



decreasing driving voltage U_S

Radehaus et al. 1987, Willebrand et al. 1992

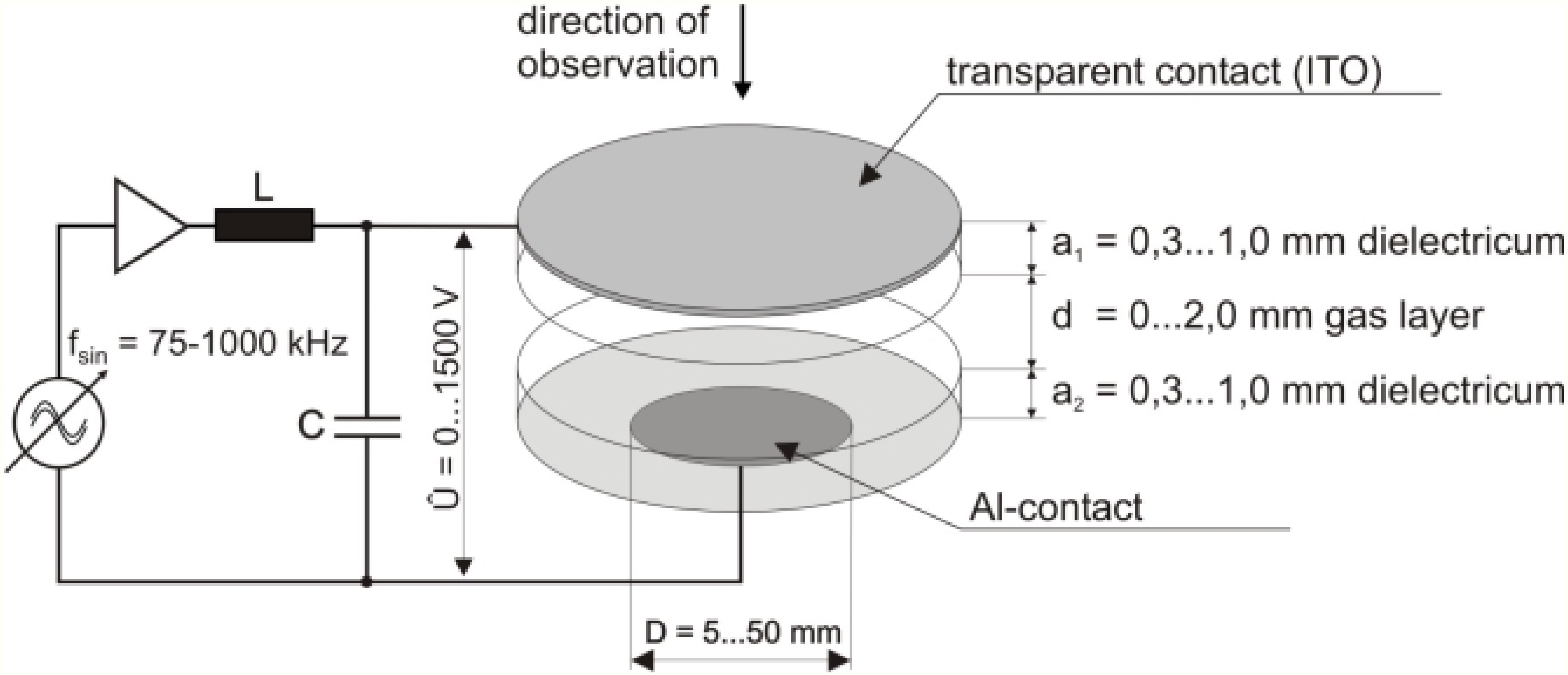
Travelling and Interacting Current Filaments in a Quasi-1-Dimensional DC Gas-Discharge System



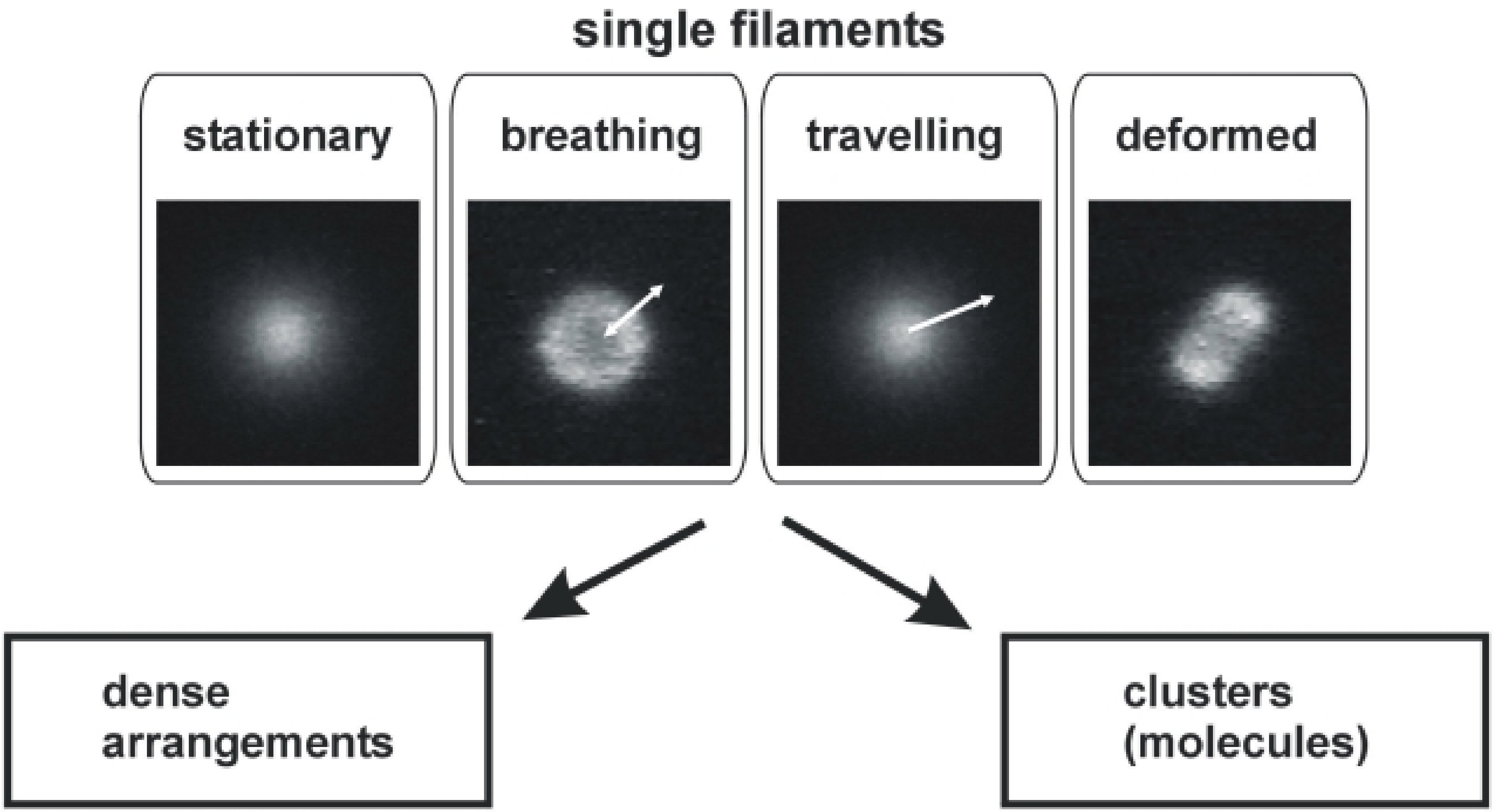
increasing driving voltage U_S

Willebrand et al. 1992

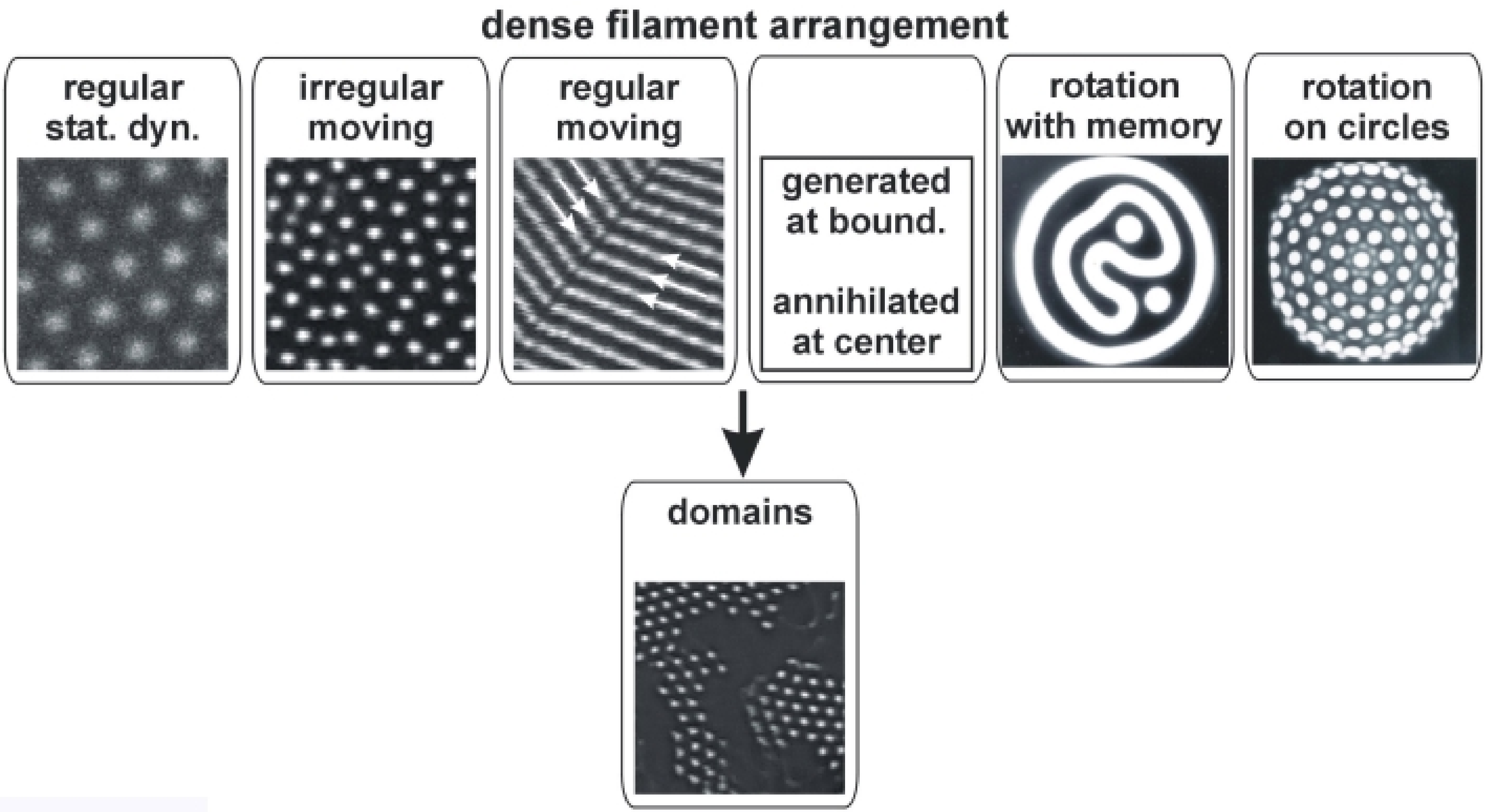
Experimental Set-Up for Measuring Self-Organized Patterns in Planar AC Gas-Discharge Systems



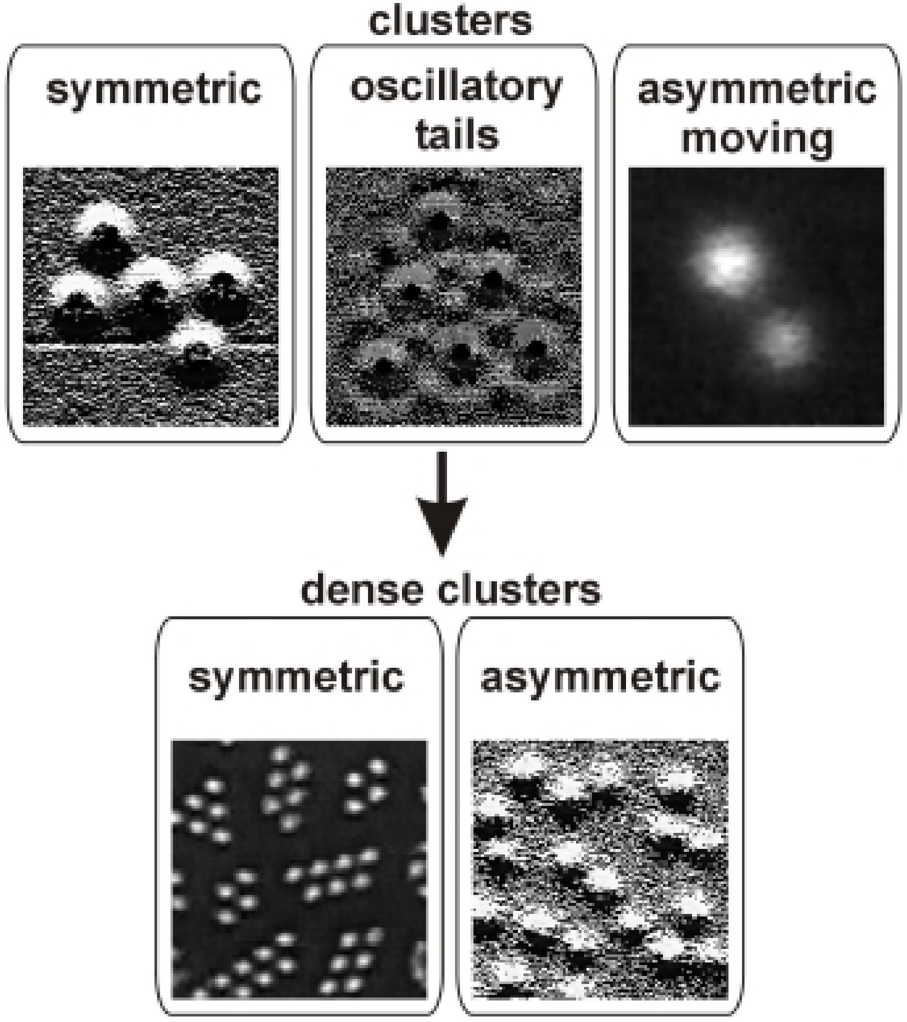
Experimentally Observed Hierarchy of Filamentary Patterns in the DBD System I



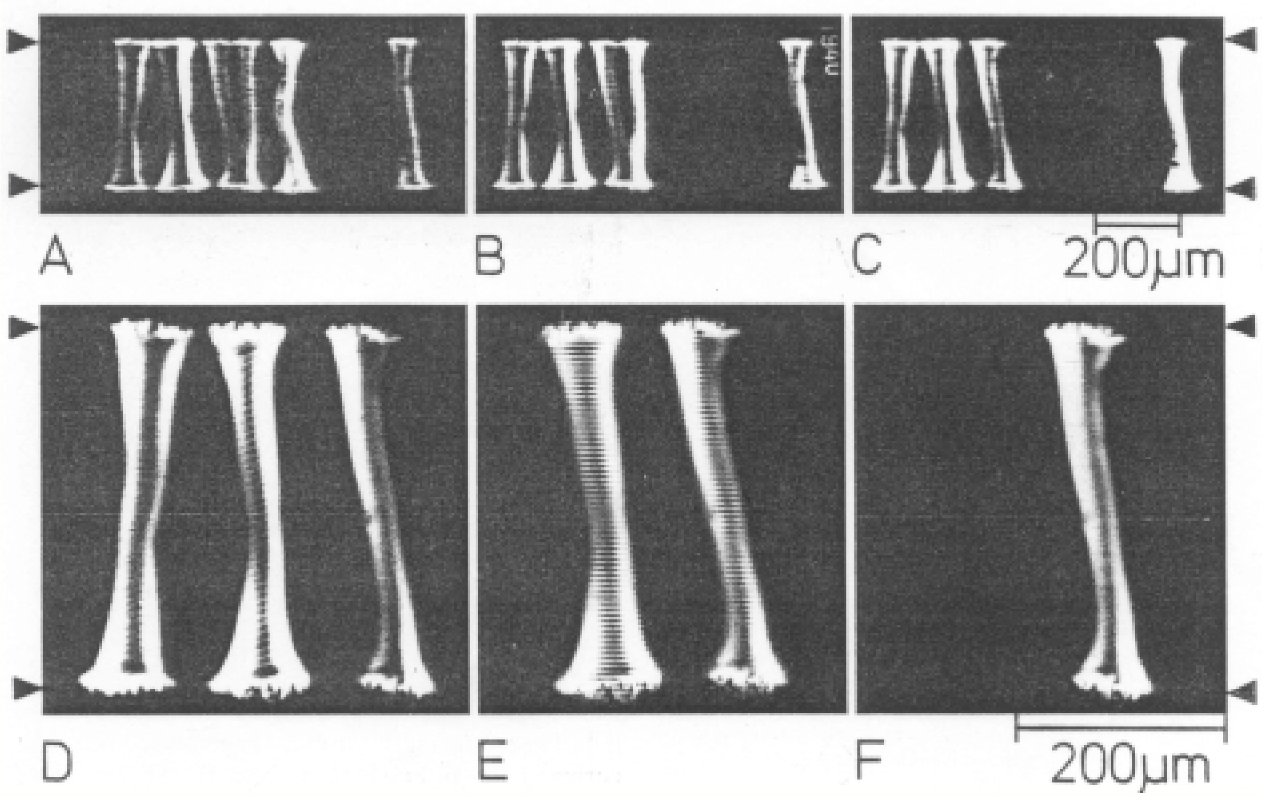
Experimentally Observed Hierarchy of Filamentary Patterns in the DBD System II



Experimentally Observed Hierarchy of Filamentary Patterns in the DBD System III

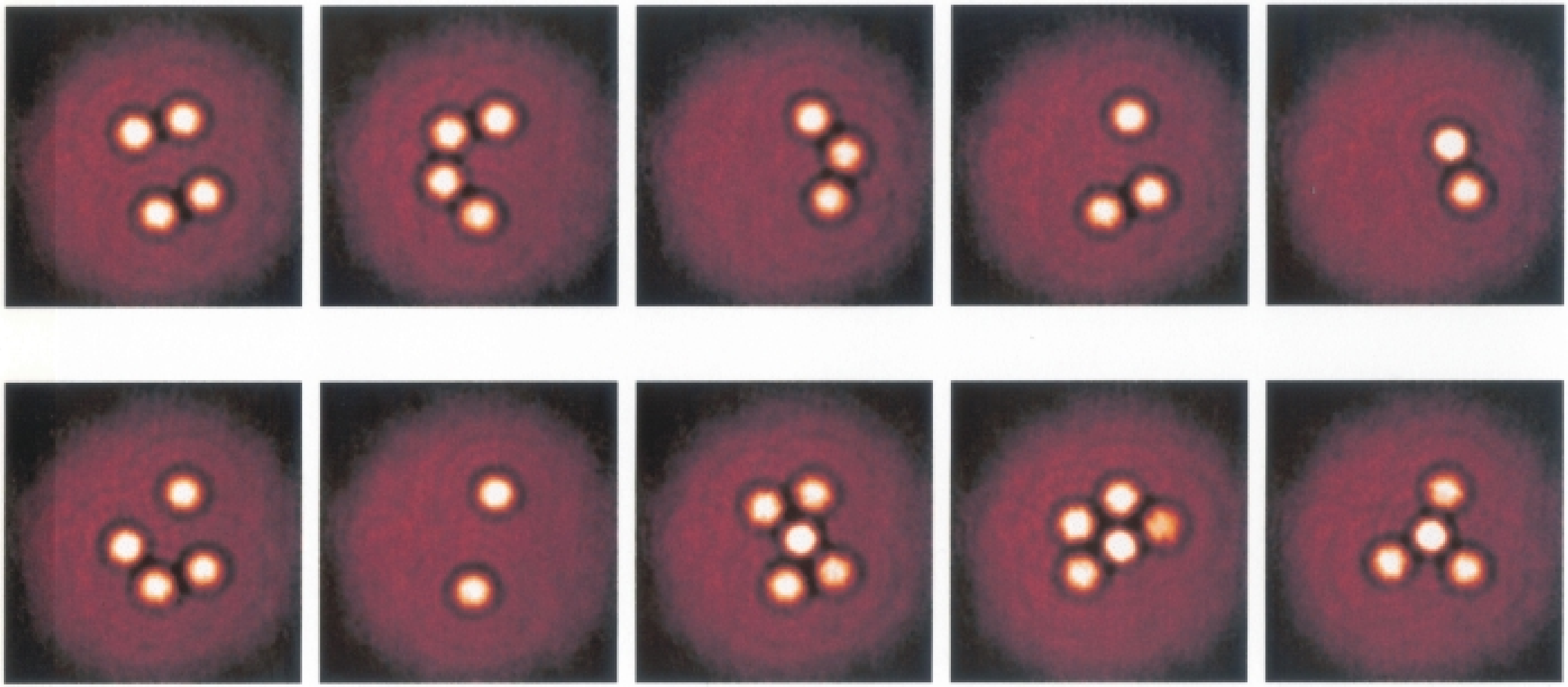


Current Filaments in n-GaAs Plates Measured at 4.2 K by Electron Microscopy



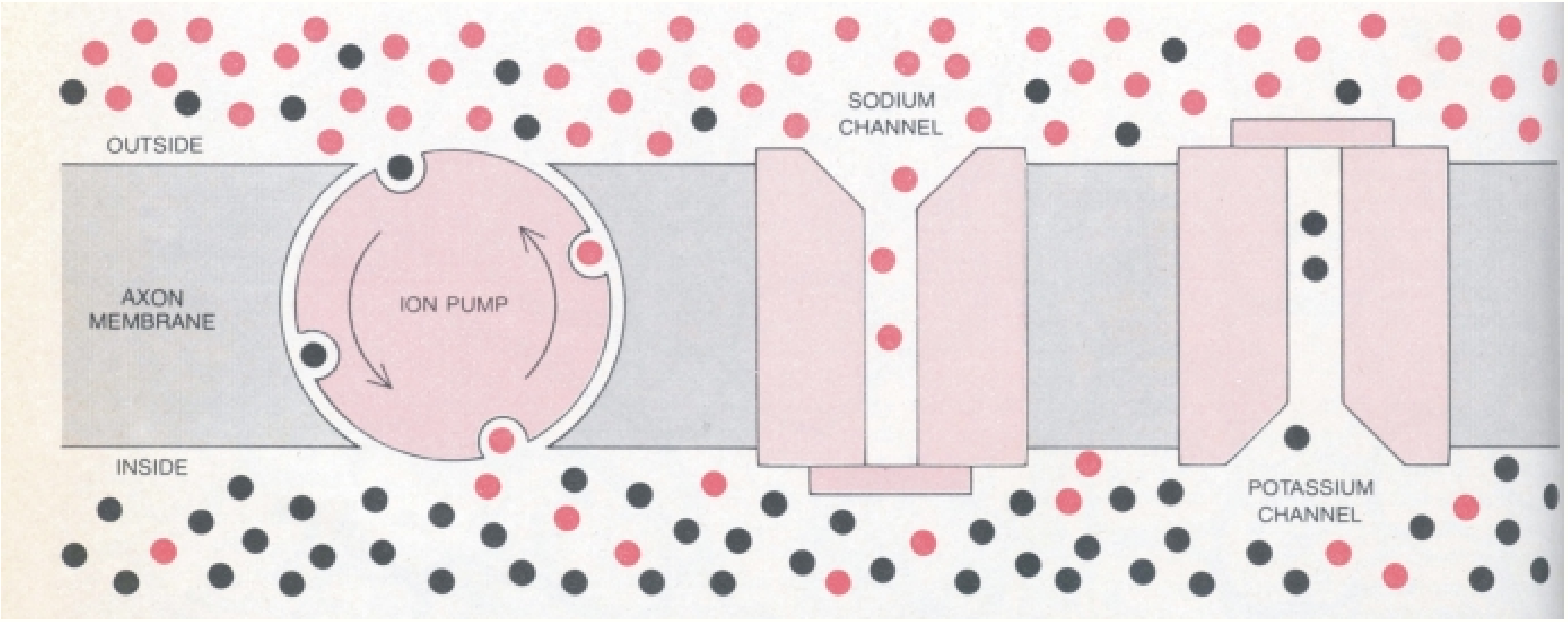
decrease of driving voltage from A to F

Clusters of Localized States Measured in a Laser Driven Na Vapour Cell



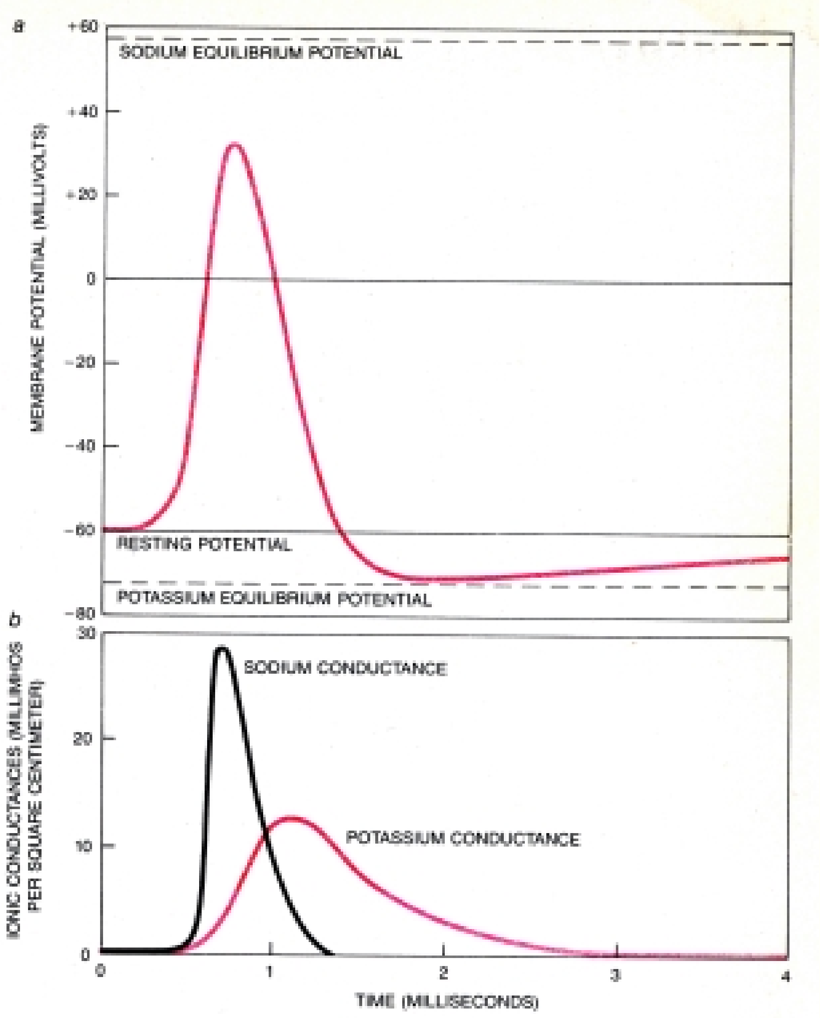
Schäpers et al. 2000

Nerve Pulse Propagation I: Mechanism for Generation of Electrical Potential Difference



Keynes 1979

Nerve Pulse Propagation II: Membrane Potential and Ion Conductivity Plotted as a Function of Time



Keynes 1979