List of Selected Papers of

Working Group Purwins

Münster, Germany

Work Related to Reaction-Diffusion Systems

(10.10.2007)

Fundamental Reaction-Diffusion Equation

$$u_t = D_u \Delta u + f\left(u,v,w\right) - \kappa_3 v - \kappa_4 w + \kappa_1 + \mu \nabla u \nabla v - \kappa_2 \int_\Omega u d\Omega$$

$$\tau v_t = D_v \Delta v + g\left(u,v,w\right) + \kappa_5 \int_\Omega u d\Omega$$

$$\theta w_t = D_w \Delta w + h\left(u,v,w\right)$$

$$f\left(u,v,w\right) = \lambda u - u^3 \quad \textit{or similar}$$

$$g\left(u,v,w\right) = u - v$$

$$h\left(u,v,w\right) = u - w$$

$$x \in \Re^{1,2,3}$$

Remarks

- The integral term represents a global coupling, if it is used it is non-zero either in the first or in the second equation; qualitatively so far, there is no observation of a big difference having the term in the first or in the second equation
- f(u,v,w) is a cubic polynomial in u or a similar function; in any case it consist of three monotonic braches (increasing, decreasing, increasing); in some cases it has been taken from the experiment in the form of the voltage current characteristic; in other cases it has been modified to get a qualitative better agreement with the experimental characteristic
- The abbreviation „Pu“ with a number behind relates to the list of publications of the research group Purwins

Definitions

RDS = above reaction-diffusion system
1k-, 2k-, 3k- = 1-, 2-, 3-component (RDS)
R^1, R^2, R^3 = 1, 2, or 3-dimensional space

1d-ENW	= 1-dimensional electrical networks
2d-ENW	= 2-dimensional electrical networks
1d-dc-GDS	= 1-dimensional dc gas-discharge system; electrodes consisting of parallel edges of planar material
2d-dc-GDS	= 2-dimensional dc gas-discharge systems; electrodes consisting of parallel planes
1d-DS	= dissipative solitons on a line
2d-DS	= dissipative solitons in the plane
3d-DS	= dissipative solitons in 3d space
SCF	= solitary current filaments; term used in relation to experiment; the projection of such a filament onto the line in 1d-systems and onto the plane in 2-d-systems is a spot that is referred to also as a 1d-DS and 2d-DS

Pu2

- Experiments, 1d-ENW
  Nearly periodic patterns
- Modelling the experimental results by 3k-RDS in R^1
  Quantitative description of nearly periodic patterns by RDS in R^1
  $D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, $\lambda \neq 0$, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 = 1$, $\kappa_4 = 0$, $\mu = 0$

These and the following experiments on ENW together with those simulating nerve pulses (Nagumo et al. 1962) demonstrate that it is possible to build analogue electronic networks that reflect many of the properties of RDS. Because of the importance of RDS in biology this might be of interest for practical application (see e.g. Pu31, Pu34, Pu36, Pu39)

Pu3

- Experiments on 2d-SiAu pin-diodes
  Stationary SCF
- Modelling the experimental results by 2k-RDS in R^2
  Qualitative description of experimental results on pin-diodes by 2k-RDS in R^1
  $D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, f(u,v,w) modified, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 \neq 0$, $\kappa_3 = 1$, $\kappa_4 = 0$, $\mu = 0$
  First time that a global coupling is introduced
  First claim that there must exist a certain universality class of systems consisting of a high ohmic monotonic layer and a highly nonlinear layer with S- (or N-) shaped current voltage characteristic of which the pattern forming properties can be described in terms of a simple RDS.

These, the following results on planar semiconductor devices and other investigations demonstrate that it is possible to build semiconductor components that behave similar to RDS and that carry among other things self-organized travelling DS. This brings also these devices near to biological information transmission and processing. Also it turns out that these systems can be controlled in an easily and fast manner optoeletronically (Pu52). Consequently also the observed phenomena in semiconductor materials may be of relevance for application in transmission and processing of data (see in particular Pu111 but also e.g. Pu31, Pu34, Pu39).

Pu5

- Experiments on 1d-dc-GDS
  Cascade of bifurcations to increasing number of stationary SCF (already in 1987!)
- **Qualitative modelling the 1d-dc-GDS experiments by 2k-RDS in R^1**
  $D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, f(u,v,w) modified, $\kappa_1 \neq 0$, $\kappa_2 \neq 0$, $\kappa_5 = 0$, $\kappa_3 = 1$, $\kappa_4 = 0$, $\mu = 0$
- Remark: Cascade of bifurcations to increasing number of SCF for increasing $\kappa_1 \sim$ (external voltage) indicates "homoclinic cycles", increasing κ_1 leads to an increase of this number and eventually to the generation of an additional DS, however after the generation the integral sets back the value (κ_2 – integral), this can be repeated in an endless manner in unbounded systems without changing the shape of the filaments (already in 1987!)

These and the following results on planar gas-discharge systems demonstrate that plasma systems under certain conditions behave like RDS (see also Pu119). Consequently on one hand they are convenient systems for the investigation of self-organized patterns in the large class of RDS. On the other hand being embedded in this class they exhibit universal behaviour. Finally it turns out that these systems can be controlled in an easily and fast manner optoeletronically (Pu29, Pu55, Pu65, Pu120). As a consequence also these systems might be possible candidates for application in information transmission and processing of data (see also Pu111).

Pu7

- Experiments on 1d- and 2d-ENW
  Various patterns
- Quantitative description of the results on 1d- and 2d-ENW by 2k-RDS in R^1 and R^2:
  $D_u = 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, f(u,v,w) modified, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 \neq 0$, $\kappa_3 = 1$, $\kappa_4 = 0$, $\mu = 0$
- Experiments on 1d-dc-GDS
  Cascade of bifurcations to increasing number of stationary SCF; see remark Pu 5
- Modelling the 1d-dc-GDS and other planar materials with S-shaped current voltage characteristic by RDS
- **Qualitative description of the experimentally observed cascade of bifurcations to increasing number of stationary SCF on 1d-dc-GDS by the 2k-RDS in R^1:**
  $D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, $\lambda \neq 0$, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 \neq 0$, $\kappa_3 = 1$, $\kappa_4 = 0$, $\mu = 0$

Pu8

- Experiments and quantitative theory on 1d-ENW
  Periodic and nearly periodic patterns
- Experiments and qualitative theory on the 1d-dc-GDS
  Cascade of bifurcations to increasing number of stationary SCF; see remark Pu 5
- Experiments and theory on pin-diodes
  Cascade of bifurcations to increasing number of stationary SCF (1d-DS); see remark Pu 5
- Qualitative description of the cascade of stationary bifurcations to increasing number of SCF 1d-dc-GDS by the 2k-RDS in R^1
  $D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, $\lambda \neq 0$, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 \neq 0$, $\kappa_3 = 1$, $\kappa_4 = 0$, $\mu = 0$

Pu9

- Experiments on 1d- and 2d-ENW
  Various periodic and non-periodic patterns
  Observation of supercritical Turing bifurcation in an experimental RDS including correct scaling law (already in 1988!)
  In addition: Homogeneous and inhomogeneous oscillations, intermittency, spatial-temporal chaos, Fitzhugh-Nagumo nerve pulses, cascade of bifurcations to increasing number of stationary SCF; see remark Pu 5
- **Quantitative description of the supercritical Turing bifurcation on 1d-ENW by the2k- RDS in R^1:**
  $D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, $(\lambda u - u^3)$ replaced by experimental characteristic, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 = 1$, $\kappa_4 = 0$, $\mu = 0$
  Quantitative description of number of stationary SCF on 1d-ENW by the RDS is also made (see R. Schmeling, Thesis, University of Münster (1994)):
  $D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, $(\lambda u - u^3)$ replaced by experimental characteristic, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 \neq 0$, $\kappa_3 = 1$, $\kappa_4 = 0$, $\mu = 0$
- Various experiments and quantitative description on 2d-ENW

P11

- Experiments on 1d-ENW
  Supercritical Turing bifurcation including correct scaling law
  Cascade of bifurcations to increasing number of stationary SCF ; see remark Pu 5
- Experiments on 1d-dc-GDS
  Cascade of bifurcations to increasing number of stationary SCF; see remark Pu 5
- Quantitative description of the supercritical Turing bifurcation on 1d-ENW by the 2k-RDS in R^1:
  $D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, $(\lambda u - u^3)$ replaced by experimental characteristic, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 = 1$, $\kappa_4 = 0$, $\mu = 0$
- Quantitative description of increasing number of stationary SCF by the RDS is also made (see R. Schmeling, Thesis, University of Münster (1994)):
  $D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, $(\lambda u - u^3)$ replaced by experimental characteristic, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 \neq 0$, $\kappa_3 = 1$, $\kappa_4 = 0$, $\mu = 0$
- Qualitative description of increasing number of stationary SCF by the RDS in the 1-dc-GDS:
  $D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, $(\lambda u - u^3)$ replaced by more realistic characteristic, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 \neq 0$, $\kappa_3 = 1$, $\kappa_4 = 0$, $\mu \neq 0$

P12

- Experiments on 1d-dc-GDS
  Detection of supercritical Turing bifurcation in GDS, since planar GDS can be considered as RDS this is the detection of a supercritical Turing bifurcation in an RDS; describes already in: H. Willebrand, Diplom-Thesis, University of Münster (1988);
  Cascade of bifurcations to increasing number of stationary SCF; see remark Pu5

Pu13

- Experiments on 1d-ENW
  Various periodic and non-periodic patterns
  Supercritical Turing bifurcation in an experimental system including correct scaling law
  Cascade of bifurcations to increasing number of stationary SCF; see remark Pu 5
- Quantitative description of the Turing-bifurcation on 1d-ENW by the 2k-RDS in R^1:

$D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, $(\lambda u - u^3)$ replaced by experimental characteristic, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 = 1$, $\kappa_4 = 0$, $\mu = 0$

Pu15

- Experiments on 1d-dc-GDS
  Cascade of bifurcations to increasing number of stationary SCF; see remark Pu 5
  Oscillations during filament generation

Observation of internal degrees of freedom of SCF corresponding to 1d-DS

Pu16

- Experiments on 1d-dc-GDS
  Periodic arrangement of SCF: coexistence of stationary and travelling patterns, oscillating periodic patterns

P17

- Experiments on 1d-dc-GDS
  Supercritical Turing bifurcation
- Theoretical analysis of Turing pattern formation

P18

- Theoretical investigation of the 2k-RDS in R^1:
  $D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, $(\lambda u - u^3)$ replaced by improved characteristic, $\kappa_1 \neq 0$, $\kappa_2 \neq 0$, $\kappa_5 = 0$, $\kappa_3 = 1$, $\kappa_4 = 0$, $\mu = 0$ or $\mu \neq 0$
  The term with $\mu \neq 0$ has mainly been introduced to take care of the experimentally observed filament splitting in the bifurcation cascade to increasing number of filaments
  Cascade of bifurcations to increasing number of stationary and travelling 1d-DS
  1d-DS splitting($\mu \neq 0$)
  1d-DS breathing (at $\mu = 0$)
  1d-DS swinging ($\mu \neq 0$)
  1d-DS disappearance at the boundary and generation of a pair in the centre (at $\mu = 0$)
  1d-DS and spatio-temporal chaos (at $\mu = 0$)
- Comparison to experimental results on SCF in pnpn-diodes, 1d-dc-GDS
- **Measurement of the supercritical Turing-bifurcation in a 1d-dc-GDS including the correct scaling law**

Since planar GDS can be considered as RDS this is the final proof of the detection of a supercritical Turing bifurcation in an RDS

P20/22/25/30/33/3 5/36/50/54/64

- Experimental investigation of SCF in 1d-pnpn semiconductors layer structures
  Cascade of bifurcations to increasing number SCF; see remark Pu 5
  Other SCF: oscillatory, rocking, period doubling, chaotic
  Frequency locking (50)
- Semi-quantitative description within the drift-diffusion approximation
- Qualitative description by the 2k-RDS in R^1:
  $D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, $(\lambda u - u^3)$ replaced by improved characteristic, $\kappa_1 \neq 0$, $\kappa_2 \neq 0$, $\kappa_5 = 0$, $\kappa_3 = 1$, $\kappa_4 = 0$, $\mu = 0$

Cascade of bifurcations to increasing number of stationary 1d-DS
Rocking 1d-DS

First semi-quantitative description of a cascade of bifurcations related to self-organized patterns in a semiconductor material using the drift-diffusion approximation

P21/26

- Experimental investigation of travelling and interacting SCF in 1d-dc-GDS
  Simple propagation
  Reflection at the boundary and at each other
  Generation and annihilation
  Periodic generation and disappearance
  Coexistence of stationary and travelling SCF
  Rocking SCF
  Period doubling,
  Chaotic motion

First clear experimental observation of particle-like behaviour of SCF (1d-DS)

Pu23/26

- Experiments on 1d-dc-GDS
  Cascade of bifurcations to increasing number of stationary and travelling SCF; see remark Pu 5
  Reflection at the boundary and at each other
  Spatio-temporal chaos
  Supercritical Turing bifurcation
- Modelling stationary and travelling 1d-DS in the 1d-dc-GDS experiments by RDS in 2k-R^1
  $D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, f(u,v,w) modified, $\kappa_1 \neq 0$, $\kappa_2 \neq 0$, $\kappa_5 = 0$, $\kappa_3 = 1$, $\kappa_4 = 0$, $\mu = 0$ or $\mu \neq 0$
  Bifurcation cascade of increasing number of stationary 1d-DS
  1d-DS breathing (at $\mu = 0$)
  1d-DS swinging ($\mu \neq 0$)
  1d-DS disappearance at the boundary and generation of a pair in the centre (at $\mu = 0$)
  1d-DS and spatio-temporal chaos (at $\mu = 0$)
  Reflection at the boundary and at each other ($\mu \neq 0$)

Pu24

- Analytical investigation of wall solutions and 1d-DS in 2k-RDS in R^1:
  $D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, f(u,v,w) modified, $\kappa_1 \neq 0$, $\kappa_2 \neq 0$, $\kappa_5 = 0$, $\kappa_3 = 1$, $\kappa_4 = 0$, $\mu = 0$

First step to understand weakly interacting DS followed by a number of papers (Pu41, Pu43, Pu 45, Pu 48) where this concept has been worked out and culminating in (Pu53, Pu60, Pu62, Pu84)

Pu27

- Theoretical investigation of wall between Hopf- and Turing-type domains in 2k-RDS in R^1:

$D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, $f(u,v,w) = \lambda u - u^3$, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 = 1$, $\kappa_4 = 0$, $\mu = 0$

Pu32

- Experimental investigation of uni- and bi-directional fronts propagation in 1d-ENW
- Quantitative description of experimental front propagation in 1d-ENW by 2k-RDS in R^1:
  $D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, $f(u,v,w)$ experimental, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 = 1$, $\kappa_4 = 0$, $\mu = 0$

Pu41

- Theoretical investigation of front dynamics in the presence of inhomogeneities in 2k-RDS in R^1 :
  $D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, $f(u,v,w) = \lambda u - u^3$ and more general form, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 = 1$, $\kappa_4 = 0$, $\mu = 0$

Pu43

- Experimental investigation of 1d-dc-GDS
  Cascade of bifurcations to increasing number of stationary SCF; see remark Pu 5
  Travelling SCF and various kinds of interaction
- Experimental investigation of 1d-ENW
  Cascade of bifurcations to increasing number of stationary SCF; see remark Pu 5
- Theoretical analysis of generation and interaction 1d-DS in terms of interacting fronts using the RDS in R^1:
  $D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, $f(u,v,w)$ similar to $\lambda u - u^3$, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 = 1$, $\kappa_4 = 0$, $\mu = 0$

With better reference to experimental and theoretical results the claim is repeated that there must exist a certain universality class of systems consisting of a high ohmic monotonic layer and a highly nonlinear layer with S- (or N-) shaped current voltage characteristic of which the pattern forming properties can be described in terms of a simple RDS.

Pu45

- Experiments on interaction of fronts with inhomogeneities in 1d-ENW
- Theoretical description of the experimental results using 1k- RDS in R^1:
  $D_u \neq 0$, $D_v = 0$, $D_w = 0$, $\tau = 0$, $\theta = 0$, $f(u,v,w) = \lambda u - u^3$ and more general form, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 = 0$, $\kappa_4 = 0$, $\mu = 0$

Pu48

- Theoretical investigation of oscillating fronts and front pairs using 2k-RDS in R^1:
  $D_u \neq 0$, $D_v \neq 0$, $D_w = 0$ and $D_w \neq 0$, $\tau \neq 0$, $\theta = 0$ and $\theta \neq 0$, $f(u,v,w) = \lambda u - u^3$ and more general form, $\kappa_1 \neq 0$, $\kappa_2 \neq 0$ and $\kappa_2 = 0$, $\kappa_5 = 0$, κ_3, κ_4 are incorporated in some more general form of $g(u,v,w)$ and $h(u,v,w)$, $\mu = 0$

First introduction of a 3k-RDS

Pu49/52

- Experiments on 2d-dc-GDS
  Hexagons

Stripes

Supercritical Turing bifurcation to stripes with correct scaling law

It is demonstrated experimentally for the first time that SCF (DS) in semiconductors materials can be manipulated and controlled by a laser beam.

Pu53

- Theoretical investigation of DS and their interacting using 3k-RDS in R^2:
  $D_u \neq 0$, $D_v \neq 0$, $D_w \neq 0$, $\tau \neq 0$, $\theta \neq 0$, $f(u,v,w) = \lambda u - u^3$ and more general form, $\kappa_1 \neq 0$, $\kappa_2 \neq 0$, $\kappa_5 = 0$, $\kappa_3 \neq 0$, $\kappa_4 \neq 0$, $\mu = 0$

  In this work it is demonstrated for the first time explicitly that the 3rd component has the important consequence to stabilize more than one independent and interacting DS in R^n, $n > 1$

  Opening of the understanding of e.g. the following properties of DS in RDS in R^n: propagation, scattering, formation of bound states, generation, annihilation, self-replication, periodic patterns consisting of nd-DS

It is demonstrated for the first time that a simple 3k-RDS for sufficient large boundary can support an arbitrary large number of stationary or travelling interacting DS in arbitrary spatial dimension. This is the basis for a universal description of various pattern forming properties in RDS that can also be observed e.g. in systems consisting of a high ohmic monotonic layer and a highly nonlinear layer with S- (or N-) shaped current voltage characteristic.

Pu60

- Theoretical investigation of the travelling bifurcation of DS using 3k-RDS in R^2:
  $D_u \neq 0$, $D_v \neq 0$, $D_w \neq 0$, $\tau \neq 0$, $\theta \neq 0$, $f(u,v,w) = \lambda u - u^3$ and more general form, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 \neq 0$, $\kappa_4 \neq 0$, $\mu = 0$

  Evaluation of the scaling laws

  Opening of the understanding of e.g. the following properties of DS in RDS in R^n: propagation, scattering, formation of bound states, generation, annihilation, self-replication

Evaluation of

Pu62

- Theoretical investigation of stationary 2d-DS and their interacting using 2k- RDS in R^2 and the formation of molecules:
  $D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, $f(u,v,w) = \lambda u - u^3$ and more general form, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 \neq 0$, $\kappa_4 = 0$, $\mu = 0$

  Stability analysis

  Derivation of an ordinary differential equation describing the dynamics of a spot during interaction in terns of its centre of mass coordinates

First step in the reduction of the RDS field equation to an ordinary differential equation describing the dynamics of a spot including its interaction in terms of the centre of mass coordinates. Discussion of the resulting simplified description of interacting DS.

Pu71

- Numerical investigation of travelling 3d-DS and their interacting using 3k-RDS in R^3 :

$D_u \neq 0$, $D_v \neq 0$, $D_w \neq 0$, $\tau \neq 0$, $\theta \neq 0$, $f(u,v,w) = \lambda u - u^3$ and more general form, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 \neq 0$, $\kappa_4 \neq 0$, $\mu = 0$
Travelling single 3d-DS
Scattering at each other
Annihilation
Scaling law for the speed

- First mention of the ordinary differential equation describing the dynamics of DS and their interaction in terms of the dynamics of the centre of mass coordinates:
  $D_u \neq 0$, $D_v \neq 0$, $D_w \neq 0$, $\tau \neq 0$, $\theta \neq 0$, $f(u,v,w) = \lambda u - u^3$ and more general form, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 \neq 0$, $\kappa_4 \neq 0$, $\mu = 0$

Pu79

- Numerical investigation of travelling 2d-DS and 3d-DS and their interacting using 3k-RDS in R^3 :
  $D_u \neq 0$, $D_v \neq 0$, $D_w \neq 0$, $\tau \neq 0$, $\theta \neq 0$, $f(u,v,w) = \lambda u - u^3$, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 \neq 0$, $\kappa_4 \neq 0$, $\mu = 0$
  Travelling single 3d-DS
  Scattering of 3d-DS at each other
  Scaling law for the speed
  Turing pattern in R^3
  Self-completion in R^3
- Discussion of the limit of an effective global coupling in a 3k-RSD in R^2:
  $D_u \neq 0$, $D_v \neq 0$, $D_w = 0$, $\tau \neq 0$, $\theta = 0$, $f(u,v,w) = \lambda u - u^3$, $\kappa_1 \neq 0$, $\kappa_2 \neq 0$, $\kappa_5 = 0$, $\kappa_3 \neq 0$, $\kappa_4 = 0$, $\mu = 0$
  Stopping of self-replication by the integral term
- Solutions of the ordinary differential equation describing the dynamics of DS and their interaction in terms of the dynamics of the centre of mass coordinates in R^3:
  $D_u \neq 0$, $D_v \neq 0$, $D_w \neq 0$, $\tau \neq 0$, $\theta \neq 0$, $f(u,v,w) = \lambda u - u^3$ and more general form, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 = 1$, $\kappa_4 \neq 0$, $\mu = 0$
  Scattering at each other, in good agreement with field equation

Pu80

- Experiments on 2d-dc-GDS
  Various stationary and dynamical patterns of which SCF are elementary constituents:
  Molecules
  Chains
  Nets
  Gas-like many-body systems based on SCF
  Generation and annihilation of 2-SCF

Pu81

- Experiments on 2d-dc-GDS
  Various stationary and dynamical phenomena and patterns of which SCF act as elementary constituents
  Crystal-, liquid- and gas-like patterns
  Indication of coexistence of condensed and gaseous phase on the same domain
  Molecule formation
  Scattering
  Generation

Pu84

- Analytical investigation of the 3k-RDS and derivation of the (reduced) ordinary differential equation describing the dynamics of DS and their interaction in terms of the dynamics of the centre of mass coordinates in R^n:
  $D_u \neq 0$, $D_v = 0$, $D_w \neq 0$, $\tau \neq 0$, $\theta = 0$, $f(u,v,w) = \lambda u - u^3$ or similar, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 \neq 0$, $\kappa_4 \neq 0$, $\mu = 0$
  Mathematical foundation of the particle picture for DS in R^n
  Among other things this work is the basis for the investigation of many-body systems consisting of DS
- Comparison between result from the field equation and the reduces equation
  Travelling single 3d-DS in R^3
  Scattering of 2d-DS at each other
  Scattering of 3d-DS at each other
  Molecule formation in R^2
  Scattering of 2d-DS at impurities

Mathematical foundation of the particle concept for propagating and weakly interacting DS with any number in any spatial dimension.

Pu92

- Numerical investigation of the 3k-RDS in R^3:
  $D_u \neq 0$, $D_v = 0$ or $\neq 0$, $D_w \neq 0$, $\tau \neq 0$, $\theta \neq 0$, $f(u,v,w) = \lambda u - u^3$, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 \neq 0$, $\kappa_4 \neq 0$, $\mu = 0$
  Travelling isolated 3d-DS
  Generation of 3d-DS combined with molecule formation

Pu98/99

- Experiments on 2d-dc-GDS
  Single trajectories of SCF with noise
  First detection of the supercritical travelling bifurcation
- Analytical investigation of the 3k-RDS in R^2:
  $D_u \neq 0$, $D_v \neq 0$, $D_w \neq 0$, $\tau \neq 0$, $\theta \neq 0$, $f(u,v,w) = \lambda u - u^3$ or similar, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 \neq 0$, $\kappa_4 \neq 0$, $\mu = 0$
  Discussion of the ordinary differential equation describing the dynamics of single 2d-DS and their interaction in terms of the dynamics of the centre of mass coordinates
  Derivation of the scaling law for the travelling bifurcation
- **Derivation of a new stochastic data analysis method in order to substract in the trajectories the noise from from the deterministic part of the speed**

Pu100/105

- Analytical investigation of the 3k-RDS in R^2:
  $D_u \neq 0$, $D_v \neq 0$, $D_w \neq 0$, $\tau \neq 0$, $\theta \neq 0$, $f(u,v,w) = \lambda u - u^3$ or similar, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 \neq 0$, $\kappa_4 \neq 0$, $\mu = 0$
  Generalization of the derivation of the deduction of the ordinary differential equation describing the dynamics of single 2d-DS and their interaction in terms of the dynamics of the centre of mass coordinates
  Rotational bifurcation and derivation of the scaling law
  Numerical investigation of molecule formation

Pu101

- Analytical investigation of the 3k-RDS in R^2:
  $D_u \neq 0$, $D_v \neq 0$, $D_w \neq 0$, $\tau \neq 0$, $\theta \neq 0$, $f(u,v,w) = \lambda u - u^3$ or similar, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 \neq 0$, $\kappa_4 \neq 0$, $\mu = 0$
  Generalization of the derivation of the deduction of the ordinary differential equation describing the dynamics of single 2d-DS and their interaction in terms of the dynamics of the centre of mass coordinates
  Travelling bifurcation in the case of change of shape and derivation of the scaling law
  Comparison between solutions of the field and the reduced equations

Pu109

- Experiments on 2d-dc-GDS
  Rotating molecules consisting of two SCF
- Numerical solutions of the3k- RDS in R^2:
  $D_u \neq 0$, $D_v \neq 0$, $D_w \neq 0$, $\tau \neq 0$, $\theta \neq 0$, $f(u,v,w) = \lambda u - u^3$ or similar, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 \neq 0$, $\kappa_4 \neq 0$, $\mu = 0$
  Interaction of two 2d-DS and formation of a rotating molecule
- Discussion of the ordinary differential equation describing the dynamics of single 2d-DS and their interaction in terms of the dynamics of the centre of mass coordinates
  Interaction of two 2d-DS and formation of a rotating molecule
  Discussion of the scaling law for the rotational bifurcation

Pu110

- Experiments on 2d-dc-GDS
  Trajectories of interacting SCF with noise
  First experimental determination of the interaction law for SCF
- Numerical investigation of the 3k-RDS in R^2:
  $D_u \neq 0$, $D_v \neq 0$, $D_w \neq 0$, $\tau \neq 0$, $\theta \neq 0$, $f(u,v,w) = \lambda u - u^3$ or similar, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 \neq 0$, $\kappa_4 \neq 0$, $\mu = 0$
  Solutions of the 3k-RDS in R^2 for the cases of non-oscillatory and oscillatory tails of 2d-DS
- **Derivation of another new stochastic data analysis method in order to determine from the interaction law for DS rom trajectories with noise**
- Discussion of the ordinary differential equation describing the dynamics of single 2d-DS and their interaction in terms of the dynamics of the centre of mass coordinates
  Comparison of the solutions of the 3k-RDS in R^2 for the cases of non-oscillatory and oscillatory tails of 2d-DS with the corresponding interaction law

Pu113

- Experiments on 2d-dc-GDS
  Stationary and noise-covered travelling SCF
  Travelling bifurcation related to change of shape
- Treatment of the 3-component 3k-RDS in R^2 in order to get a scaling law for the speed:
  $D_u \neq 0$, $D_v \neq 0$, $D_w \neq 0$, $\tau \neq 0$, $\theta \neq 0$, $f(u,v,w) = \lambda u - u^3$ or similar, $\kappa_1 \neq 0$, $\kappa_2 = 0$, $\kappa_5 = 0$, $\kappa_3 \neq 0$, $\kappa_4 \neq 0$, $\mu = 0$
  Scaling law and relation to change of shape
- Discussion of the ordinary differential equation describing the dynamics of single 2d-DS and their interaction in terms of the dynamics of the centre of mass coordinates
  Interaction of two 2d-DS and formation of a rotating molecule
  Discussion of the scaling law for the rotational bifurcation

- Discussion of the new stochastic data analysis method in order to substract in the trajectories the noise from the deterministic part of the speed

Pu118

Restricted review on some experimental and theoretical aspects of DS including the particle picture and the interaction of particles. Heuristic derivation of the field equation, eneralized derivation of the particle equations, discussion of mechanisms of pattern formation leading to stable DS and reflection on universality of the RDS.

- Analytical and numerical treatment of various versions of the 2k- and 3k-RDS in $R^{1,2,3}$
  Relevanz of the 3k-RDS
  Activator inhibitor systems
  Turing patterns
  Localized solitary patterns
  Various numerical solutions of isolated and interacting DS
  Analytical treatment in order to obtain scaling laws for travelling and other bifurcations
  Analytical treatment in order to obtain a particle description of DS
  Discussion of interaction law
- Experimental realization of DS in the form of SCF on ENW and electrical transport layer structures
- Experimental results on 2d-dc GDS
  Isolated SCF
  Trajectories with noise, travelling bifurcation and interaction law

Pu119

- Analytical treatment of the gas-discharge specific drift-diffusion equation describing the planar 2d-GDS using the experimental parameters
  Derivation of a 2k-RDS equation near to the ignition of gas-discharge
  Proof that for lateral pattern in the current distributions drift can be neglected

This is the first step to prove analytically that with respect to lateral pattern formation GDS can be considered as RDS. Therefore this is also the first step to a rigorous theoretical foundation of the experimentally observed universal behaviour.