9.
Reaction-Diffusion Equations

(references refer to the list of publications given in chapter 12)

9.1 General remarks

The subject of the present chapter are numerical and analytical solutions of the following 3-component reaction-diffusion equation

$$\partial_t u = d_u^2 \Delta u + f(u) - \kappa_3 v - \kappa_4 w + \kappa_1 - \kappa_2 \int_\Omega u \, d\Omega + \mu (\nabla u)(\nabla u), \quad (1)$$

$$\tau \partial_t v = d_v^2 \Delta v + u - v - \kappa_1{}' + \kappa_2{}' \int_\Omega v \, d\Omega, \quad (2)$$

$$\Theta \partial_t w = d_v^2 \Delta w + u - w, \quad (3)$$

$$u = u(x, y; t); \;\; v = v(x, y; t); \;\; w = w(x, y; t)$$

with appropriate boundary conditions usually being of Neumann zero flux type or periodic type. For the derivation of the equation with $\mu = 0$ we refer to the chapter A Model for Pattern Formation. The term $\mu(\nabla \mu)(\nabla \mu)$ has been introduced in some work to facilitate the generation of LSs by splitting, when increasing the driver κ_1 or κ'_1. The above equation is an extension of the FitzHugh-Nagumo equation and in the case of $\mu = 0$ frequently we will use the abbreviation "FHN equation".

In the general context of reaction-diffusion equations the individual quantities in the above equation have following meaning:

u — **dependent variable acting as activator in some region of phase space of the local system,**

v, w — **dependent variables acting always as inhibitor; among other things 2 inhibitors in addition to 1 activator guarantee the existence of one or several stable travelling LSs at given time,**

$f(u)$ — **nonlinear function similar to the form $\lambda u - u^3$; it is monotonically increasing for large |u| and possibly decreasing for small |u|**

$\lambda > 0$	**autocatalytic constant assuring u to act as activator in some region of phase space of the local system,**
$\tau, \Theta \geq 0$	**overall relaxation times of v and w normalized to that of u**
$d_u, d_v, d_w \geq 0$	**diffusion lengths of u, v, w,**
κ_1, κ_1'	**drivers (ether** $\kappa_{1,}\kappa_2 = 0$ **or** $\kappa_1', \kappa_2' = 0$**),**
$\kappa_2, \kappa_2' \geq 0$	**global couplings (ether** $\kappa_{1,}\kappa_2 = 0$ **or** $\kappa_1', \kappa_2' = 0$**),**
$\kappa_{3,}\kappa_4' \geq 0$	**strength of action of inhibitors v, w with respect to the evolution of u,**
$\mu \geq 0$	**occasionally introduced to generate DS splitting while increasing** κ_1 **or** κ_2**.**

As reported in [Pu084, Pu118] in the absence of global coupling and for the case of localized structures (LSs) in the form of dissipative solitons (DSs) as solutions, near to the supercritical bifurcation from a stationary to a travelling DS the FHN equation can be reduced to a many particle equation in the form of a set of ordinary differential equations. E.g. in the case of $\mu, d_v, \Theta = 0$ **and the absence of global coupling the many particle equation reads as:**

$$\partial_t \vec{p}_i = \vec{\alpha}_i + F(|\vec{p}_i - \vec{p}_j|) \frac{\vec{p}_i - \vec{p}_j}{|\vec{p}_i - \vec{p}_j|}, \tag{4}$$

$$\partial_t \vec{\alpha}_i = (\tau - \tau_c)\kappa_3^2 \vec{\alpha}_i - \frac{1}{\kappa_3} Q |\vec{\alpha}_i|^2 \vec{\alpha}_i + \kappa_3 F(|\vec{p}_i - \vec{p}_j|) \frac{\vec{p}_i - \vec{p}_j}{|\vec{p}_i - \vec{p}_j|}, \tag{5}$$

$$\tau_c = \frac{1}{\kappa_3}, \tag{6}$$

$$Q = \frac{\langle (\partial_x^2 u_0)^2 \rangle}{\langle (\partial_x u_0)^2 \rangle}. \tag{7}$$

Here $\vec{p}_i$ **is the positioning vector of the i-the particle (DS) depending on space and time,** u_0 **denotes the** u **component of the stationary DS obtained from the solution of the field equation at the bifurcation point** $\tau_c = 1/\kappa_3$ **and the function** F **describes the mutual interaction of the DSs *i* and *j*.** F **can be calculated from the stationary DS at the bifurcation point. For a given set of parameters, the vector** $\vec{\alpha}_i$ **is a measure for the displacement of the centre of the distribution of the slow inhibitor component with respect to the fast activator** u **of the freely travelling DS. One concludes immediately from the particle equation that for a single isolated DS *i* (with vanishing** F**) we can write**

$$\partial_t \vec{p} = \vec{\alpha}, \tag{8}$$

$$\partial_t \vec{\alpha} = (\tau - \tau_c)\kappa_3^2 \vec{\alpha} - \frac{1}{\kappa_3} Q |\vec{\alpha}|^2 \vec{\alpha}. \tag{9}$$

Solution with finite constant speed $\partial_t \vec{p}=\vec{\alpha}=\vec{v}_0$ follow directly from (8,9) and obey the scaling law

$$v_0^2=\frac{\kappa_3^3}{Q}(\tau-\tau_c). \tag{10}$$

The kind of scaling law (10) should also arise with other parameters than τ, with respect the breathing bifurcation (see e.g. [Pu126]) and the bifurcation with change of shape (see e.g. [Pu113]).

We note that the particle equation (4-7) has also been extended to include the influence of inhomogeneities [Pu084].

Another topic of wide range interest is the determination of the deterministic part of the dynamics of dependent variables that carry noise [Pu098;Pu099;Pu131]. This work has been performed in close collaboration with R. Friedrich and his research group at the Institut für Theoretische Physik at the University of Münster. In order to develop the stochastic data analysis method for the deterministic part of the dynamics let us assume a noisy system in which the deterministic dynamics of a single DS is governed by the particle equation (8,9). With certain assumptions concerning the noise it is possible to set up an appropriate Langevin equation in the form

$$\dot{\vec{v}}(t)=\underline{\vec{h}}(\vec{v}(t))+\underline{R}(\vec{v}(t))\vec{\Gamma}(t), \tag{11}$$

where $\vec{v}(t)$ denotes the speed, $\underline{h}(\vec{v}(t))$ the deterministic acceleration and $\underline{R}(\vec{v}(t))\vec{\Gamma}(t)$ stands for the contribution of noise. On this background a formalism has been derived to determine the deterministic acceleration in dependence of the speed as follows [Pu098; Pu099]

$$h_v(v)\approx\frac{1}{\Delta t}\left\langle\frac{[\vec{v}\,'(t+\Delta t)-\vec{v}\,'(t)]\vec{v}\,'(t)}{v\,'(t)}\right\rangle\Bigg|_{v'(t)\approx v} \tag{12}$$

with

$$h_v(v)=|\vec{h}_v(\vec{v})|,\;\; v'(t)=|\vec{v}\,'(t)|. \tag{13}$$

To apply equation (12) the range of possible experimental speeds has to be subdivide into intervals I_n. Then *v´(t)*, *v´(t + Δt)* are the velocity values for discrete values of subsequent time steps obtained from the experimental DS trajectories. In (12) only differences *v´(t + Δt) - v´(t)* have to be considered for which the value of *v´(t)* is contained in I_n. E.g. in 1-dimensional space the acceleration *h*(*v(t))* is evaluated from the experimental trajectories as is depicted in the following fig. 9.1

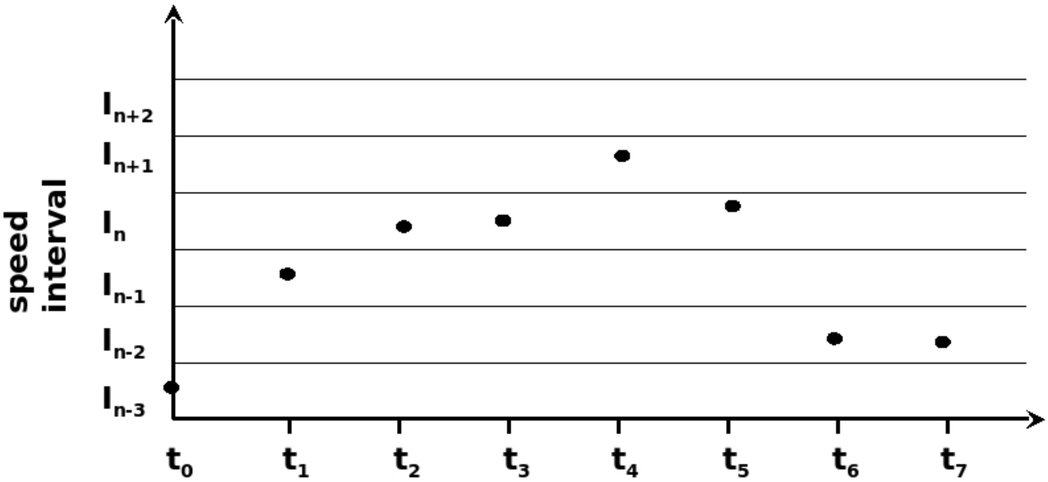

fig. 9.1

by writing

$$h_v(v(I_n)) \approx \frac{1}{3}\left[\frac{v(t_3)-v(t_2)}{\Delta t}+\frac{v(t_4)-v(t_3)}{\Delta t}+\frac{v(t_6)-v(t_5)}{\Delta t}\right].$$

Finally, by plotting the acceleration *h*(*v)* as a function of v and searching for appropriate zeros in this plot the stable solutions for the speed of the underlying deterministic system described by equation (1-4) can be determined.

In a similar way the law for mutually interacting DSs can be evaluated in the presence of noise [Pu110].

With respect to dissipative solitons (DSs) a short review is presented in [Pu118] and a rather extensive review will be submitted for publication by [Purwins, Amiranashvili and Bödeker in 2009].

9.2 Graphical representation of selected results

The following is a series of figures reflecting main analytical and numerical results that have been obtained in relation to the investigation of the FHN equation (1–3) with 1, 2 or three components in 1-, 2- or 3-dimensional space. - We refer also to the theoretical work reported in relation to electrical networks that are the subject of the chapter Electrical Networks: Experiment and Theory.

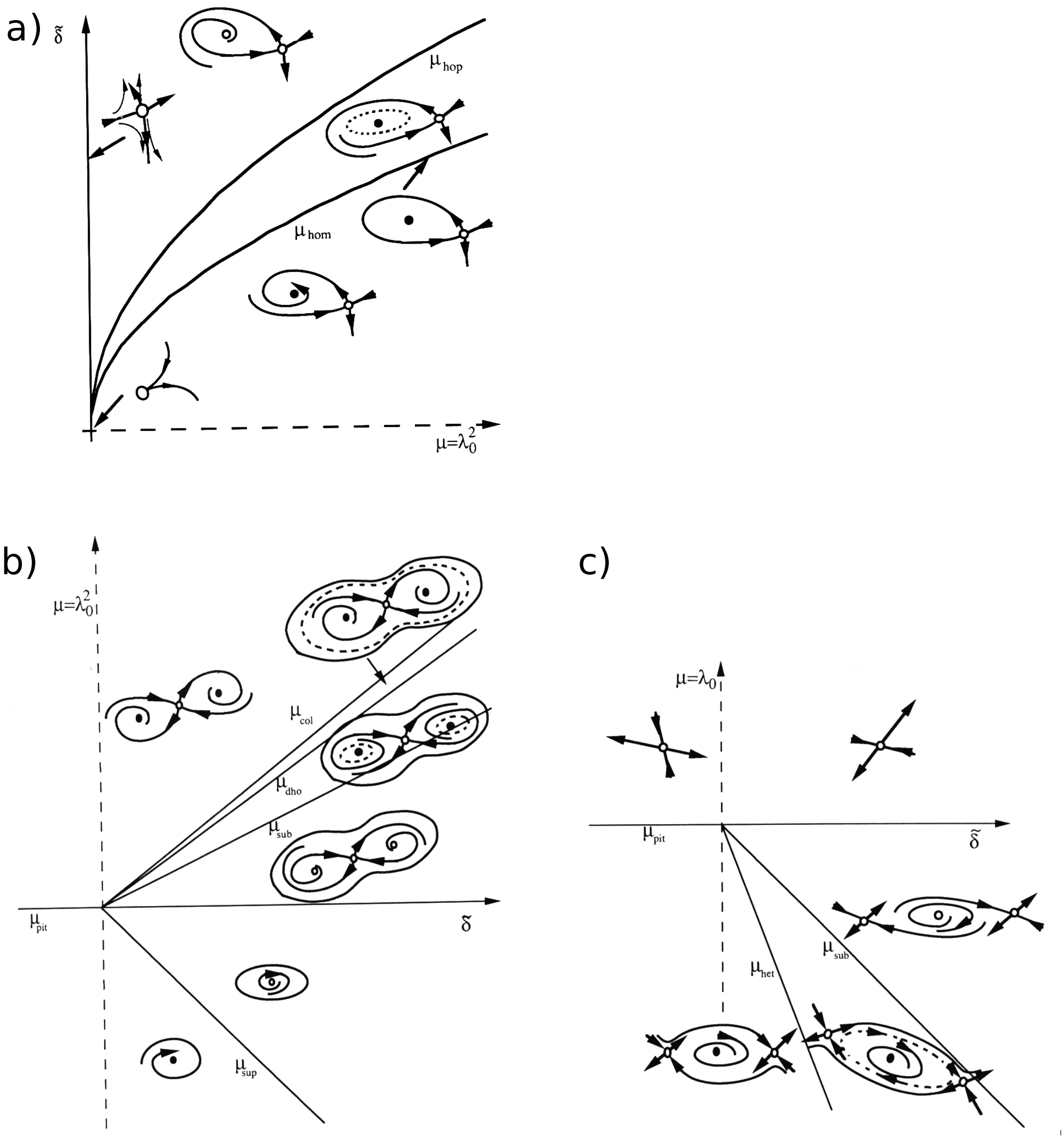

Fig. 9.2a-c

Fig. 9.2a-c

Bifurcation diagrams obtained from the analytical investigation of the FHN equation (1-3) with 2-components in R^1 without global coupling in the case of the interaction of a front with an inhomogeneity near to pinning: Unfolding of the Takens-Bogdanov bifurcation (a), unfolding of the symmetric Takens-Bogdanov bifurcation of type I (b) and of type II (c). Here μ is the square of the eigenvalue of the corresponding 1-component system and $\tilde{\delta}$ is linearely related to τ. [Pu041] - see also [P. Schütz, „Strukturen mit großen Amplituden in Reaktions-Diffusions-Systemen vom Aktivator-Inhibitor-Typ", Thesis, Institut für Angewandte Physik, University of Münster (1995)]

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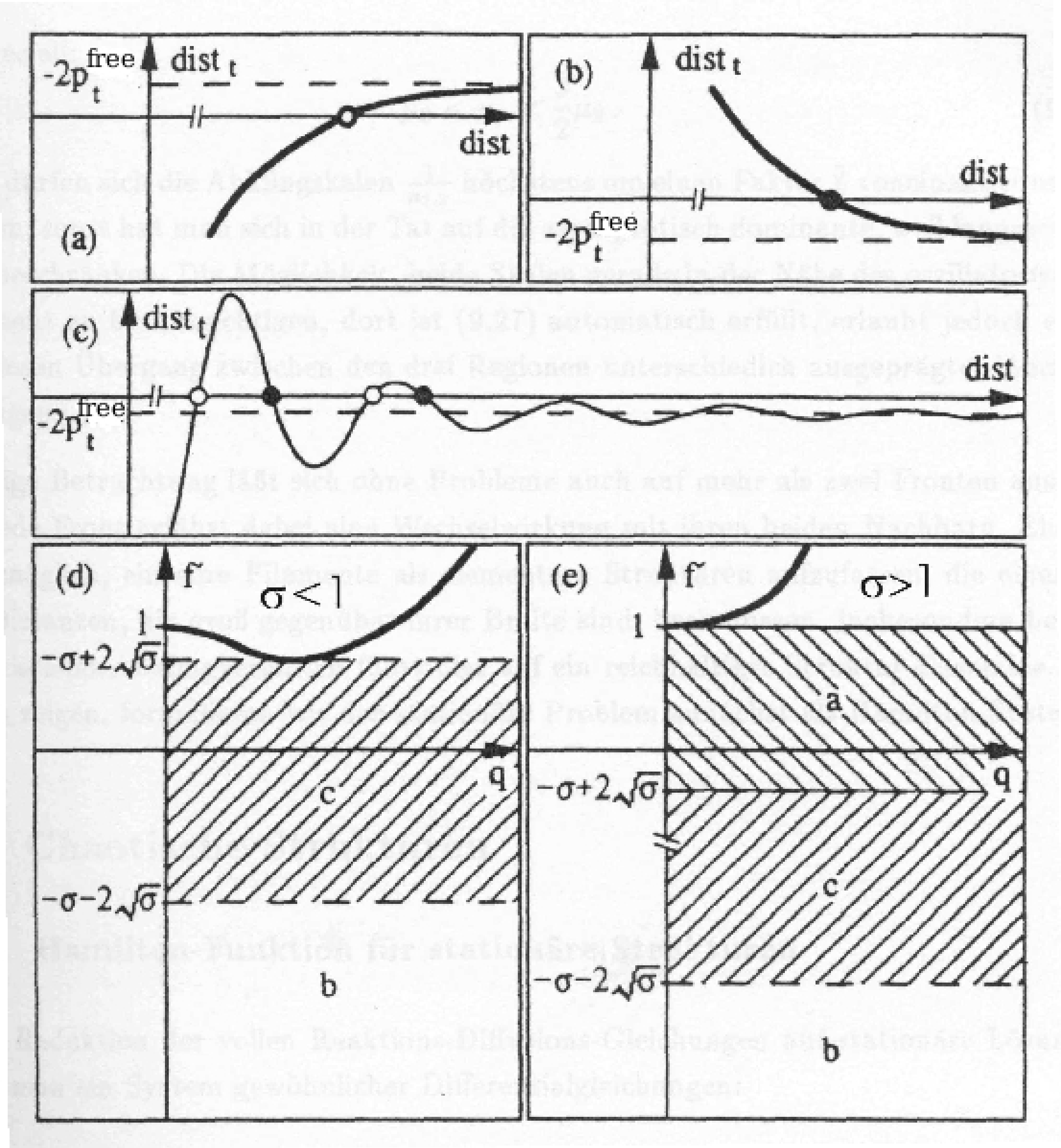

Fig. 9.3a-e

Fig. 9.3a-e

Schematic presentation of the results for the analytical investigation of the relative speed $\partial_t(dist) = dist_t$ in dependence of the distance *dist* for a front and an antifront being the solutions of the FHN equation (1-3) with 2 components in R^1 without global coupling and with vanishing inhibitor time constant τ (a-c). $2p_t^{free}$ denotes the relative speed of the free fronts (large dist), full dots denote stable zeros for the speed and consequently stable stationary bound states (DSs) and open dots indicate unstable states. Depending on the parameters the front-anti-front interaction is asymptotically attractive (a), asymptotically repulsive (b) or periodic in nature (c). In the latter case an unlimited number of different DSs with increasing discrete distances of the interacting fronts is possible. (d) and (e) relate the results (a-c) to the destabilization of the high value homogeneous stationary state (u^+, v^+) resulting from the stable fixed point of the spatially uncoupled system. To this end in (d, e) we represent the corresponding lower stability curves in the form of $f'(u^+, v^+)$ over the wave vector q by using $\sigma = d^2_u / d^2_v$ with $\sigma < 1$ (d) and $\sigma > 1$(e). The striped areas labelled by a, b and c indicate parameter ranges corresponding to the figs. (a - c) respectively. - The underlying ideas present a far reaching concept for the understanding of the interaction of fronts and DSs in R^1. Once a stable front-antifront pattern is generated in turn a given tail of such a DS can be considered as part of a front that is able to interact with the tail of another DS just the way two fronts would do. Therefore the developed concept for the interaction of fronts can also be applied to the interaction of DSs. [Pu043] - compare to: experiement e.g. figs. 3.14, 4.19, 4.20, 4.21, 5.8, 5.12; theory figs. 9.4, 9.5, 9.11, 9.12, 9.18, 9.19, 9.20, 9.30 - see also [Pu018; M. Bode, „Beschreibung strukturbildender Prozesse in eindimensionalen Reaktions-Diffusions-Systemen durch Reduktion auf Amplituden-gleichungen und Elementarstrukturen", Thesis, Institut für Angewandte Physik, University of Münster (1992); Pu024; R. Schmeling, „Experimentelle und numerische Untersuchung von Strukturen in einem Reaktions-Diffusions-System anhand eines elektrischen Netzwerkes", Thesis, Institut für Angewandte Physik, University of Münster (1994)]

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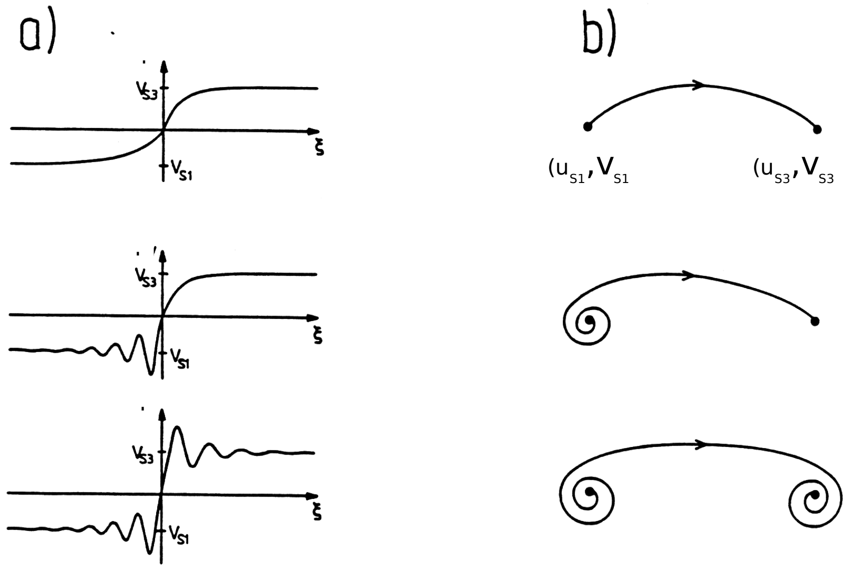

Fig. 9.4a,b

Schematic presentation of front solutions of the FHN equation (1-3) with 2 components in R^1. In (a) from top to bottom three kinds of fronts may be observable having two monontonic tails, one monotonic and one oscillatory tail and two oscillatory tails. In b) the related possible pairs of values of *(u, v)* are depicted in phase space spanned by *u* and *v*. [Pu018] - see also fig. 9.3.

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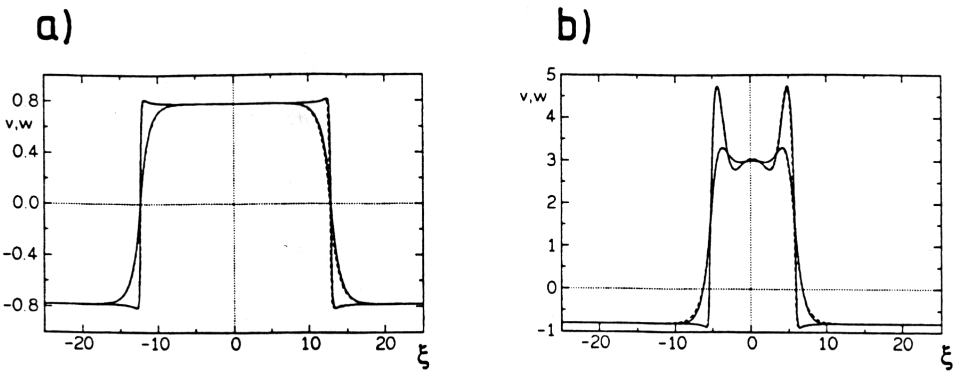

Fig. 9.5a,b

Comparison between the numerical (dashed line) and analytical solution (continuous line) for activator *u* (here denoted by *v*) and the inhibitor *v* (here denoted by *w*) of the FHN equation (1-3) with 2 components in R^1 and with global coupling for a front-antifront pattern. In both cases the broader distribution corresponds to the inhibitor. Analytical and numerical results are almost undistinguishable. In (a) and (b) the tails of the interacting fronts are non-oscillatory and oscillatory, respectively. [Pu024] compare to: experiement e.g. figs. 3.3, 3.5, 3.6; theory figs. 9.3, 9.4,9.11, 9.12, 9.18, 9.19, 9.20, 9.30 - see also e.g. [M. Bode, Thesis (1992); R. Schmeling, Thesis (1994)]

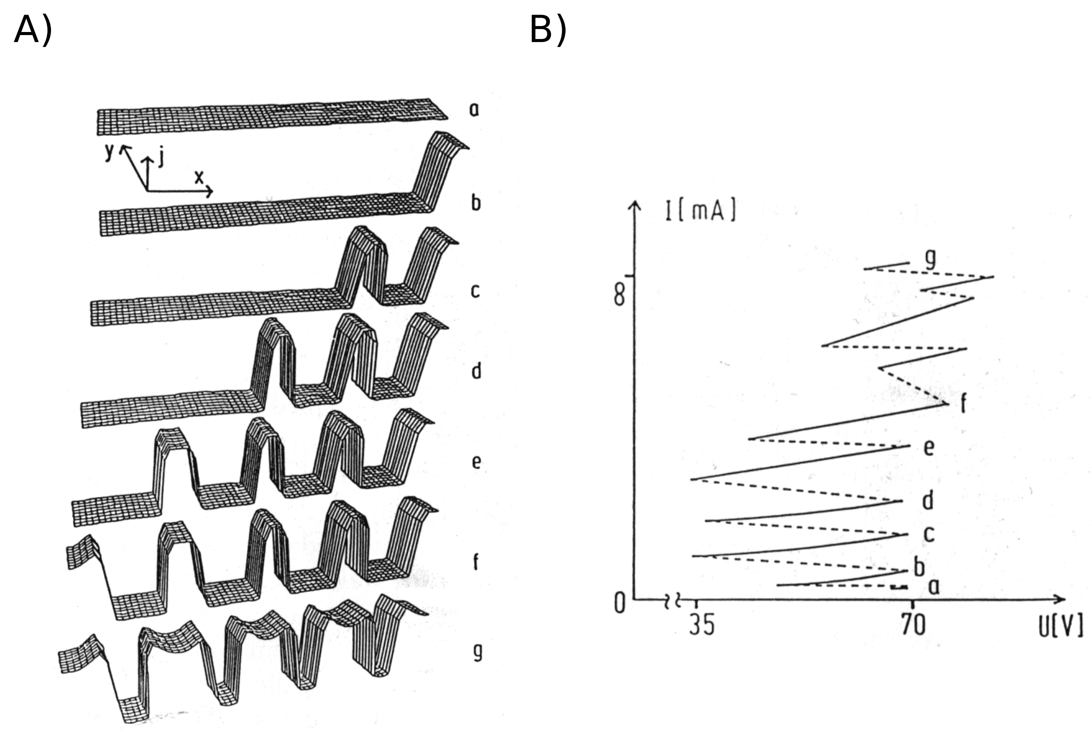

Fig. 9.6A,B

Early numerical results for a cascade of stable numerical solutions for the FHN equation (1-3) with 2 component in R^1 and with global coupling for $\kappa_2 = 0$. In (A), with increasing parameter κ'_1 the distribution of the component $j \sim u$ displays an increasing number of localized structures (LSs). (B) shows the global characteristic where $I \sim \langle u \rangle$ is plotted versus $U \sim (\kappa'_1 - \kappa'_2\langle u \rangle)$. One observes jumps to distinct branches with increasing number of LSs. Positions marked by letters a to f correspond to the same letters as those in (A). This is an early example for what nowadays is referred to as snaking. [Pu008] - In terms of the 2-layer model of the chapter A Model for Pattern Formation the relation between I and U corresponds to the global current – voltage characteristic describing the overall current I through the device 2.1 in dependence of the voltage drop at the device, namely $(U_0 – IR_0)$. - compare to: experiment figs. 3.12, 4.5, 4.6, 4.16, 5.7, 5.18, 7.3; theory figs. 3.12, 9.10 - see also e.g. [Pu003; Pu007; Dohmen, „Entwicklung von Modellgleichungen zur Beschreibung nichtlinearer elektrischer Systeme und Untersuchung der Lösungsvielfalt mit analytischen und numerischen Methoden", Thesis, Institut für Angewandte Physik, University of Münster (1991); R. Schmeling, Thesis (1994); Pu018; Pu022]

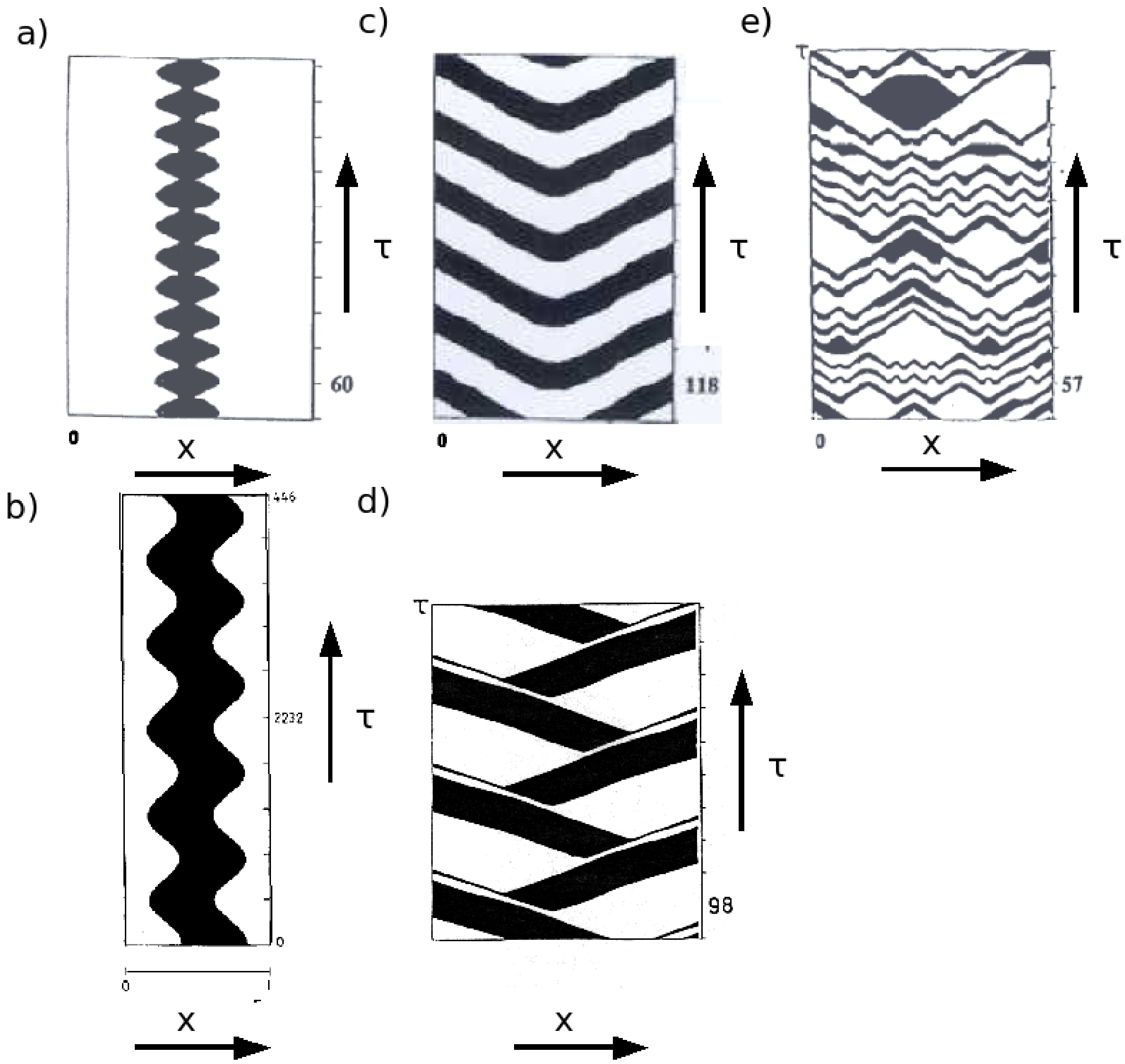

Fig. 9.7a-d

Stable dynamical numerical solutions for *u* of the FHN equation (1-3) with 2 components in R^1 for LSs reflecting the excitation of internal degrees of freedom. The representation is given in a space time plots with the space coordinate x as the abscissa and the time τ as ordinate. The following phenomena are presented: breathing DS without global coupling (a); pendulating LS including global coupling (b); periodic generation of a pair of LSs in the centre, travelling to the boundary and subsequent annihilation including global coupling (c); similar to (b) but generation due to splitting (d); spatio-temporal chaotic motion including global coupling (e). [Dohmen, Thesis, (1991); Pu018; Pu022; Pu025] - compare to: experiment fig. 4.8, 7.2; theory 8.1, 8.2

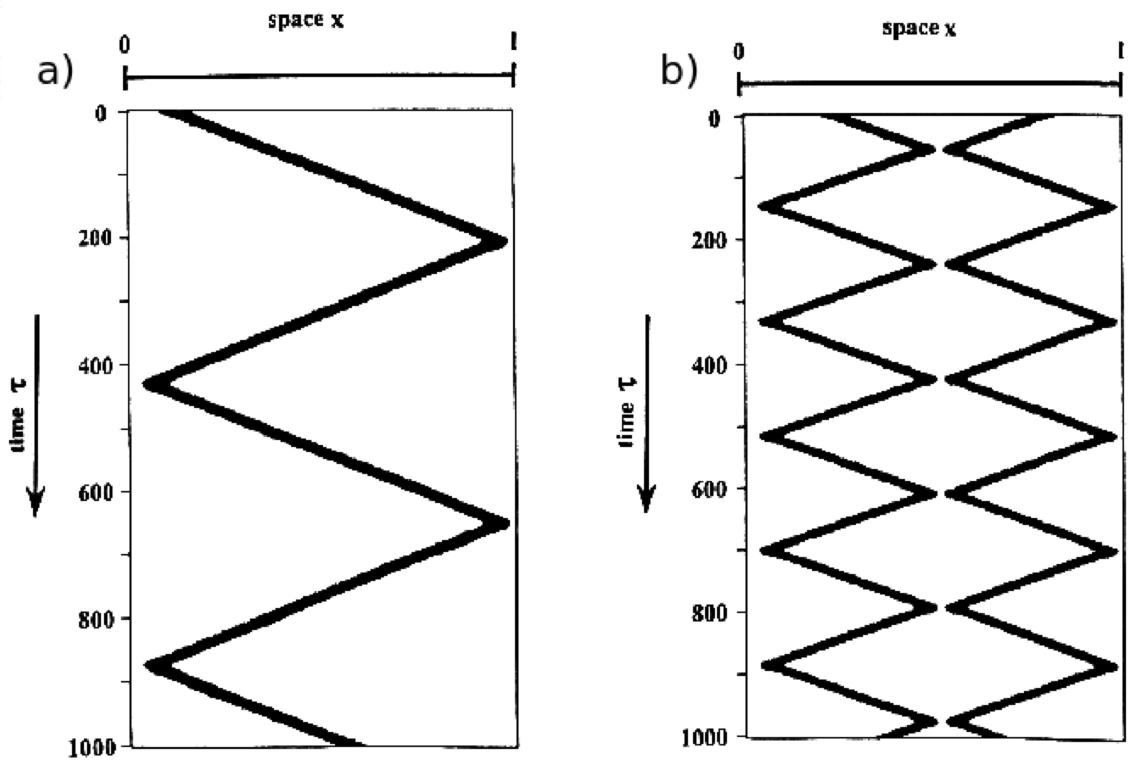

Fig. 9.8a,b

Stable numerical solutions for *u* of the FHN equation (1-3) with 2 components in R^1 with global coupling and very small but finite parameter μ: single LS being reflected a the boundary (a) and reflection of LSs at each other (b) [Pu026] – compare to: experiment e.g. figs. 3,13, 4.7, 4.18, 4.19, 5.9, 7.2; theory figs. 6.2, 8.1, 9.13, 9.14, 9.15

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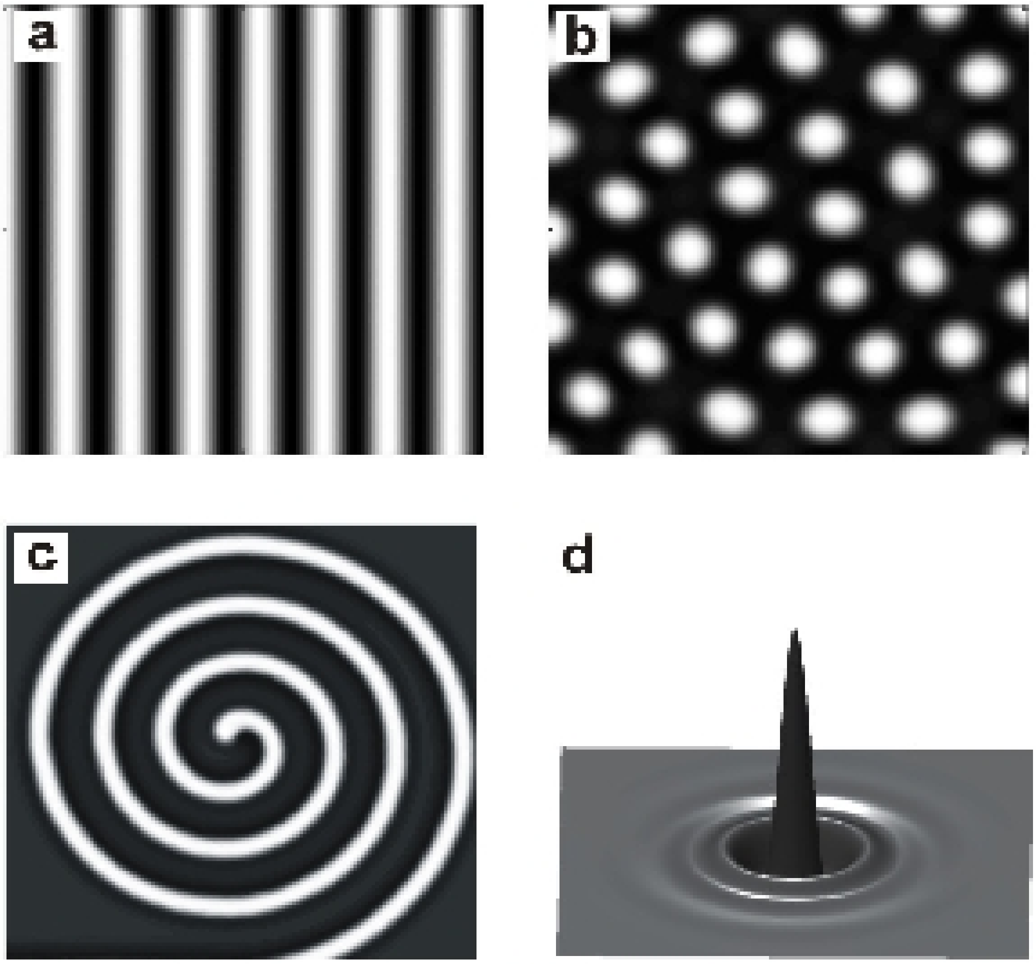


Fig. 9.9a-d

Stable numerical solutions for *u* of the FHN equation (1-3) in R^2 with 3 components without global coupling. Among other things, according to the chosen parameters one observes stationary stripes (a), hexagons (b), rotating outrunning spirals (c) and single DS (d) [Pu127]. - compare to: experiment
4.11, 4.13, 4.14, 5.4, 5.5, 5.6; with respect to DSs see below

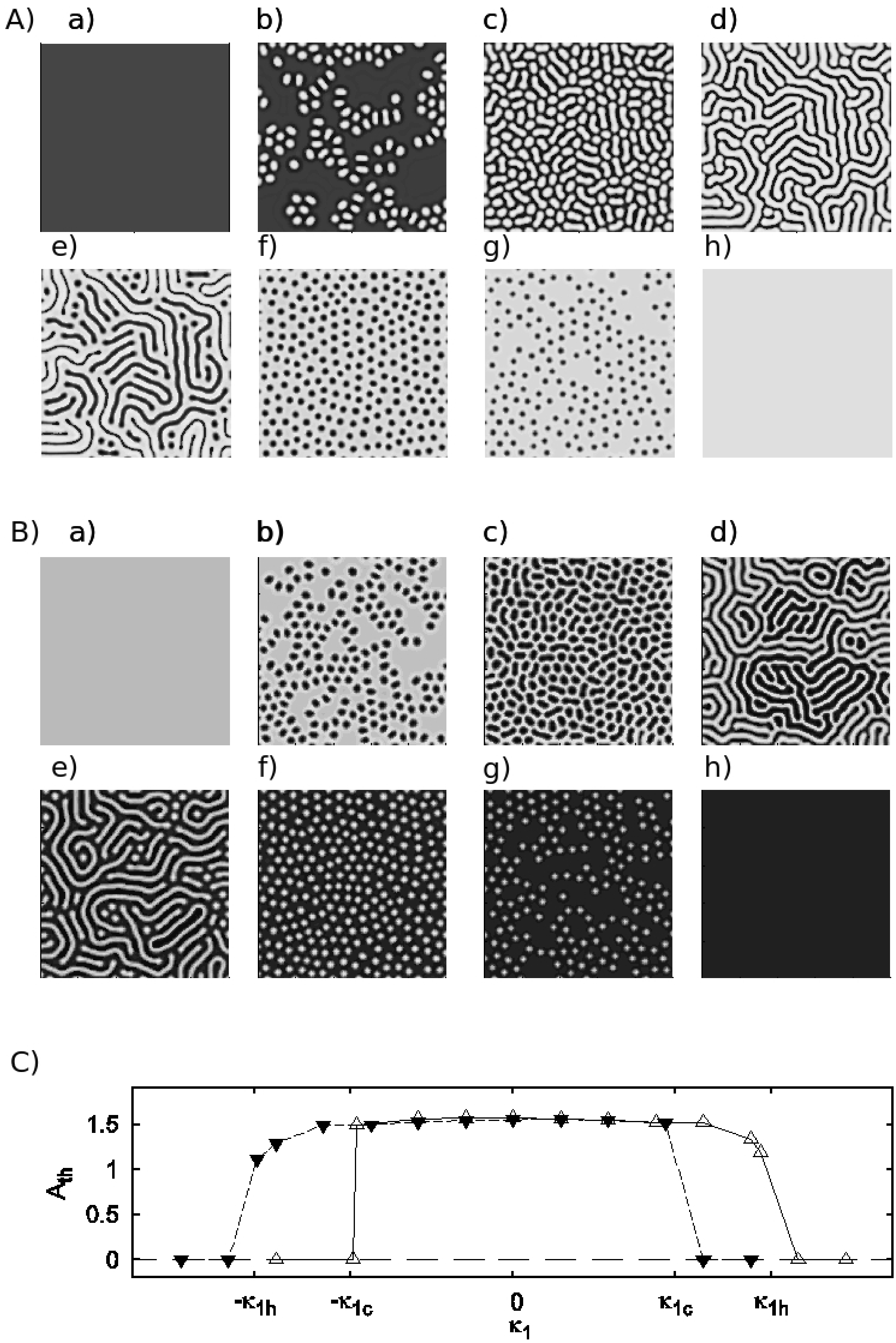
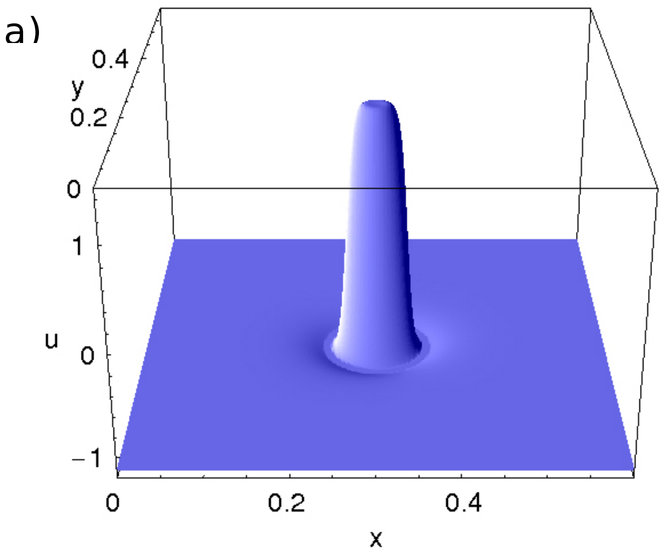

A)
a)
b)
c)
d)
e)
f)
g)
h)
B)
a)
b)
c)
d)
e)
f)
g)
h)
C)
A_{th}
1.5
1
0.5
0
$-\kappa_{1h}$
$-\kappa_{1c}$
0
κ_{1c}
κ_{1h}
κ_1

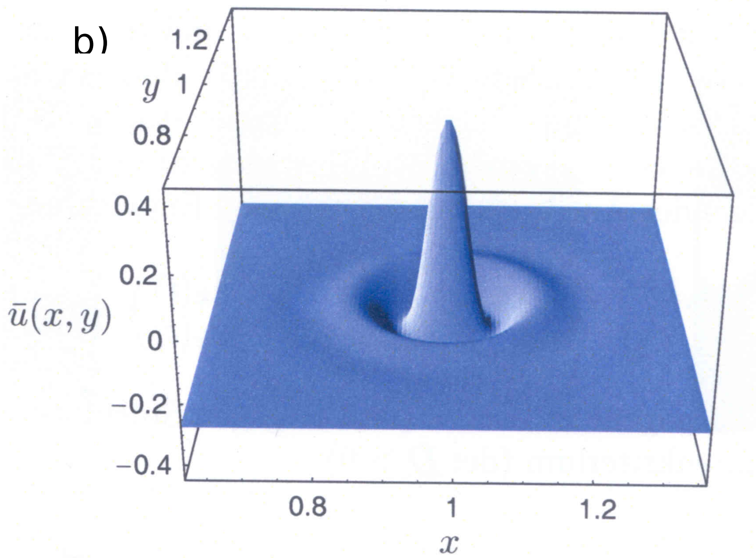
Fig. 9.10A-C

Fig. 9.10A-C

Stable numerical solutions for *u* for the 3-component FHN equation (1-3) in R^2 with κ_1', $\kappa'_2 = 0$. By increasing κ_1 essentially one observes the following bifurcation cascade in (A): Homogeneous stationary state with a low value of *u* (a) → bright LSs (b) → intermediate patterns (c - e) → dark LSs (f - g) → homogeneous stationary state with a high value of *u* (h). In (B) the reverse scenario is observed by decreasing κ_1. The evolution of the peak to background value for the *u* component in (C) clearly reveals the strong hysteresis when increasing (continuous line) and decreasing κ_1 (dotted line). Though theory assumes a dc driver and should apply directly only to dc systems it is amazing to observe the similarity to experimental results obtained from ac systems. - The observations represent a generic scenario for the formation of LSs by changing the strength of the driver. [Pu128] - compare to: experiment figs. 3.12, 4.5, 4.6, 4.16, 5.7, 7.3, 7.10; theory 3.12, 9.6



non-ocillatory tails



oscillatory tails

Fig. 9.11a,b

Stable numerical solutions for *u* of the 3-component FHN equation (1-3) in R^2 without global coupling. According to the chosen parameters one observes isolated stationary DSs with non-oscillatory (a) and with oscillatory tails (b) [Pu118] - compare to: experiment figs. 3.13, 3.14, 4.17, 4.21, 5.8; theory 9.3, 9.4, 9.5, 9.12, 9.18, 9.19, 9.20 - see also e.g. [Pu003; Pu007; R. Dohmen, Thesis (1991); M. Bode, Thesis (1992); Pu018, Pu024, Pu043, Pu063; Pu100; Pu118]

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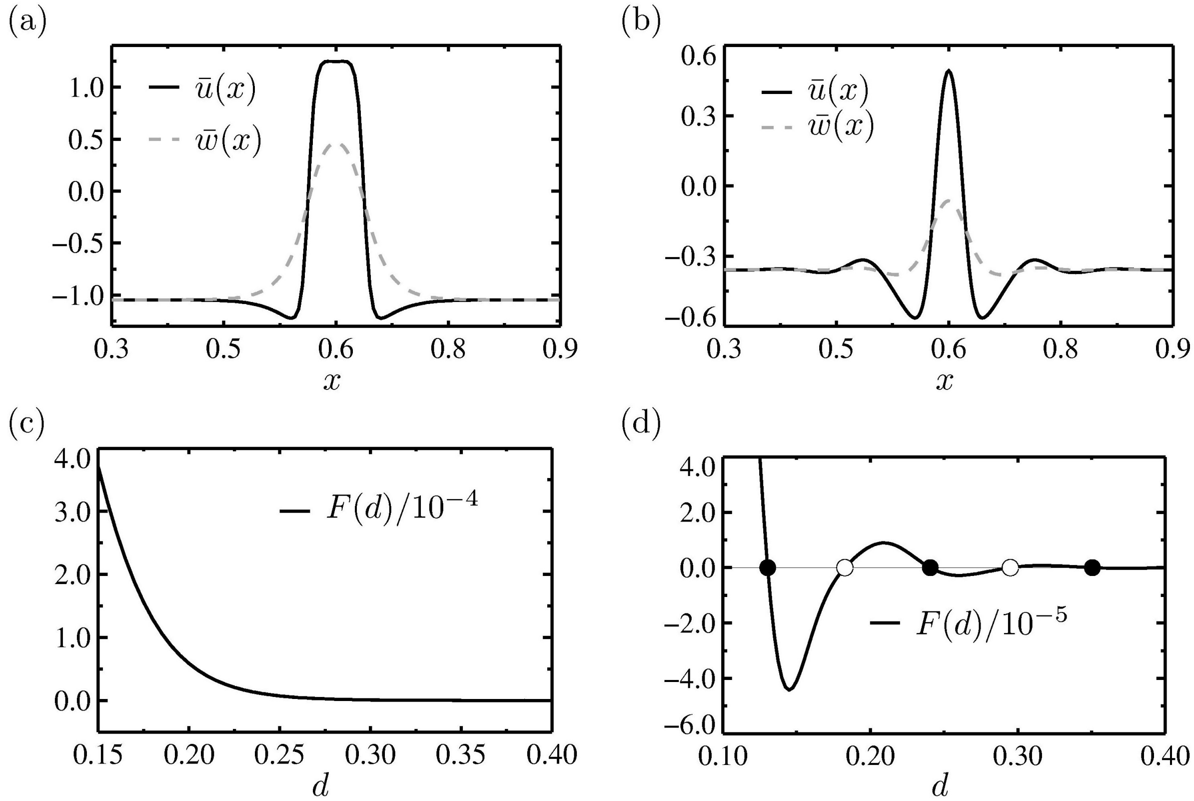

Fig. 9.12a-d

Typical stable numerical solutions of the 3-component FHN equation (1-3) in R^2 without global coupling similar to fig. 9.11. The ordinate in (a,b) represents the cross sections $\bar{u}(x)=\bar{v}(x)$ and $\bar{w}(x)$ through the centre of the DS for isolated DSs with non-oscillatory (a) and oscillatory tails (b), respectively. The related interaction laws in the particle equation (4,5) are depicted in (c) and (d). [Pu118] - for more details see fig. 9.11

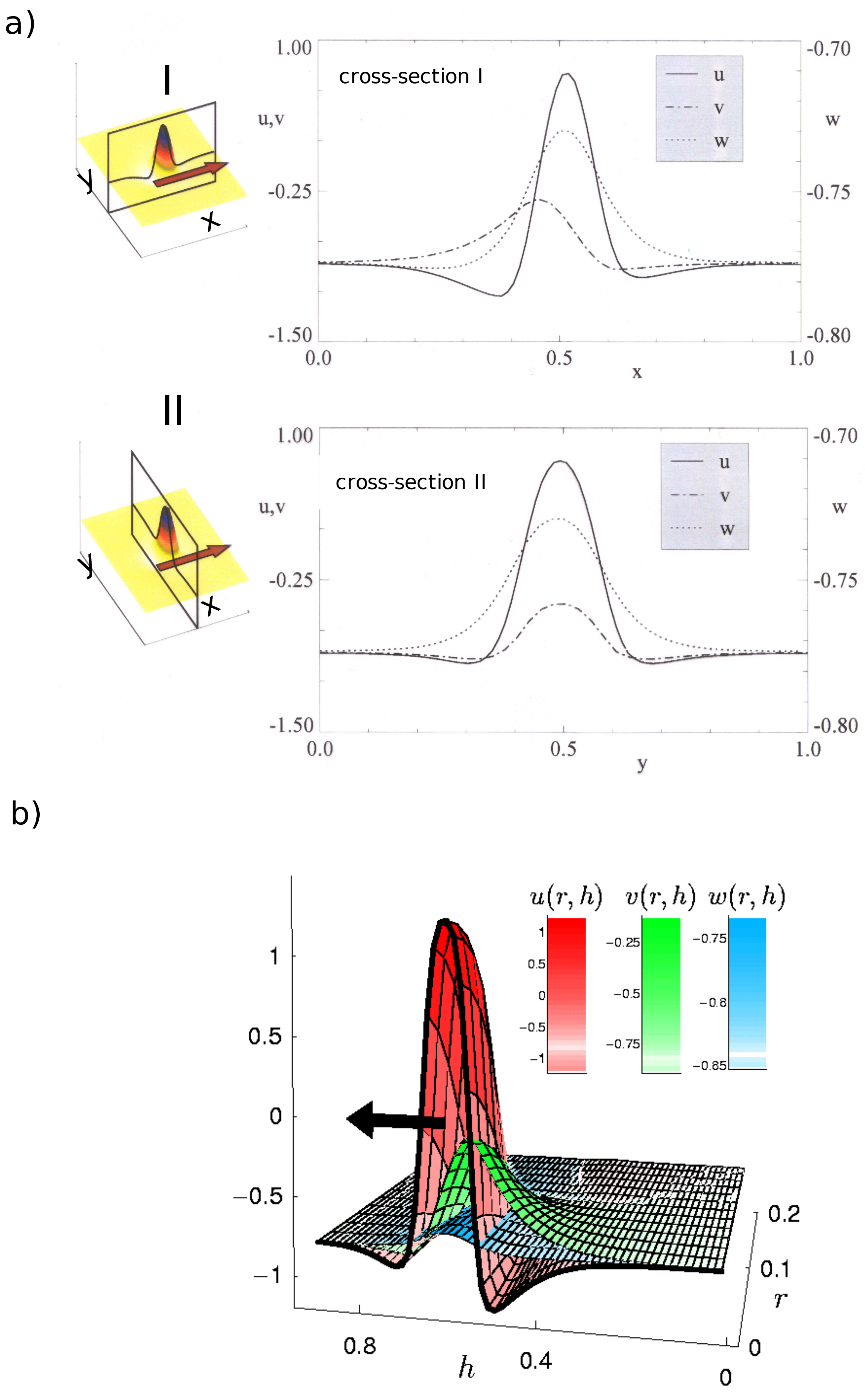
a)
I
y
x
cross-section I
u
v
w
1.00
-0.25
-1.50
u,v
-0.70
-0.75
-0.80
w
0.0
0.5
1.0
x
II
y
x
cross-section II
u
v
w
1.00
-0.25
-1.50
u,v
-0.70
-0.75
-0.80
w
0.0
0.5
1.0
y
b)
$u(r,h)$ $v(r,h)$ $w(r,h)$
1
0.5
0
−0.5
−1
−0.25
−0.5
−0.75
−0.75
−0.8
−0.85
0.8
h
0.4
0
0.2
0.1
r
0

Fig. 9.13a,b

Fig. 9.13a,b

Stable numerical solutions of the 3-component FHN equation (1-3) without global coupling for an isolated travelling DS in R^2. In (a) in the upper figure the direction of propagation and the centre of the DS lie in the cross section plane, in the lower figure the direction is cut vertically [Pu053]. - In (b) we depict some other DS [Pu092]. - In more than one spatial dimension a third component is essential for the stabilization of a travelling DS. The displacement of the activator u with respect to the slow inhibitor v is a measure for the speed of propagation. - compare to: experiment figs. 4.18, 4.19, 5.9, 5.14, 6.2, 7.2; theory figs. 8.1, 9.8, 9.14, 9.15, 9.28 - see also e.g. [R. Dohmen, Thesis (1991); M. Bode, Thesis (1992); Pu060, Pu071, Pu084, Pu101, Pu105]

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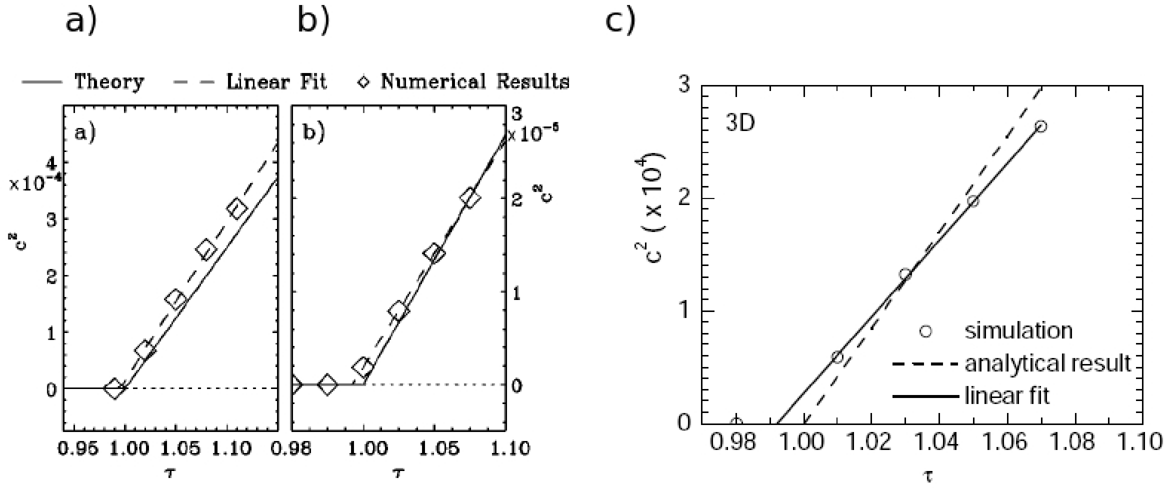

Fig. 9.14a-c

Analytical (theoretical) results compared to the numerical solutions (simulations) of the 3-component FHN equation (1-3) without global coupling in the case of the destabilization of an isolated DSs to a travelling one. In the present case one observes a supercritical bifurcation from a rotational symmetric stationary DS to a travelling one in 1-dimensional (a) [Pu060], 2-dimensional (b) [Pu060] and 3-dimensional space (c) [Pu071] with τ as bifurcation parameter. The square of the speed c scales linearly with the bifurcation parameter τ. - compare to: experiment figs. 4.18, 5.9, 7.2; theory figs. 8.1, 9.21- see also e.g. [Pu071; Pu101; Pu105; Pu109; Pu113]

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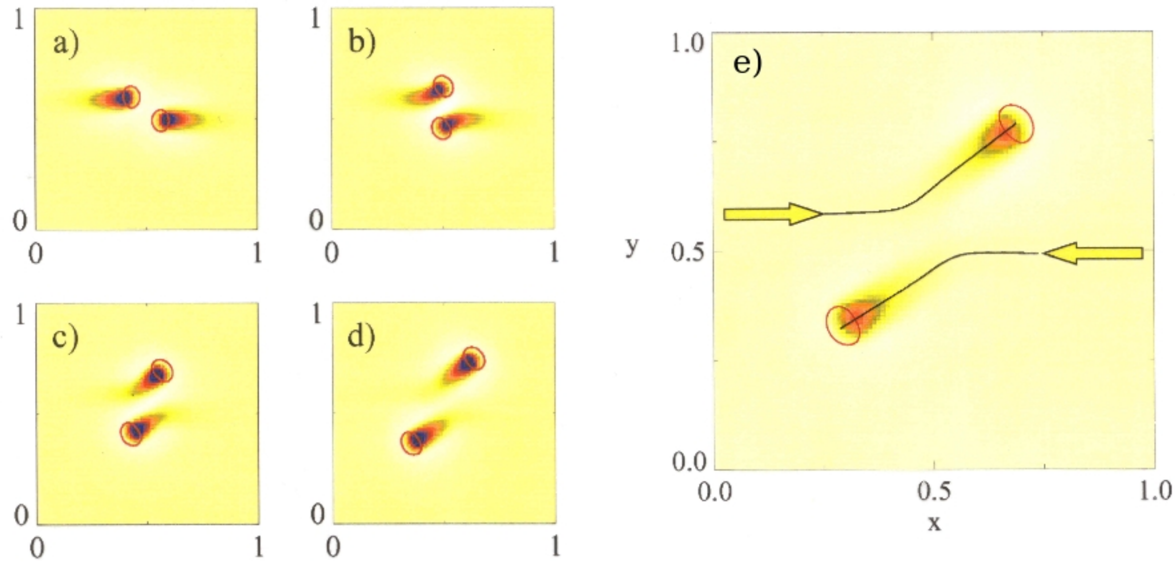

Fig. 9.15

Numerical solutions of the 3-component FHN equation (1-3) in R^2 without global coupling for the case of scattering of two travelling DSs: (a – d) represent the evolution in time and (e) indicates the corresponding trajectories. [Pu053; C. P. Schenk, "Numerische und analytische Untersuchung solitärer Strukturen in zwei- und dreikomponentigen Reaktions-Diffusions-Systemen" Thesis, Institut für Angewandte Physik, University of Münster (1999)] - compare to: experiment figs. 3.13, 4.19, 7.2; theory figs. 8.4, 9.8, 9.16, 9.17, 9.29, 9.32 - see also e.g. [Pu084; Pu118]

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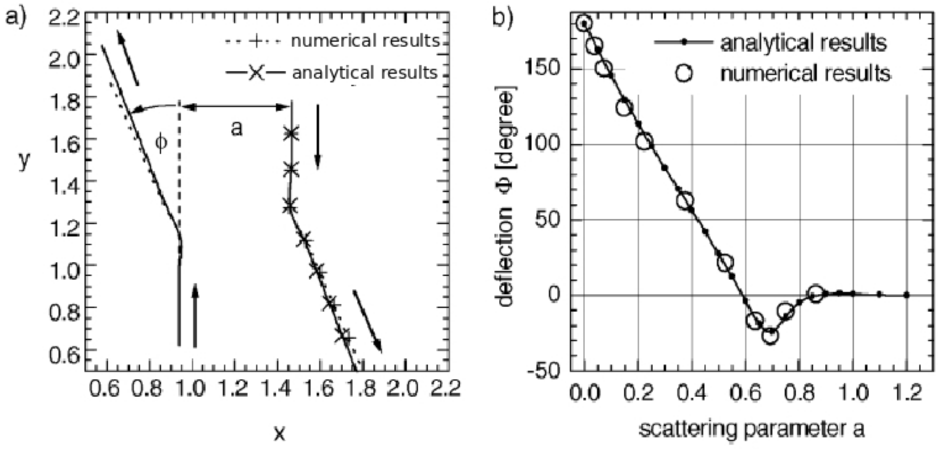

Fig. 9.16a,b

Interaction behaviour obtained from the numerical solution of the 3-component FHN equation (1-3) without global coupling in R^2 (referred to as simulation) for the case of scattering of two travelling DS in R^2 and comparison with the solutions of the corresponding particle equation (4,5) (referred to as analytical results). In (a) we represent the trajectories of two antiparallel incoming DSs with offset a. In (b) the deflection angle Φ is plotted in dependence of the offset (scattering parameter a). [Pu084] - compare to: experiment figs. 3.13, 4.19, 4.19, 7.2; theory figs. 8.4, 9.8, 9.15, 9.17, 9.29, 9.32

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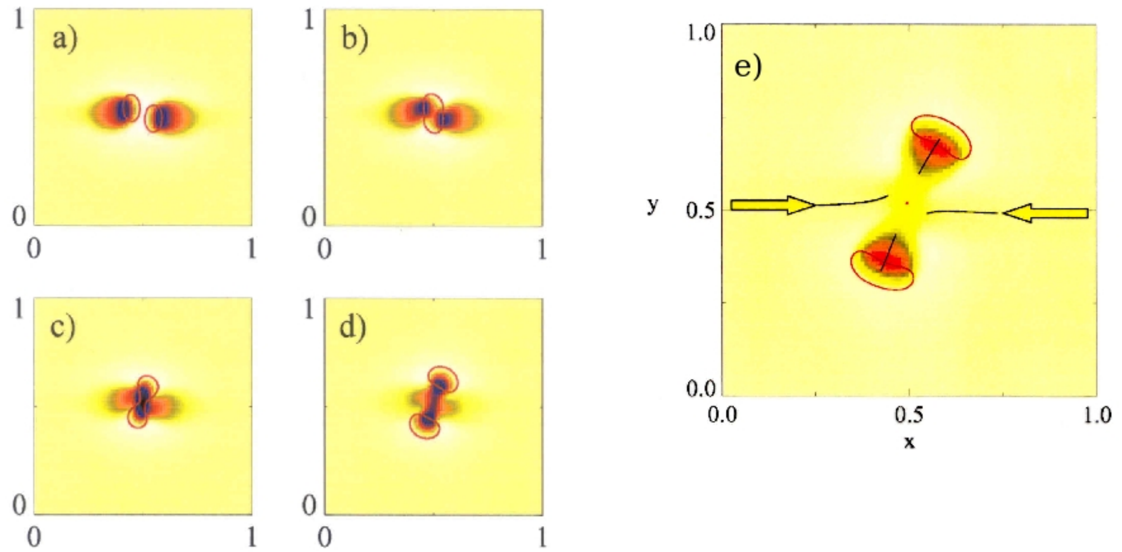

Fig. 9.17

Numerical solutions of the 3-component FHN equation (1-3) in R^2 without global coupling for the case of the interpenetration of two travelling DSs. (a–d) represent the evolution in time and (e) indicates the corresponding trajectories. Despite strong interaction in this case the number of DSs is conserved. [Pu053; C. P. Schenk, Thesis (1999)] - compare to: experiment figs. 3.13, 4.19, 4.19, 7.2; theory figs. 8.4, 9.8, 9.15, 9.16, 9.29, 9.32

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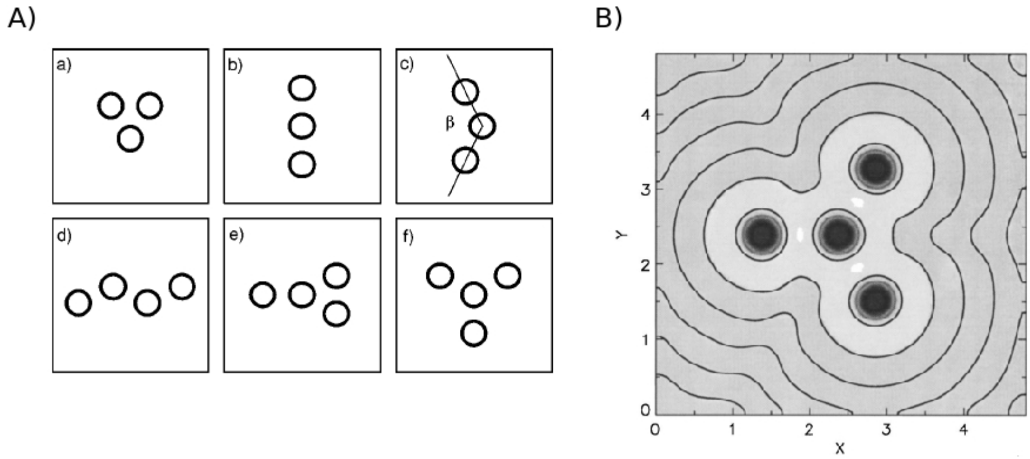

Fig. 9.18A, B

Stationary numerical solutions for *u* of the FHN equation (1-3) in R^2 with 2 components for interacting stationary DSs in the case of the formation of bound states (molecules): In (A) except for (b) the solutions are stable. The molecule in (B) corresponds to the case (f) in (A). In (B) *u* takes the same values on the black lines thereby reflecting the oscillatory behaviour in the surrounding of a given individual DS. [Pu062] - compare to: experiement figs. 3.14, 4.19, 4.20, 4.21, 5.8, 5.12; theory 9.3, 9.12, 9.19, 9.20, 9.30 - see also e.g. [Pu018; Pu084; Pu100; Pu105; Pu0109; Pu118]

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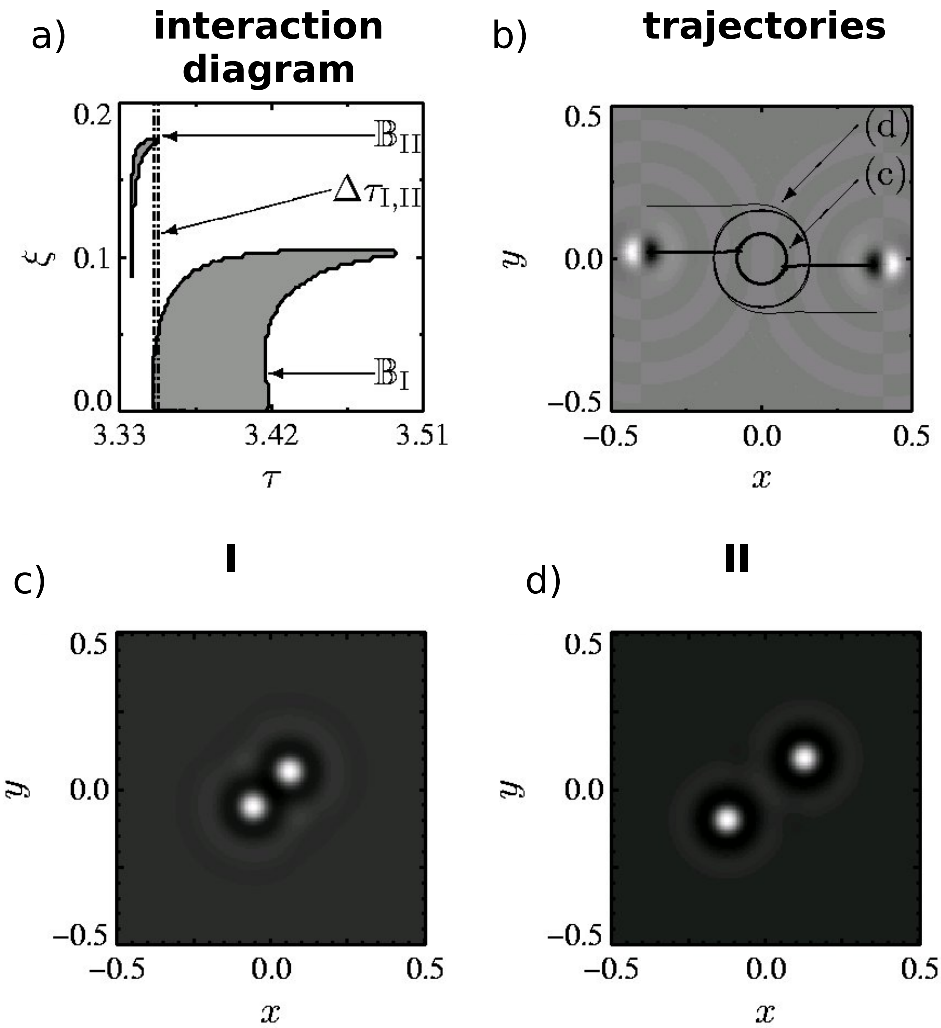

Fig. 9.19a-d

An example for an interaction diagram obtained from the particle description equation (4,5) in R^2 is depicted in (a) in which two anti-parallel travelling DSs with off-set ξ collide and generate a molecule. From (a) one concludes that in dependence of τ regions B_I and B_{II} may exist with stable rotating 2-DS molecules as solution having different inter-DS distances. For a narrow band $\Delta_{\tau I,II}$ of values of τ these states can coexist. This is also indicated in (b). In (c) and (d) snapshots of two stable rotating molecules with different inter-DS distance are displayed using a value of τ laying in $\Delta\tau_{I,II}$ when solving numerically the 3-component FHN (1-3) equation without global coupling. [Pu109] - compare to: experiment figs. 3.14, 4.18, 4.20, 4.21, 5.8, 5.12; theory 9.3, 9.12, 9.18, 9.20, 9.30 - see also the additional references in fig. 18

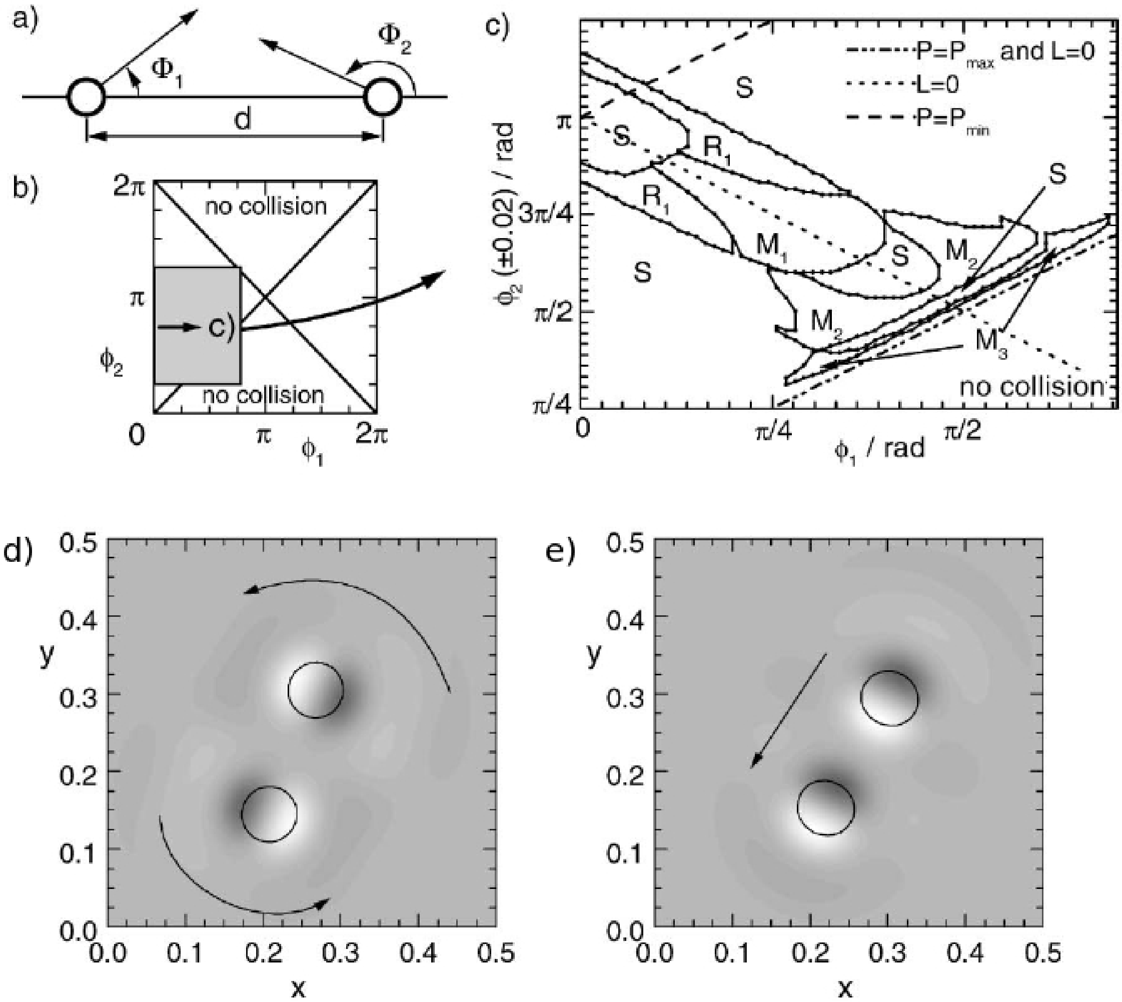

Fig. 9.20a-e

Results from the numerical solutions of the particle equation (4,5) and the corresponding 3-component FHN equation (1-3) in R^2 without global coupling for the case of the formation of rotating and travelling 2-DS molecule. The scattering diagram (c) is obtained from equation (4,5) in R^2 while (a) illustrates the choice of the initial conditions by defining the angles of direction of propagation Φ_1, Φ_2 with respect to the line interconnecting the two DSs. In (b) nontrivial collision geometries are referred to that are relevant for (c). In (c) we have the following nomenclature: S = scattering; R_i, M_i = formation of 2-DS molecules with rotational (translational) motion where different i denote different inter-DS distance of the molecule long after collision. In (d, e) we represent numerical solutions of the 3-component FHN equation (1-3) for the parameters chosen in (c) depicting rotational and translational motion of 2-DS molecules. [Pu084] - compare to: experiment figs. 3.14, 4.18, 4.20, 4.21, 5.8, 5.12; theory 9.3, 9.12, 9.18, 9.19, 9.30

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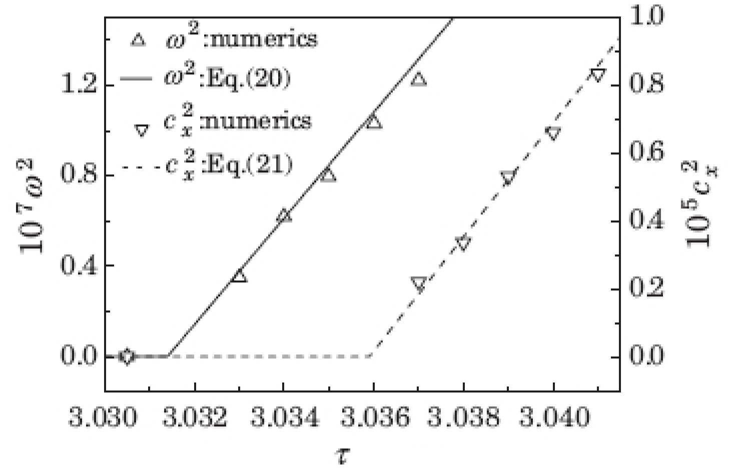

Fig. 9.21

Analytical (continuous curves) versus numerical treatment (triangles) of the 3-component FHN equation (1-3) in R^2 without global coupling for the case of the destabilization of a 2-DS molecule: One observes a supercritical rotational bifurcation and the square of the angular velocity ω scales linearly with the bifurcation parameter τ. Apparently, the rotational bifurcation precedes the travelling bifurcation (translational motion with speed c_x of the molecule in direction of the symmetry axis of rotation). [Pu100]. - compare to: experiement figs. 4.18, 5.9; theory fig. 9.14

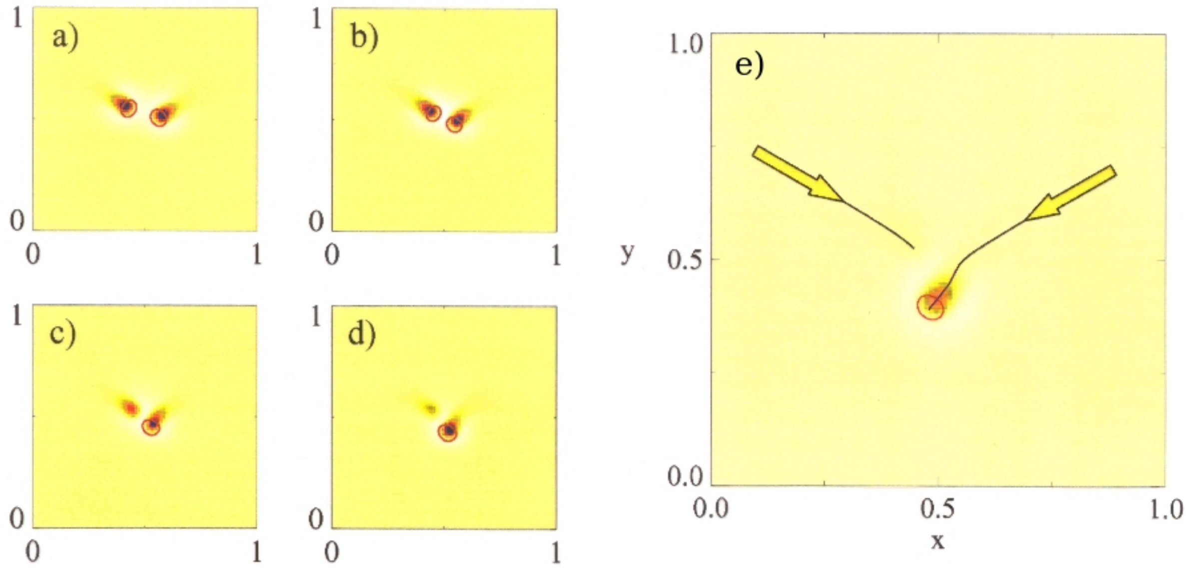

Fig. 9.22

Numerical solutions of the 3-component FHN equation (1-3) in R^2 without global coupling in the case of the interaction of two travelling DS with annihilation of one DS. The representation is done in the x-y plane. [Pu053; C. P. Schenk, Thesis (1999)] - compare to: experiment figs. 4.7; 4.23, 5.13; theory 9.31, 9.34 - see also e.g. [Pu043; Pu053; Pu118]

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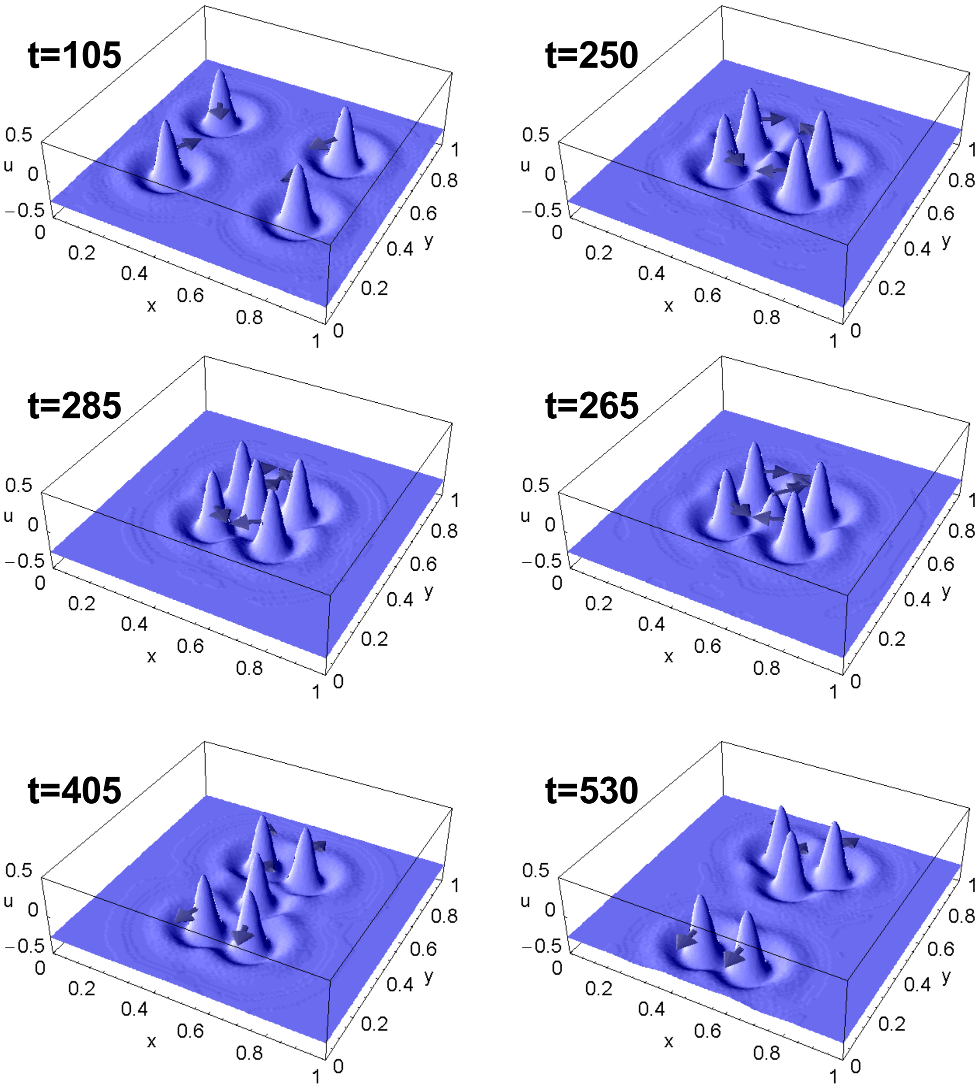

Fig. 9.23

Numerical solutions of the 3-component FHN equation in R^2 without global coupling in the case of the interaction of four travelling DSs in R^2 with generation of one DS. The variable u is represented in dependence of the position in the x-y plane. [Pu118] - compare to experiment figs. 4.7, 4.22, 5.13; theory 9.34 - see also e.g. [Pu079; Pu092]

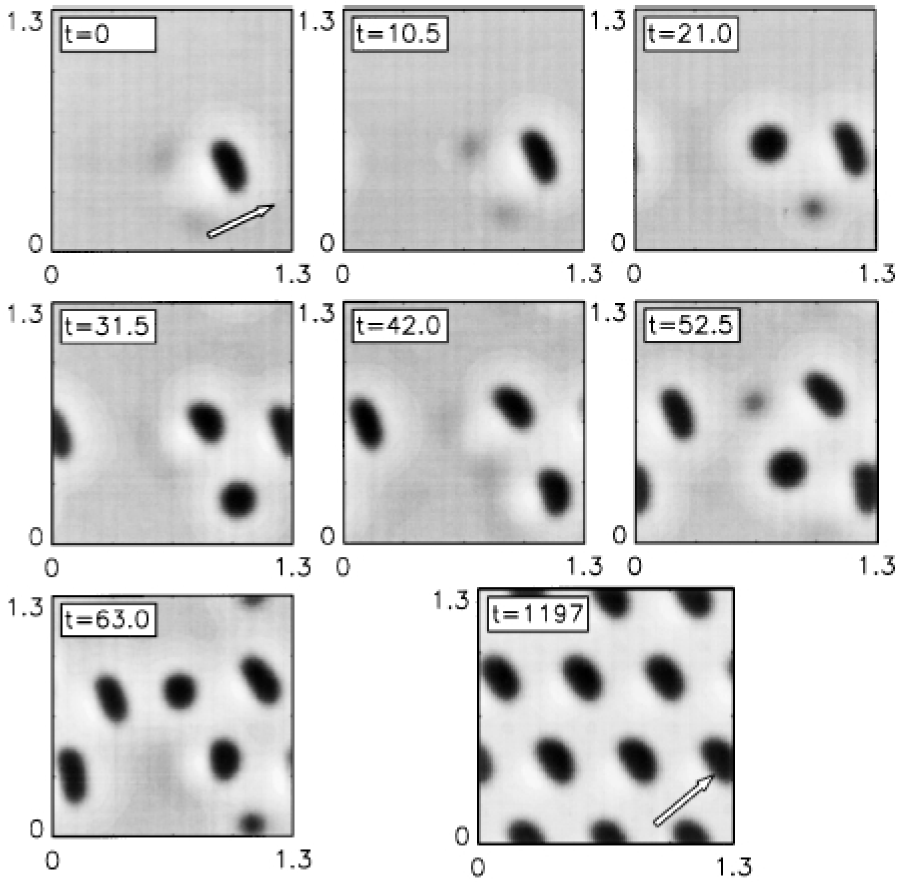

Fig. 9.24

Numerical solutions for u of the 3-component FHN equation in R^2 without global coupling for the case of self-completion. Starting from a single stable moving DS solution with oscillatory tails the DS is destabilized by a small parameter change. In the course of time more and more LSs are generated in the tails of already existing ones. This evolution comes to an end when the domain is covered completely with a hexagonal arrangement of LSs . The final state is a stable moving hexagonal pattern. [53]. - A similar scenario can be obtained by starting from a single stable DS solution with oscillatory tails in the presence of sufficient temporal, spatial or spatio-temporal noise. An example for a final stable stationary hexagonal pattern can also be found in [C. P. Schenk, Thesis (1999)] - compare to: experiment fig. 4.24; theory fig. 9.33. - see also e.g. [Pu079; Pu092]

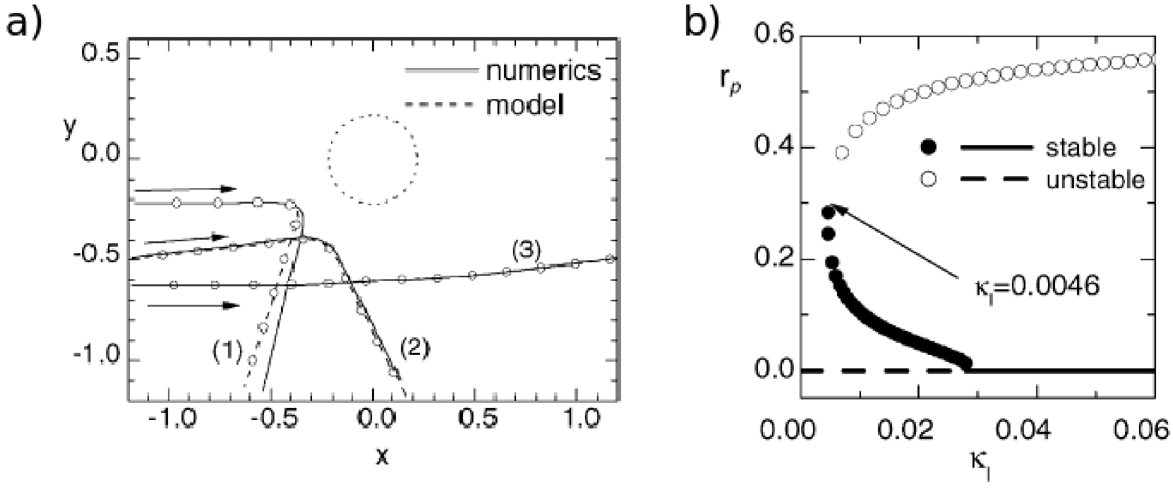

Fig. 9.25a,b

Interaction of an isolated DS with an impurity in R^2. In (a) we plot the trajectories in the case of scattering. The trajectories are obtained on one hand numerically by solving the 3-component FHN equation (1-3) without global coupling adding a local rotationally symmetric impurity [Pu084] and on the other hand from solving numerically the corresponding particle equation (4,5) [Pu084] (referred to as model). The impurity is indicated by a circle. The different trajectories in (a) correspond to different initial conditions while the parameters are kept fixed. [Pu084] - In (b) the bifurcation diagram is represented for a typical situation in the case that a single DS is captured and rotates with radius r_p around the centre of the impurity. In the figure r_p of the limit cycle is plotted versus κ_I which is proportional to the strength of the impurity. Apparently there exists a lower threshold for κ_I below which the limit cycle vanishes. The results in (b) are obtained from the particle equation. [Pu084]

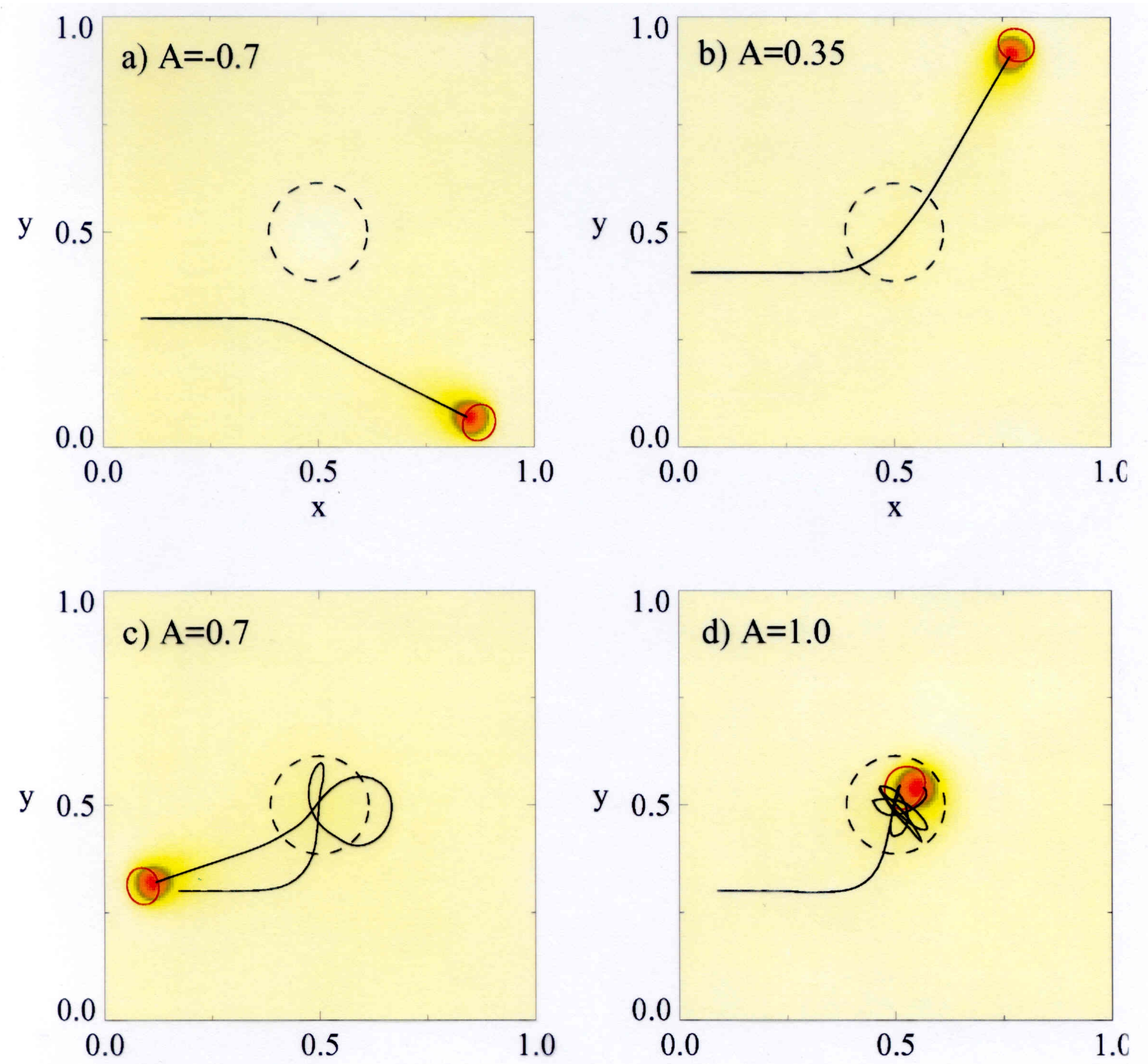

Fig. 9.26

Numerical solutions of the 3-component FHN equation (1-3) in R^2 without global coupling for the case of the interaction of a travelling DS with an impurity. "A" is a measure for the strength of the latter: simple scattering (a), interpenetration (b), reflection (c) and capturing (d). [Pu053; C. P. Schenk, Thesis (1999); Pu084]

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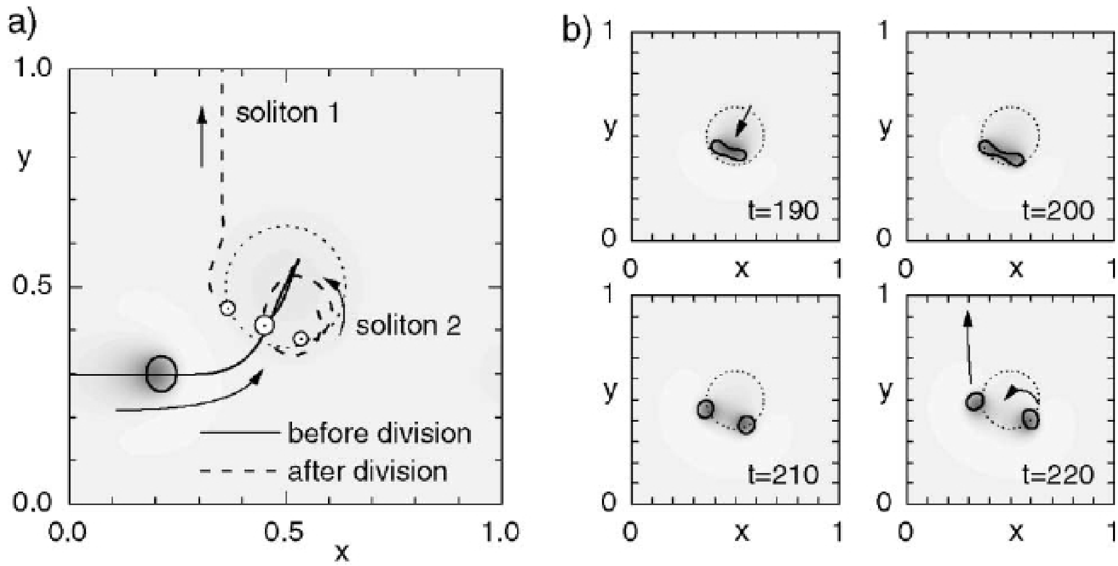

Fig. 9.27a,b

Numerical solutions of the 3-component FHN equation (1-3) in R^2 without global coupling for the case of the interaction of a single travelling DS with an impurity in R^2 with subsequent generation of another DS: in (a) the trajectories are represented for ingoing and out going DSs. From (b) it becomes clear that in the course of the interaction with the impurity an additional LS is generated. From the two DSs finally one leaves the impurity while the other one is trapped. [Pu053; C. P. Schenk, Thesis (1999); Pu084]

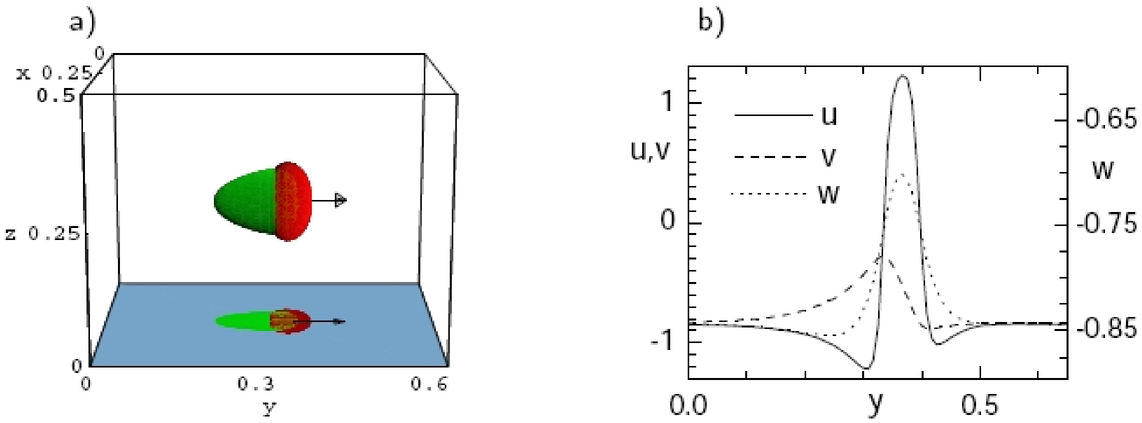

Fig. 9.28a,b

Numerical solutions of the 3-component FHN equation (1-3) in R^3 without global coupling for a travelling DS: Iso-surfaces of the fast activator *u* (red) and slow inhibitor *v* (green) (a) and corresponding cross section through the centre of the DS (b). [Pu071] - - compare to: experiment figs. 4.18, 4.19, 5.9, 5.14, 6.2, 7.2; theory figs. 8.1, 9.8, 9.13, 9.14, 9.15, 9.28

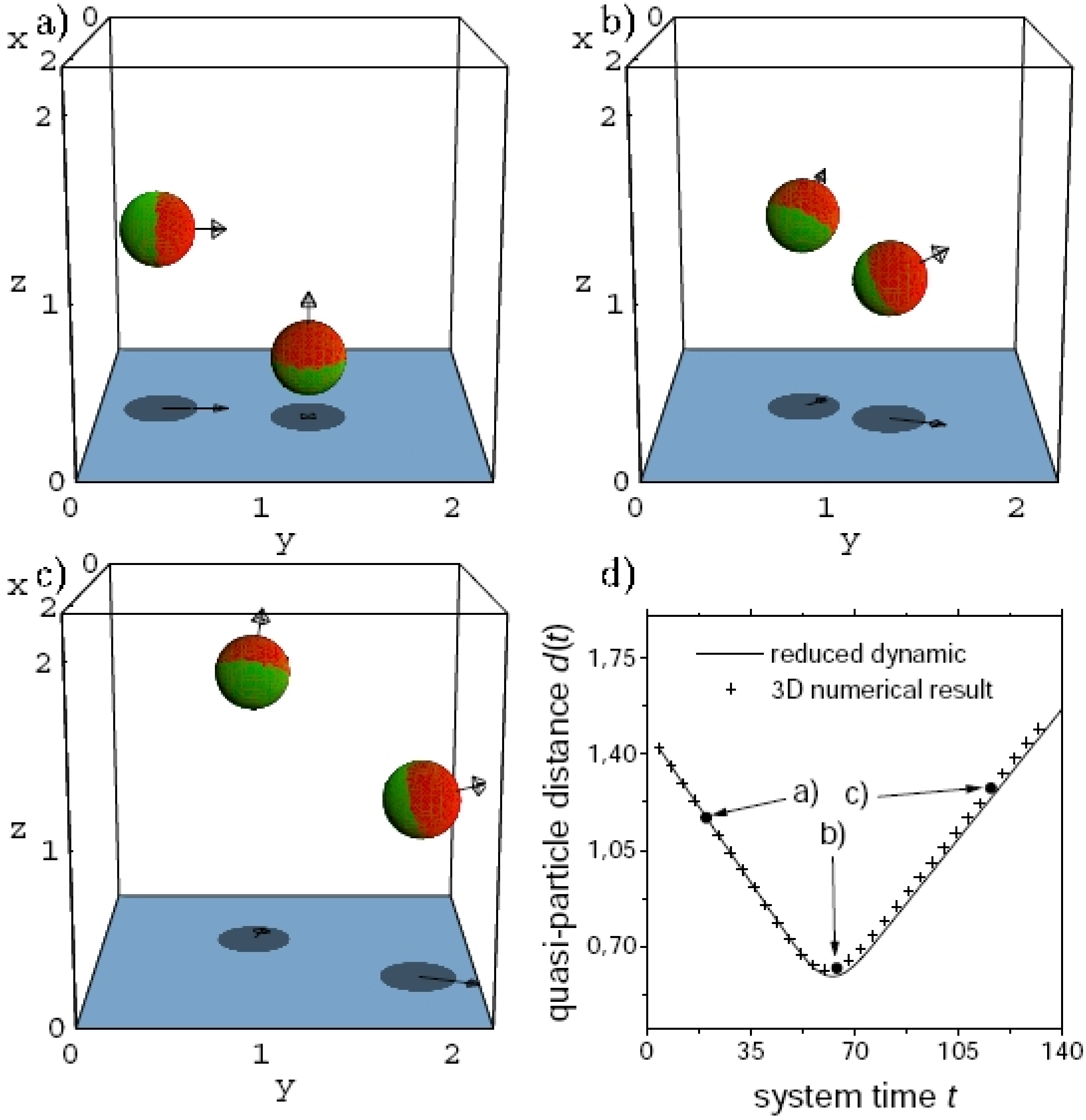

Fig. 9.29a,d

Numerical solutions of the 3-component FHN equation (1-3) in R^3 without global coupling in the case of scattering of two travelling DSs at each other: iso-surfaces of the fast activator *u* are indicated by red and the slow inhibitor *v* by green colour. (a – c) describe the evolution of the two DSs in the course of time and (d) displays the inter-DS distance as a function of time. [Pu079] - compare to: experiment figs. 3.13, 4.19; theory figs. 8.4, 9.8, 9.15, 9.16, 9.17, 9.29, 9.32

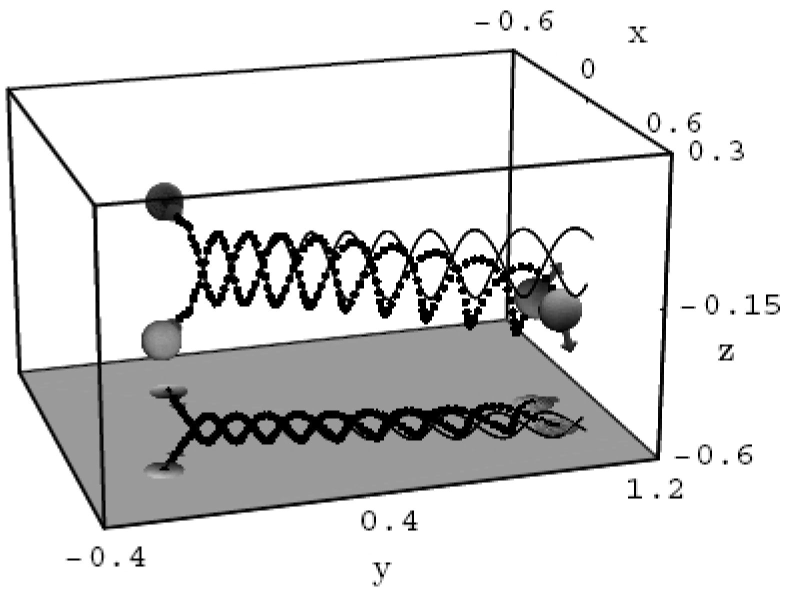

Fig. 9.30

Numerical solutions of the 3-component FHN equation (1-3) in R^3 without global coupling for the evolution of a 2-DS molecule that propagates along and rotates around the axis vertical to the connecting straight line of the centres of the two individual DSs. [Pu118] - compare to: experiment figs. 3.14, 4.19, 4.20, 4.21, 5.8, 5.12; theory 9.3, 9.12, 9.18, 9.19, 9.20

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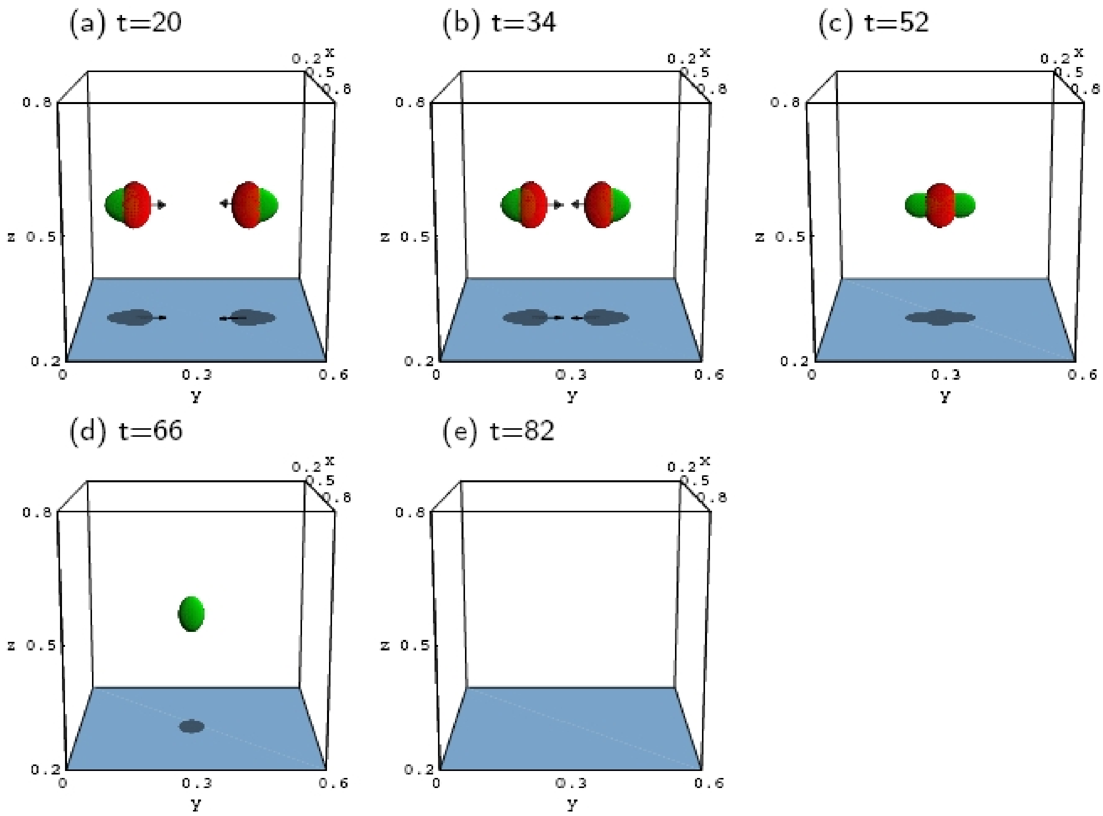

Fig. 9.31a-e

Numerical solutions of the 3-component FHN equation (1-3) in R^3 without global coupling for the evolution of two travelling DSs in R^3 both undergoing annihilation in the course of a collision: iso-surfaces of the fast activator *u* (red) and slow inhibitor *v* (green). [Pu071] compare to: experiment figs. 4.7; 4.23, 5.13; theory 9.22, 9.34

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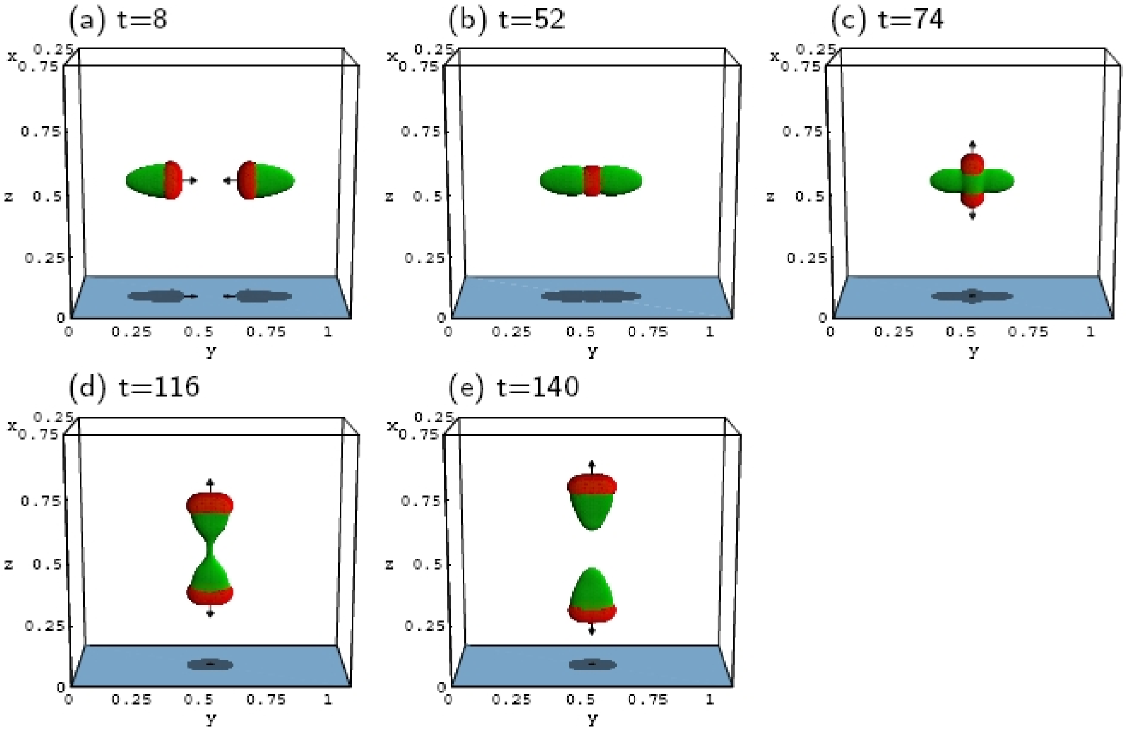

Fig. 9.32a-e

Numerical solutions of the 3-component FHN equation (1-3) in R^3 without global coupling in the case of the evolution of two travelling DSs both undergoing interpenetration in the course of a collision (a-e). The collision process leads to two travelling DSs with antiparallel speed vertical to the trajectories of the incoming DSs. The iso-surfaces of the fast activator *u* are indicated by red color and slow the inhibitor *v* by green. [Pu071] - compare to: experiment figs. 3.13, 4.19, 7.2; theory figs. 8.4, 9.8, 9.15, 9.16, 9.17, 9.29

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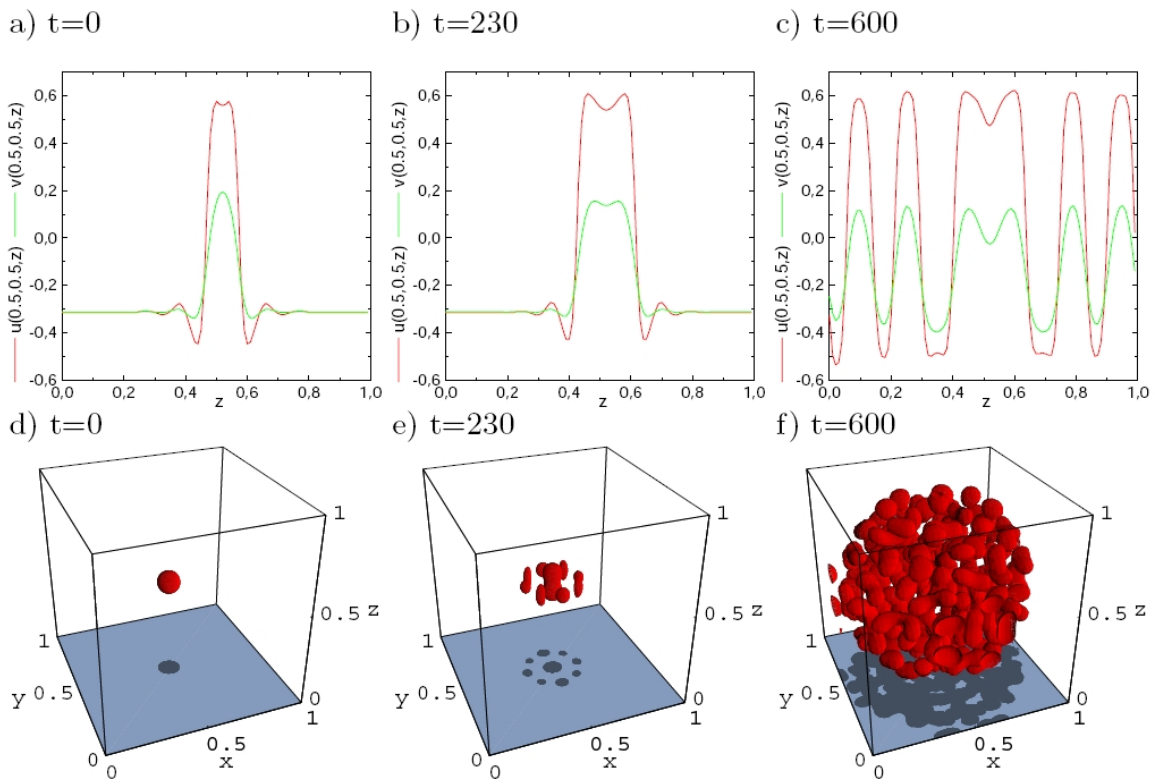

Fig. 9.33a-f

Numerical solutions of the 3-component FHN equation (1-3) in R^3 without global coupling in the case of self-completion. For appropriate parameters near to a Turing bifurcation a single LS with oscillatory tails is set as initial condition. The evolution of the cross sections of the *u* and *v* distributions are represented in (a-c) and in (d-f) the corresponding *u* distributions are represented in R^3. The LS is unstable with respect to dumb-bell like modes and in the courses of time more and more LSs are generated in the maxima of the tails of already existing one. In the final end the whole domain is filled with equidistant LSs. [Pu079] - compare to: experiment fig. 4.24; theory fig. 9.24

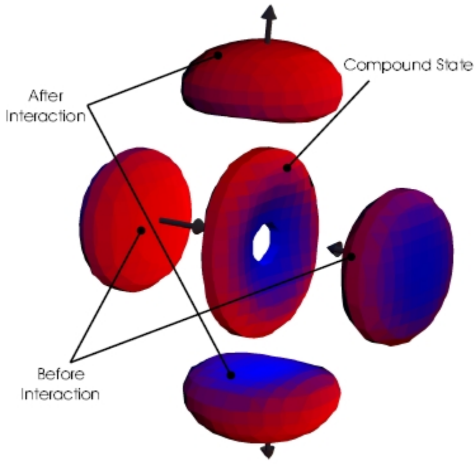

Fig. 9.34

Numerical solutions of the 3-component FHN equation (1-3) in R^3 without global coupling: in the course of a central collision of two DSs with interpenetration a torus is generated that in the present case decays into two DSs that escape vertically to the direction of the incoming DSs. [Pu079] - In other cases the decay of the torus may be related to annihilation or generation of DSs. [Pu079] - This is an interesting mechanism for the change of the number of DSs in the course of a collision process.

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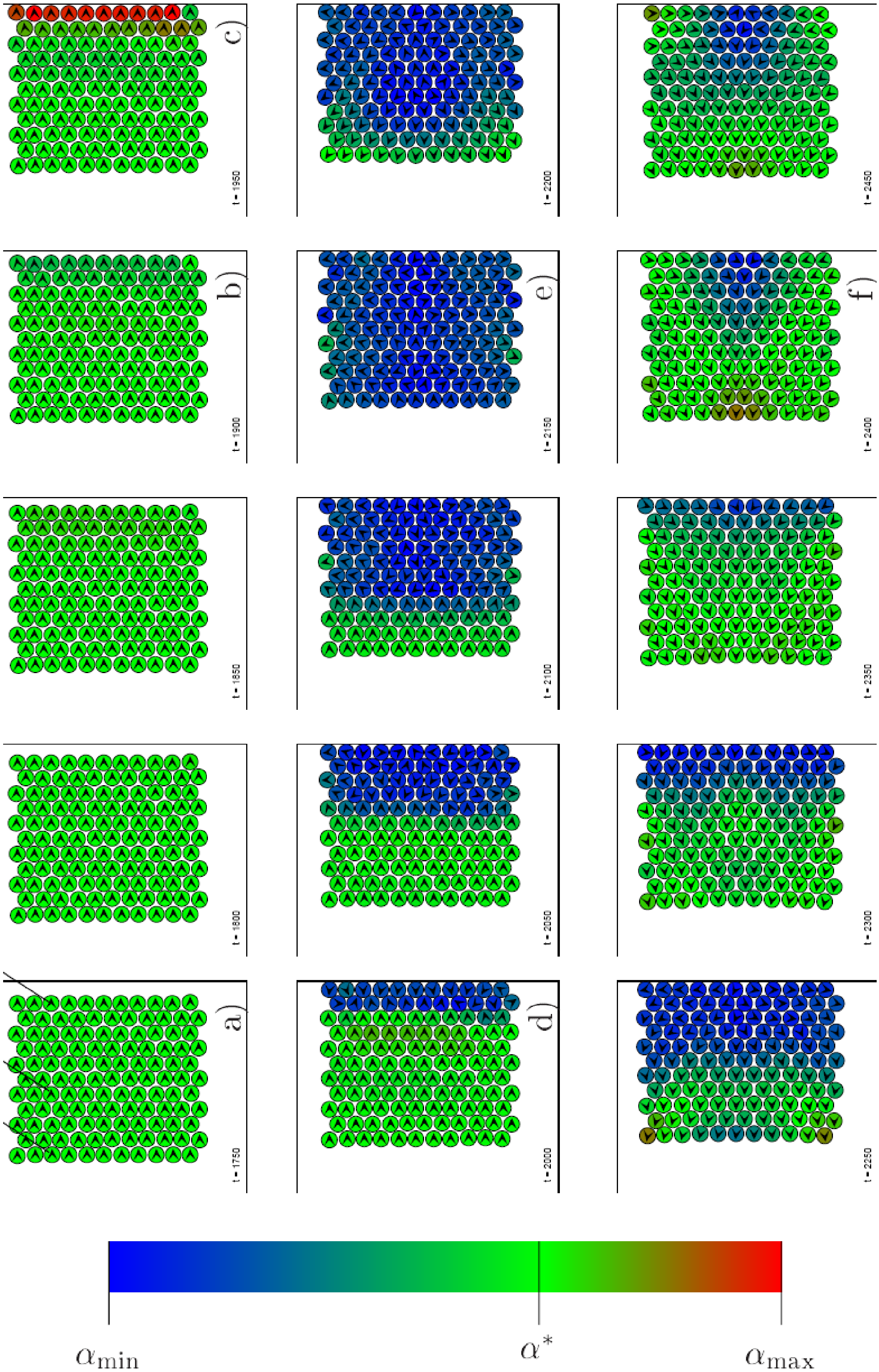

Fig. 9.35

Fig. 9.35

Numerical solutions of the particle equation (4,5) in R^2 in the case of a cluster of 121 DSs initially travelling in the same direction (a). The signs >, < within the individual spheres indicate the direction of motion of the individual DSs while the absolute of the speed scales with α^* and is coded by colour. Apparently a wave is generated that propagates in the cluster. [M. C. Röttger, "Numerische Untersuchungen zur reduzierten Dynamik dissipativer Solitonen in einem drei-komponentigen Reaktions-Diffusions-System", Diplom-Arbeit, University of Münster (2003)]

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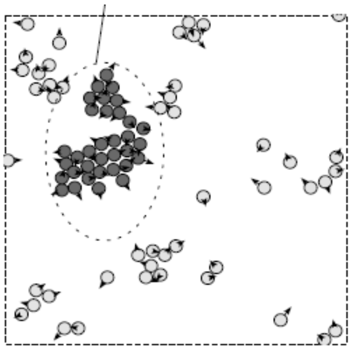
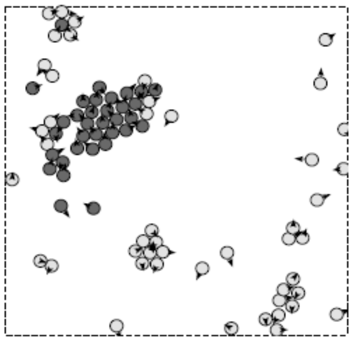
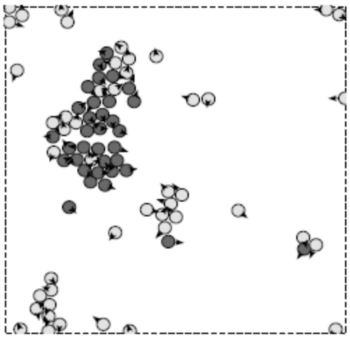
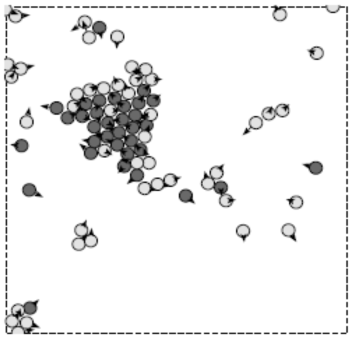


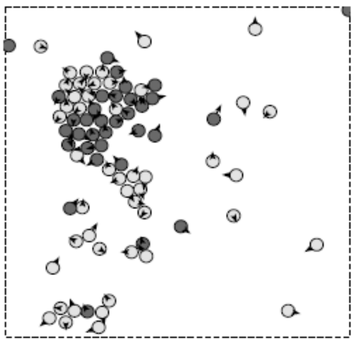


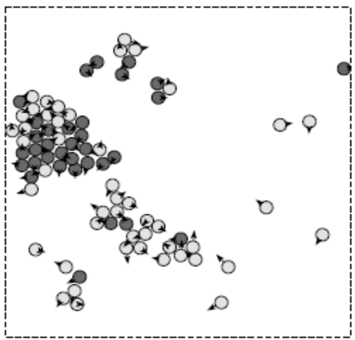
a) $t = 100000$ b) $t = 110000$



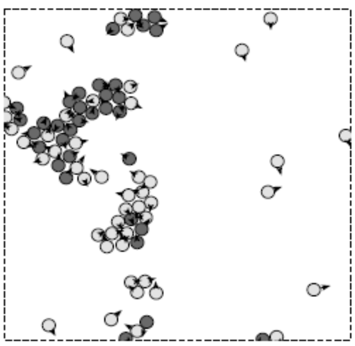


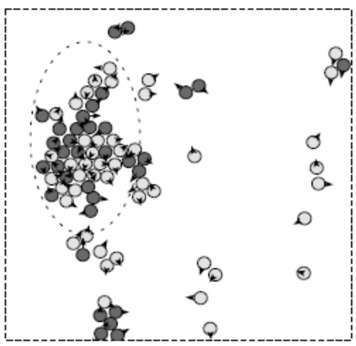
c) $t = 120000$ d) $t = 130000$





e) $t = 140000$ f) $t = 150000$





g) $t = 160000$ h) $t = 170000$

Fig. 9.36a-h

Fig. 9.36a-h

Evolution of an ensemble of 81 DSs obtained from the numerical solution of the particle equation (4,5) in R^2. Apparently, in the course of time (a-d) due to collision processes an equilibrium of clusters with more or less unbounded DSs is established. The arrows indicate the direction of motion of the DSs. The full circles indicate those DSs that initially where bound to the largest cluster. [M. C. Röttger, "Numerische Untersuchungen zur reduzierten Dynamik dissipativer Solitonen in einem drei-komponentigen Reaktions-Diffusions-System", Diplom-Arbeit, University of Münster (2003)] compare to: experiment figs. 4.26, 5.7, 5.8, 5.15

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pattern phenomenon	n-d FHN	1-d ENW	2-d ENW	1-d dc	2-d dc	2-d ac
stationary DS	1,2,3	x	x	x	x	x
stationary molecule	1,2,3				x	x
rotating molecule	2,3	-		-	x	
travelling molecule	1,2,3	x		x		
interaction: generation of DS	2,3			x	x	x
interaction: annihilation of DS	1,2,3	x		x	x	x
interaction: scattering of DS	1,2,3	ref		ref	x	x
stationary periodic stripes	(1),2	(x)	x	(x)	x	x
stationary hexagonal pattern	2	-	x	-	x	x
rotating spiral	2	-		-	x	x
Turing bifurcation	1,2	x		x	x	(x)?
bifurcation cascade for DSs	1,2,3	x	x	x	x	(x)
drift bifurcation of DSs	1,2,3				x	x
self-replication	1,2,3	x			x	
gas-, liquid-like state made of DSs	2			x	x	x
drift of periodic stripe pattern	1,2	x		x	x	x
drift/ rotating hex. pattern	2	-		-	x	x
defect: stripe/ hex. pattern		-		-	x	x

Fig. 9.37

Survey of some important solutions and phenomena obtained from the 3-component FHN equation (1-3) in n-dimensional space (n=1,2,3) and comparison to the corresponding experimental findings. The 2nd column gives the dimension of the space in which the FHN equation has been solved, the 3rd and 4th column refer to the 1- and 2-dimensional electrical networks, the 5th and 6th column to 1- and 2-dimensional dc gas-discharge systems, and the last column to quasi 2-dimensional ac gas-discharge systems. [Pu131] - The table could be extended also by other systems like certain chemical solutions, semiconductor devices and biological systems. In the columns 3 to 7 the sign x indicates experimental observation, the blanc fields denote the absence of experimental observation, ref means reflection and the sign --- denotes impossibility of patterns. - The table demonstrates the universal behaviour of a certain class of electrical transport systems that, with respect to pattern formation can be embedded in the much large class of reaction-diffusion systems.

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9.3 Listing of main results

With respect to the abbreviations used in the following listing of observed phenomena we refer to the Introduction.

Pu002: **Berkemeier, Dirksmeyer, Klempt, Purwins (1986)**
proof that analogue electronic circuits can be described quantitatively by r-d equations with respect to the formation of self-organized patterns - see also: Electrical Networks: Experiment and Theory

Pu003: **Radehaus, Kardell, Baumann, Jäger, Purwins (1987)**
isolated stationary and travelling LSs (DSs)
theo: 2-k + gc, R^2 - num: stat DS
bifurcation: Turing bifurcation and snaking
theo: 2-k + gc, R^2 - num: (stat hom) ↔ (stat period), Turing; snaking
periodic pattern in R^2
theo: 2-k + gc, R^2 – num: stat periodic
electrodynamic derivation of the 2-component FHN equation including global coupling - claim: many planar semiconductor devices can be considered as r-d systems with respect to lateral patter formation if it is in contact with an effective layer with S-shaped (current density)-(voltage) characteristic - claim: the high ohmic layer with monotonic (current density)-(voltage) characteristic supports spatially inhomogeneous patterns - stressing the importance of the activator inhibitor principle with local activation and lateral inhibition - first report of snaking - see also: Semiconductors: Experiment", Semiconductors: Theory

Pu005: **Radehaus, Dirksmeyer, Willebrand, Purwins (1987)**
isolated stat DSs
theo: 2-k + gc, R^1 - num: stat DSs (mentioned)
bifurcation: snaking
theo: 2-k + gc, R^1 - num: snaking (mentioned)
first claim, that the planar dc gas-discharge devices can be considered as r-d systems with respect to lateral patter formation - see also: DC Gas-Discharge Systems: Experiment

Pu006: **Baumann, Symanczyk, Radehaus, Purwins, Jäger (1987)**
filament diameter in good agreement with numerical results obtained from the 2-component FHN equation - see also: Semiconductors: Experiment, Semiconductors: Theory

Pu007: **Purwins, Klempt, Berkemeier (1987)**

isolated stationary DSs

theo: 2-k + gc, R^1 stat DS

periodic pattern in R^1

theo: 2-k, R^1, discrete - num: stat. nearly periodic

miscellaneous patterns

theo: 2-k , R^2 - num: hom oscillations; stat approximately centre symmetric circles

bifurcation: snaking

theo: 2-k + gc, $R^{1,2}$ – snaking

2-component FHN system as the basic model for the universal behaviour of a relatively large class of biological, chemical and physical systems – snaking in reaction-diffusion systems - see also: Electrical Networks, DC Gas-Discharge Systems: Experiment, Semiconductors: Theory

Pu008: **Purwins, Radehaus C. Radehaus, and J. Berkemeier (1987)**

isolated stationary DSs

theo: 2-k + gc, $R^{1,2}$ - stat DS

periodic pattern in R^1

theo: 2-k, R^1, discrete - stat. nearly periodic

bifurcation: snaking

theo: 2-k + gc, $R^{1,2}$ – snaking

see [Pu007] - detailed derivation of the 2-component FHN equation for the electronic network and the corresponding generalized 2-layer model - extended discussion of the basic reaction-diffusion model - see also: Electrical Network: Experiment and Theory s, DC Gas-Discharge Systems: Experiment

Pu009: **Purwins, Radehaus (1987)**

isolated stationary and travelling DSs

theo: 2-k + gc, R^1 - num: stat DSs

bifurcation: Turing

theo: 2-k , R^1 – anal: stat (hom) ↔ (stat periodic), supercritical Turing bif

periodic pattern in R^1

theo: 2-k , R^1 - anal: stat periodic

miscellaneous non LS (DS) patterns

theo: 2-k , R^2 - stat approximately centresymmetric circles

the quantitative theoretical description of the experimentally observed snaking in 1-dimensional electrical networks is performed without any fitting in [R. Schmeling, Thesis, University of Münster (1994); Purwins, Amiranshvili, Bödeker (2009)] by solving the corresponding FHN equation with 2 components - on the same basis the supercritical Turing bifurcation being detected on an electronic network is reproduced quantitatively including the correct scaling law - see also: Electrical Networks: Experiment and Theory

Pu011: **Purwins, Radehaus, Dirksmeyer, Dohmen, Schmeling, Willebrand (1989)**

isolated stationary DSs

theo: 2-k, $\mu \neq 0$ + gc , R^1 - num: stat DS

splitting of DSs

theo: 2-k, $\mu \neq 0$ + gc, R^1 – num: splitting

bifurcation: snaking

theo: 2-k, $\mu \neq 0$ + gc, R^1 – num: snaking

bifurcation: Turing

theo: 2-k , R^1 - anal: (stat hom) ↔ (stat periodic), supercritical Turing bif

periodic pattern in R^1

theo: 2-k , R^1 - anal: stat periodic

straight foreword description of stationary DSs and snaking using the FHN equation with 2 components - focus on the importance of the - activator-inhibitor principle for the investigated FHN equations and the formation of localized structures in physical systems - splitting of filaments while increasing the external driver is achieved by choosing $\mu \neq 0$ - see also: Electrical Networks: Experiment and Theory, DC Gas-Discharge Systems: Experiment

Pu013: Dirksmeyer, Schmeling, Berkemeier, Purwins (1990)

isolated stationary DSs

theo: 2-k + gc, R^1 – num: stat (bright) DS; stat (dark ,invers) DSs

bifurcation: snaking

theo: 2-k + gc, R^1 – num: snaking

bifurcation: Turing

theo: 2-k , R^1 – anal: stat (hom) ↔ (stat periodic), supercritical Turing bif (quant)

periodic pattern in R^1

theo: 2-k , R^1 - anal: stat periodic (quant); num: front travelling through a stat hom state leaving behind a stat periodic pattern , “Turing fronts” (quant)

discussion of the electrical network with respect to the introduction of “current diffusion” - first quantitative description of the experimentally observed stat dark (invers) DSs on 1-dimensional electrical networks without any fitting in [R. Schmeling, Thesis, University of Münster (1994)] by solving the corresponding FHN equation with 2 components - see remarks Pu009 - see also: Electrical Networks: Experiment and Theory

Pu017: Radehaus, Willebrand, Dohmen, Niedernostheide, Bengel, Purwins (1992)

bifurcation: Turing

theo: modified 2-k + gc , R^1 - anal: (stat hom) ↔ (stat periodic), supercritical and subcritical Turing bif

periodic pattern in R^1

theo: modified 2-k + gc , R^1- anal: period

treatment of the experimentally observed supercritical and subcritical Turing bifurcation in terms of a modified FHN equation with 2 component using the centre the manifold theory - see also: DC Gas-Discharge Systems: Experiment

Pu018: **Niedernostheide, Dohmen, Willebrand, Schulze, Purwins (1992)**

isolated stationary and travelling DSs

theo: 2-k + gc, R^1 - num: stat DSs, trav DSs

isolated breathing and pendulating LSs (DSs)

theo: 2-k, R^1 – num: breath DSs

theo: 2-k+ gc, R^1 – num: pend DSs

splitting of LSs (DSs)

theo: 2-k, $\mu \neq 0$ + gc, R^1 - num: splitting of DSs while increasing the external driver

bifurcation: snaking

theo: 2-k + gc, R^1 - num: snaking

bifurcation: Turing

theo: 2-k, R^1 - anal: (stat hom) ↔ (stat periodic), supercritical and subcritical Turing bif

theo: 2-k + gc, R^1 - num: (stat hom) ↔ (stat periodic), supercritical and subcritical Turing bif

periodic pattern in R^1

theo: 2-k, R^1 - anal: stat periodic

theo: 2-k + gc, R^1 - num: stat periodic

analytical construction of DSs (for details see [Pu024]) - see also: DC Gas-Discharge Systems: Experiment, Semiconductors: Experiment

Pu022: **Niedernostheide, Arps, Dohmen, Willebrand, Purwins (1992)**

isolated stationary and travelling DSs

theo: 2-k + gc, R^1 - stat, trav Ds

isolated pendulating DSs

theo: 2-k + gc, R^1 - pend DSs

bifurcation: complex dynamical bifscenario, snaking

theo: 2-k + gc, R^1 - bif scenario: (stat DS) ↔ (pend DS) ↔ (trav DS); snaking

reasonable qualitative description of the observed static and dynamic DS behaviour in p-n-p-n semiconductor devices in term of the FHN model with 2 components - see also: Semiconductors: Experiment

Pu023: **Willebrand, Niedernostheide,Dohmen, Purwins (1993)**

isolated stationary and travelling DSs

theo: 2-k, $\mu \neq 0$ + gc, R^1 – num: stat DS; num: simultaneous periodic annihilation of two trav DSs at opposite boundaries and simultaneous generation of a pair of counter propagating DSs in the centre; similar but irregular behaviour

isolated breathing and pendulating DSs

theo: 2-k , R^1 - num: breath

theo: 2-k, $\mu \neq 0$, R^1 + gc, - num: pend DS

splitting of DSs

theo: 2-k, $\mu \neq 0$ + gc, R^1 - num: stat DS splitting due to parameter change

bifurcation: snaking

theo: 2-k, $\mu \neq 0$ + gc , R^1 - num: snaking

bifurcation: Turing

theo: 2-k, R^1 - anal: (stat hom) ↔ (stat periodic) , supercritical Turing bif

summary - material partly contained in previous work - see also: DC Gas-Discharge Systems: Experiment

Pu024: **Dohmen, Niedernostheide, Willebrand, Purwins (1993)**

isolated stationary and travelling DSs

theo: 2-k + gc, R^1 - construction of stable stat DSs from interacting front-antifront pairs with oscillatory tails

fronts

theo: 2-k + gc, R^1 (piecewise linear nonlinearity) - stat front; front- antifront interaction due to oscillating tails

Pu025: **Niedernostheide, Dohmen, Willebrand, Kerner, Purwins (1993)**

isolated stationary DSs

theo: 2-k + gc, R^1 - num: stat DS

isolated breathing and pendulating DSs

theo: 2-k + gc, R^1 - num: breath DSs; pend DSs; num: DS with simultaneous dynamics of breath and pend

bifurcation: involving breathing and pendulating

theo: 2-k + gc, R^1 - bif diagrams including: (stat) ↔ (breath), (stat) ↔ (pend), (stat) ↔ (breath+pend), (pend) ↔ (breath+pend), (breath) ↔ (breath+pend); corresponding co-dimension 2 bif

writing the semiconductor specific model developed in [Pu020[in terms of the FHN equation with 2 components - stressing the similarity for semiconductor devices, electrical networks and gas-discharge systems with respect to lateral pattern formation- **see also: Semiconductors: Theory**

Pu026: **Willebrand, Or-Guil, Schilke, Purwins, Astrov (1993)**

isolated travelling DSs

theo: 2-k + gc, $\mu \neq 0$, R^1 - num: trav DS

interaction of DSs: reflection

theo: 2-k + gc, $\mu \neq 0$, R^1 - num: ref (repulsion) of trav DS at the boundary and at each other

solutions of the FHN equations with 2 components in 1-dimensional space reproduce experimentally observed reflection of travelling DSs at the boundaries, periodic reflection of two DSs at each other and irregular motion of several DSs trying to avoid each other - **see also: DC Gas-Discharge Systems: Experiment**

Pu027: **Heidemann, Bode, Purwins (1993)**

fronts

theo: 2-k , R^1 - front travelling through a hom oscillating state leaving behind a stat periodic pattern ("Turing fronts") and vice versa ("Hopf-front") (semi-quant); derivation of a pair of complex Ginzburg-Landau (GL) equations in the vicinity of the codimension-2 bif where: (stat hom), (stat periodic), (hom oscillations) meet; quant agreement between numerical solutions of r-d and GL equation; analysis of font-phase diffusion interaction

theoretical analysis of "the propagation of "Turing"- and "Hopf-fronts" - detection of the interplay of front propagation and phase diffusion - **derivation of a pair of complex G-L equations near to the codimension-2 point where the following states meet: stat hom, stat periodic, hom oscillations - see also: Electrical Networks: Experiment and Theory**

Pu032: **Bode, Reuter, Schmeling, Purwins (1994)**

fronts

theo: 2-k, R^1 - anal, bif: (uni-directional front propagating) ↔ (bi-directional front propagation), (quant)

quantitative analytical description of bi-directional front propagating and of the corresponding bifurcation behaviour - see also: Electrical Networks: **Experiment and Theory**

Pu041: **Schütz, Bode, Purwins (1995)**

fronts

theo: 1-k, R^1 + local inhom - anal expression for front speed near to pinning: front pinning

theo: 2-k, R^1 + local inhom - anal expression for front speed near to pinning: front pinning, ref, local oscillation

bifurcation

theo: 1-k, R^1 + local inhom - saddle-node bif and corresponding normal form

theo: 2-k, R^1 + local inhom - saddle-node Takens-Bogdanov, symmetric Takens-Bogdanov bif, corresponding normal forms

analytical description of the interaction between fronts and a local inhomogeneities including stationary front pinning, local oscillations and reflection

Pu043: **Bode, Purwins (1995)**

isolated stationary and travelling DSs

theo: 2-k, R^1 - stat, trav, anal: construction of mutually interacting stat front- antifront DS assuming the existence two stab stat hom states, basis for the interaction of front-antifront DSs with impurities; generalization to systems with one stab stat hom state, basis for the concept of a description of DSs in terms of "centre of mass" coordinates (particle description)

isolated breathing and pendulating DSs

theo: 2-k, R^1 + gc - anal: basis for breathing DSs, pend DSs

interaction of DSs: formation of molecules

theo: 2-k, R^1 - anal: interaction of front-antifront DSs (molecule formation), generalization to systems with one stab stat hom state

fronts

theo: 2-k, R^1 - anal: stat and trav fronts, oscillating fronts, front speed for hom systems and inhom ones, front impurity interaction, front-front interaction

bifurcation DSs including snaking

theo: 2-k, R^1 + gc - snaking; importance of gc for snaking

this paper is a milestone because it summarizes the continuous improvement of the analytical description of fronts and DSs in the investigated r-d systems - it clearly stresses the fundamental approach that emerges from [Pu32, Pu41] and which is presented systematically in [M. Bode, "Beschreibung strukturbildender Prozesse in eindimensionalen Reaktions-Diffusions-Systemendurch Reduktion a Amplituden-gleichungen und Elementarstrukturen", Thesis, University of Münster (1992)] - the new concept describes stationary fronts or DSs in homogeneous and rotational symmetric systems where the destabilization is due to the excitation of a dynamical degree of freedom that is related to the appearance of a positive (real) eigenvalue when driving some parameter beyond the bifurcation point - the crucial point is that at the bifurcation point the degeneracy of the corresponding eigenstate with the symmetry eigenmode (e.g. the Goldstone mode) leads to one ordinary eigenfuction and a degenerate one, both of which are the starting point for the perturbation theory - in the latter approach the symmetry mode (e. g. the Goldstone mode) is taken account of by introducing some speed that reflects the motion of the front or DS (e.g. the translational motion) - at the same time the outlined approach is the basis for a particle description of DSs near to the bifurcation from a stationary to a to a dynamical state (travelling bifurcation) - **see also: Electrical Networks, "DC Gas-Discharge Systems: Experiment**

Pu045: Kulka, Bode, Purwins (1995)

fronts

theo: 1-k, R^1 - anal: front speed near to pinning in the presence of inhomogeneities (quant)

analytical determination of the speed of a front under the influence of an inhomogeneity using the concept of front speed combined with multiple scale techniques, the latter being decribed in [Pu041, Pu043;M. Bode, Thesis, University of Münster (1992)] - quantitative agreement between theory and experiments on electrical networks - remarks: 1. on logic AND operation elements realized by propagating pulses on appropriate transmission lines, 2. on measuring time dependent correlation via propagating fronts being important e.g. for velocity filtering in the field of visual information processing, 3. on the extension of the concept of front speed to 2-component systems allowing for local oscillations of the front around an inhomogeneity - see also: Electrical Networks: Experiment and Theory

Pu046: **Or-Guil, Ammelt, Niedernostheide, Purwins (1995)**

isolated stationary LSs

theo: 2-k + gc, R^2 - num: stat LSs with oscillatory tails

interaction of LSs (DSs): formation of molecules

theo: 2-k , R^2 - num: stat n-DSs molecules

theo: 2-k + gc, R^2 - num: stat n-LSs

hexagonal patterns in R^2

theo: 2-k , R^2 - num: stat hex pattern made of LSs

fronts

theo: 2-k , R^2 - num: generically trav fronts

theo: 2-k + gc, R^2 - num: stat fronts

waves: target pattern, rotating spirals

theo: 2-k+ gc , R^2 - target pattern (repeatedly outrunning centro-symmetric waves);

theo: 2-k, R^2 - rotating spirals and moving fractions of the former; spiral defect chaos

many body LS- (DS-)systems

theo: 2-k + gc, R^2 - num: stat hexagons consisting of LSs ; LSs arrange in several large clusters with hex symmetry inside, cluster size increases with increasing driver κ_1

miscellaneous patterns

theo: 2-k , R^2 - hom oscillations

bifurcation: Turing, Hopf, snaking

theo: 2-k, R^2 - (stat hom) ↔ (stat hex LS pattern), subcritical Turing bif; (stat hom) ↔ (oscillating hom), Hopf bif.

theo: 2-k + gc, R^2 - snaking, LSs arrange in several large clustern with hex symmetry inside, cluster size increases with increasing driver κ_1; (stat hom) ↔ (expending target pattern)

various numerical solution of the FHN equation with 2 components in 2-dimensional space - particular emphasis on global coupling, the latter favouring snaking, it also favours expanding target patterns with respect to homogeneous oscillations - comparison with experimental results obtained from planar ac gas-discharge systems and ac driven ZnS:Mn films - see also: AC Gas-Discharge Systems: Experiment, Semiconductors: Experiment

Pu048: **Woesler, Schütz, Bode, Or-Guil, Purwins 1996**

essentially an analytical stability analysis for fronts in a 3-component FHN systems

Pu053: **Schenk, Or-Guil , Bode, Purwins (1997)**

isolated stationary and travelling LSs (DSs)

theo: 3-k , R^2 - num: stat, trav DS of any number

self completion based on LSs (DSs)

theo: 3-k , R^2 - num: formation of a moving hexagonal pattern made of DSs

interaction of LSs (DSs): scattering

theo: 3-k , R^2 - num: scattering including interprenetation

generation, annihilation of LSs (DSs): due to interaction

theo: 3-k , R^2 - num: an of 1 or 2 DSs in the course of a collision of 2 DSs

for the first time a 3-component FHN system is introduced in order to realize an arbitrary number of stable isolated travelling DSs with mutual

interaction in $R^{1,2,3}$ - individual stationary and travelling DSs as well as scattering, annihilation and generation are observed numerically - discussion of intermediate states appearing in the course of DS interaction

Pu060: **Or-Guil, Bode, Schenk, Purwins (1998)**

isolated stationary and travelling LSs (DSs)

theo: 3-k , $R^{1,2,3}$ - stat, trav DSs

bifurcation of LSs (DSs) including snaking

theo: 3-k , $R^{1,2,3}$ - anal: supercritical bif (stat) ↔ (trav DSs) , scaling law, normal form

analytical investigation of the 3-component FHN equation in $R^{1,2,3}$ near to the bifurcation from of stationary DS to a travelling one using multiple scale perturbation theory - stability analysis for stationary and travelling DSs - derivation of the normal form of the bifurcation containing the centre coordinate of the involved DS as dependent variable; this equation represents also a dynamical equation for a single DS in the particle picture - analytical expression for the speed of propagation - an essential ingredient of the presented theory is the fact that in the treated bifurcation scenario a discrete real eigenvalue crosses the imaginary axis; together with the symmetry modes this leads to the generation of generalized eigenvectors at the bifurcation point which in turn are responsible for the onset of propagation

Pu063: **Ammelt, Astrov, Purwins (1998)**

stripes in R^2

theo: 2-k - stat stripes

hexagons in R^2

theo: 2-k - stat hex pattern; hex defect structures

bifurcation

theo: 2-k - (stat hom) ↔ (stat hex) ↔ (stat stripes)

qualitative reproduction of the experimental findings - see also: **AC Gas-Discharge Systems: Experiment**

Pu062: **Schenk, Schütz, Bode, Purwins (1898)**

isolated stationary and travelling DSs

theo: 2-k , R^2 - non-oscillatory and oscillatory tails of individual stat DSs, stability

interaction of LSs (DSs): formation of molecules

theo: 2-k , R^2 - anal: interaction law for any number of mutually interacting DSs in terms of relative speed presenting a "particle" description; interaction with the boundary; formation of stat n-DSs ; num: stat DSs, comparison of speed to results from anal (quant), formation of stat n-DSs and stability

theo: 3-k , R^2 - general conclusion: by adding a third component it is always possible to make the molecules travel

analytical description of the interaction of stationary DSs and related formation of n-DS patterns FHN equations with 2 and 3 components in R^2 - derivation of particle equations in terms of ordinary differential equations describing the dynamics of the centre of mass of the interacting DSs - formation of stable stationary n-DS patterns as solutions of the 2-component system - formation of stable stationary and travelling n-DS patterns as solutions of the 2-component system

Pu071: Schenk, Liehr, Bode, Purwins (2000)

isolated stationary and travelling DSs

theo: 3-k , R^3 - num, anal: stat, trav DSs, speed in dependence of time constant

interaction of DSs: scattering and reflection

theo: 3-k , R^3 - comparison num solution field equation, num solution particle equation: scattering in the course of weak interaction; speed in dependence of relaxation time, godd agreement - num field equation: head-on collision of counter propagating DSs with merging, formation of a torus, subsequent emission of two counter propagation DSs vertical to the incoming ones

generation, annihilation of DSs: due to interaction

theo: 3-k , R^3 - num field equation: disappearance of two trav DSs after their collision

bifurcation of DSs

theo: 3-k, R^3 - (stat DS) ↔ (trav DS), drift bifurcation, good agreement between results from num field equation and num particle equation

many body DS systems

theo: 3-k, R^3 - in the absence of a global coupling many DSs can exist simultaneously

first calculation of the propagation and interaction of DS in 3-dimensional space based on solution of the 3-component FHN equation - numerical solutions for propagation of isolated DSs, for scattering at each other and for their annihilation - presentation of the ordinary differential equation containing the centre coordinates of interacting DSs describing the propagation and weak interaction of DSs near to the travelling bifurcation, see in particular [Pu084] - discussion of intermediate state appearing in the course of DS interaction

Pu079: Liehr, Bode, Purwins (2001)

isolated stationary and travelling DSs

theo: 3-k , R^3 - num: stat, trav DSs

self completion based on DSs

theo: 2-k + gc, R^2 - num self completion scenario just beyond the subcritical Turing bif possibly ending up with partial filling of the domain because of gc, generation of new LSs in the oscillatory tails of already existing ones

theo: 3-k, R^3 - num self completion scenario just beyond the subcritical Turing bif: generation of new LSs in the oscillatory tails of already existing ones; num self completion scenario: generation of new LSs in the oscillatory tails of already existing ones due to dumbbell-like deformation of existing LSs

interaction of DSs: scattering

theo: 3-k , R^3 - num field equation, num particle equation: scattering (quant); num: head-on collision of counter propagating DSs, formation of a torus, subsequent emission of two counter propagation DSs vertical to the incoming ones

generation, annihilation of LSs (DSs): due to interaction

theo: 3-k , R^3 - num field equation, gen of DSs in the course of collision of two DSs: gen of additional DSs in oscillatory tails or formation of a torus with subsequent disintegration into more than two DSs; num field equation, an of DSs in the course of collision of two DSs: an of one or both DSs

investigation of the mechanisms of DS generation in the context of self-completion in $R^{2,3}$ in the 3-component FHN equation, discussion of intermediate state appearing in the course of DS interaction - oscillatory tails of DSs are responsible for self-completion and generation of additional DSs in the course of DS collision - detection of formation of dumb-bell like deformations being responsible for self-completion and generation of additional DSs in the course of DS collision

Pu083: **Purwins, Astrov, Brauer, Bode (2001)**

summary - comparison of the solutions of the 2- and 3-component reaction-diffusion system with previous experimental results on planar gas-discharge systems

Pu084: **Bode, Liehr, Schenk, Purwins (2002)**

isolated stationary and travelling DSs

theo: 3-k , $R^{2,3}$ - num: stat, trav DSs

interaction of DSs: scattering, reflection and pinning at inhomogeneities

theo: 3-k , R^2 - comparison num, anal: scattering of two DSs at each other (quant); num: head-on collision of counter propagating DSs with merging and subsequent emission of two counter propagation DSs vertical to the incoming DSs

theo: 3-k , R^2 + inhom – comparison num, anal: scattering of a single DS (quant); num: collision of one propagating DSs with the inhomogeneity, formation of a stable local circular motion of a single DS and emission of a second one

theo: 3-k , R^3 - num, anal: scattering(quant); num: head-on collision of counter propagating DSs with merging, formation of a torus, and subsequent emission of two counter propagation DSs vertical to the incoming ones

interaction of DSs: formation of molecules

theo: 3-k , R^2 - anal: stat, trav, rot n-DSs

generation, annihilation of DSs: due to interaction or spontaneously

theo: 3-k , R^2 - num: collision of two propagating DSs under some angle with merging and subsequent emission of one DS

theo: 3-k , R^3 - num: head-on collision of counter propagating DSs with merging, formation of a torus, and subsequent emission of more than two propagating DSs; num: head-on collision of counter propagating DSs that both disappear

straight forward extension of [Pu060]: including DS interaction in $R^{2,3}$ as solution of the 3-component FHN equation - analytical treatment of weak perturbations of stationary and travelling DSs in $R^{2,3}$ which includes interaction with each other, with the boundary and with inhomogeneities - reduction of the field equation to a particle equation with an additive interaction term containing the position coordinates of the involved DSs as dependent variables thereby representing the theoretical foundation of the particle concept for DSs in the presence of interaction - oscillatory interaction terms can lead to n-DS patterns - the particle equations, for the first time allow for a description of DS many systems provided the number of particles is conserved (weak interaction) - description of the following phenomena in $R^{2,3}$ using the particle equations in the absence of inhomogeneities: stationary and travelling DSs, scattering,

formation of stationary, travelling and rotating n-DS patterns - corresponding description in the presence of inhomogeneities: pinning and localized oscillations as well as scattering of DSs - in addition as numerical solutions of the 3-component system one discusses: annihilation and generation in the absence and in the presence of inhomogeneities and various complex phenomena related to DS interaction - discussion of intermediate states appearing in the course of DS interaction

Pu092: **Liehr, Moskalenko, Röttger, Berkemeier, Purwins (2002)**

isolated stationary and travelling DSs

theo: 3-k, R^2 - num: trav DSs

theo: 3-k, R^3 - num: trav DSs

interaction of DSs: formation of molecules

theo: 3-k, R^2 - num: trav 2-DS, trav 3-DS

generation, annihilation of DSs: due to interaction or spontaneously

theo: 3-k, R^2 - num: the collision four trav DSs results in the gen of a fifth one forming a 2-DS and 3-DS pattern

theo: 3-k, R^3 - num: the collision three trav DSs results in the gen of a fourth one

generation of DSs in $R^{2,3}$ as solution of the 3-component FHN equation - discussion of intermediate states appearing in the course of DS interaction

Pu098: **Bödeker, Röttger, Liehr, Frank, Friedrich, Purwins (2003)**

Pu099: **Liehr, Bödeker, Röttger, Frank, Friedrich, Purwins (2003)**

isolated stationary and travelling LSs (DSs)

theo: 3-k, R^2 - stat, trav DSs

interaction of DSs: scattering and reflection

theo: 3-k, R^2 - stat, trav DSs; num: DSs with non-oscillatory and oscillatory tails and corresponding interaction laws; interaction of two anti-parallel moving DSs

bifurcation of DSs : travelling bif

theo: 3-k, R^2 - (stat DS) ↔ (trav DS)

repetition of material from [Pu084]: presentation of the 3-component FHN system, particle description near to the travelling bifurcation point, normal form for the travelling bifurcation - setting up the Langevin equation by considering the experimentally observed DSs as particles - development of a new stochastic data analysis method to extract from the experimentally determined stochastic trajectories the intrinsic speed - good agreement between the theoretical and the experimental scaling law for the speed in the vicinity of the travelling bifurcation point **- see also:** **AC Gas-Discharge Systems: Experiment**

Pu100: **Moskalenko, Liehr, Purwins (2003)**

isolated stationary and travelling LSs (DSs)

theo: 3-k, R^2 - stat, trav DSs

interaction of LSs (DSs): formation of molecules

theo: 3-k, R^2 - stat, trav 2-DS

bifurcation of DSs: trav/rot bif

theo: 3-k, R^2 - anal: (stat 2-DS) ↔ (trav 2-DS) or (rot 2-DS)

analytical treatment of the 3-component FHN system in R^2 near to the transition of a non-spherical DS from a stationary to a travelling or a

rotating DS - **critical parameter for a change from the travelling to the rational bifurcation - the square of the drift velocity or the square of the angular velocity scale lineárely with the bifurcation parameter**

Pu101: **Gurevich, Bödeker, Moskalenko, Liehr, Purwins (2003)**
isolated stationary and travelling LSs (DSs)
theo: 3-k, $R^{2,3}$ - stat, trav DSs
bifurcation of DSs: trav bif
theo: 3-k, R^2 - anal: (stat DS) ↔ (trav DS), drift bifurcation
analytical treatment of the 3-component FHN system in $R^{2,3}$ near to the transition of a stationary DS to a travelling DS due to a change of the shape of the DS

Pu105: **Liehr, Moskalenko, Purwins (2003)**
material also contained in [Pu100]

Pu107: **Gurevich, Liehr, Amiranashvili, Purwins (2004)**
according to the FHN model with 2 components for dc gas-discharge systems one variable is the current density (activator) and the other one the voltage drop at the high ohmic layer (inhibitor) - justification of the third component in the 3-component FHN system: by comparing experimental results for the surface charge and related experimental properties with estimates from theory it is demonstrated that it is reasonable to consider the surface charge at the (high ohmic layer)-(gas discharge space) interface as the second inhibitor that has been introduced in [Pu053] when setting up the 3-component FHN system

Pu109: **Liehr, Moskalenko, Astrov, Bode, Purwins (2004)**
isolated stationary and travelling DSs
theo: 3-k, R^2 - stat, trav DSs
interaction of DSs: formation of molecules
theo: 3-k, R^2 - stat, rotating 2-DS (molecule); formation of a 2-DS by collision of two trav DSs, well defined distance of molecule depends on impact parameters
bifurcation of DSs
theo: 3-k, R^2 - (stat2-DS) ↔ (rotating 2-DS)
discussion of the 3-component FHN equation - solving directly the field equation and by solving the particle equation: reproduction of the experimentally observed rotating pair of two DSs (molecule); good quantitative agreement between the results from the field and the particle equations - good quantitative agreement also for the study case of two DSs that approach each other in 2-dimensional space and that interact and finally form the stable rotating molecule

Pu110: **Bödeker, Liehr, Röttger, Frank, Friedrich, Purwins (2004)**
interaction of DSs: scattering and reflection
exp: 2d-dc-GDS, $R_0 \neq 0$ - determination of the stochastic trajectories of two interacting DSs
theo: 3-k, R^2 - num: DSs with non-oscillatory and oscillatory tails, corresponding interaction laws
starting from the 3-component FHN system and the particle description near to the travelling bifurcation point - setting up the Langevin equation

in the particle picture (see [Pu098,Pu099]) – extending the Langevin equation to interacting DSs - development of a new additional stochastic data analysis method to extract from the experimentally determined stochastic trajectories the law for the mutual interaction of two DSs - good consistency with other data is obtained for the derived interaction law - see also: AC Gas-Discharge Systems: Experiment

Pu111: **Costello, Adamatzky, Ratcliffe, Zanin, Liehr, Purwins (2004)**
self-organized Voronoi diagrams from Pu090 are put into relation to corresponding solutions the 2-component FHN system in R^2 with two stable stationary hom states - front propagation such that a high activator state extends, initial values such that at predefined positions (points in the later Voronoi net) a relatively high activator values is set - propagating fronts come to a rest at lines that define the Voronoi net - great similarity to results on experimental system - see also: AC Gas-Discharge Systems: Experiment

Pu113: **Gurevich, Bödeker, Moskalenko, Liehr, Purwins (2004)**
isolated stationary and travelling LSs (DSs)
theo: 3k, R^2 - stat, trav DS
bifurcation of DSs
theo: 3k, R^2 - (stat DS) ↔ (trav DS),
determination of the intrinsic speed of an isolated DS from its stochastic trajectory by following the procedure of [Pu098, Pu099] - in contrast to the case of [Pu098, Pu099] the change of shape of the DS is essential for the supercritical travelling bifurcation - derivation of the scaling law - see also: DC Gas-Discharge Systems: Experiment

Pu114: **Purwins, Bödeker, Liehr (2005)**
summary - comparison of the experimental results on quasi 2-dimensional dc gas-discharge systems with solutions of the FHN equation with 2 and 3 components - see also: DC Gas-Discharge Systems: Experiment

Pu118: **Purwins, Bödeker, Liehr (2005)**
review of previous works on DSs in FHN systems - mechanisms of pattern formation in 2-component systems: Turing patterns, localized structures, activator-inhibitor principle - derivation of the 2- and 3-component FHN equation by starting from an electronic equivalent circuit - numerical solutions of the 3-component FHN equation for isolated DS: DSs with non-oscillatory and oscillatory tails - numerical solutions of the 3-component FHN equation for two interacting DSs: scattering, formation of rotating molecules, annihilation, generation - analytical treatment of the 3-component FHN equation: normal form for the drift bifurcation, reduction of the field equation to a particle equation for a single particle and for mutually interacting particles, calculation of the particle interaction law - numerical and analytical work: rotating molecule in R^3 propagating along the axis of rotation - experimental results on DSs in dc gas-discharge: isolated DSs with non-oscillatory and oscillatory tails, drift bifurcation, interaction law, generation and annihilation - see also: Electrical Networks: Experiment and Theory, AC Gas-Discharge Systems: Experiment

Pu126: **Gurevich, Amiranshvili, Purwins (2006)**

isolated stationary and travelling DSs

theo: 3-k, R^2 - stat, DSs

isolated breathing DSs

theo: 3-k, R^2 - breath DSs

bifurcation of DSs

theo: 3-k, R^2 - anal: (stat DS) ↔ (breath DS), normal form being of the Hopf type, determination of the coefficients

analytical and numerical treatment of the 3-component FHN system in R^2 near to the transition of a stationary DS to a breathing one - evaluation of the corresponding normal form - good agreement between analytical and numerical results

Pu127: **Purwins, Amiranashvili (2007)**

summary - considering simple patterns: e.g. isolated DS, stripes, hexagons and rotating spirals - considering patterns of higher complexity with DSs as elementary building blocks: e.g."molecules" and "many body systems" in the form of crystal-, liquid-, gas-like arrangements, chains and nets - universal experimental behaviour for a certain class of systems containing: planar ac and dc gas-discharge systems, electrical networks, semiconductor layer systems, chemical solutions and biological systems - theoretical definition of the corresponding universality class: writing down a 3-component reaction-diffusion system serving as a kind of normal form for the qualitative description of the experimentally observed self-organized patterns - illustration of the formation of DSs in planar electrical transport systems on the basis of the 2-component reaction diffusion equation - see also: Electrical Networks: Experiment and Theory, DC Gas-Discharge Systems: Experiment, AC Gas-Discharge Systems: Experiment, Gas-Discharge: Theory, Semiconductors: Experiment, Semiconductors: Theory

Pu128: **Stollenwerk, Gurevich, Laven, Purwins (2007)**

isolated stationary and travelling DSs

exp: 2d-ac-GDS - intrinsically stat, or slowly drifting bright (on a dark background) and dark DSs (on a bright background); simultaneous existence of the former DSs with bright and dark periodic stripes respectively

theo: 2-k + gc, R^2 - qualitative reproduction of the experimental phenomena

stripes in R^2

exp: 2d-ac-GDS - periodic stripes

theo: 2-k + gc, R^1 - qualitative reproduction of the experimental phenomena

many body DS systems

exp: 2d-ac-GDS - dense arrangement of DSs

theo: 2-k + gc, R^1 - qualitative reproduction of the experimental phenomena

miscellaneous patterns

exp: 2d-ac-GDS - dense arrangement of DSs and stipes on the same domain;

theo: 2-k + gc, R^1 - qualitative reproduction of the experimental phenomena

bifurcation of DSs including snaking

exp: 2d-ac-GDS - increasing the driving frequency: (stat hom dark) ↔ (isolated bright DS) ↔ (dense arrangement of bright DSs) ↔ (simultaneous existence of dense arrangement of bright DSs together with periodic bright stripes) ↔ (periodic bright stripes/ periodic dark stripes) ↔ (simultaneous existence of dense arrangement of dark DSs together with periodic dark stripes) ↔ (simultaneous existence of dense arrangement of dark DSs together with dark periodic stipes) ↔ (dense arrangement of dark DSs) ↔ (isolated dark DSs) ↔ (stat hom bright)

theo: 2-k + gc, R^1 qualitative reproduction of the experimental phenomena

by the solutions of the 2-component reaction-diffusion system qualitative reproduction of the following experimentally observed bifurcation scenario: (stat hom dark) ↔ (isolated bright DS) ↔ (dense arrangement of bright DSs) ↔ (simultaneous existence of dense arrangement of bright DSs together with periodic bright stripes) ↔ (periodic bright stripes/ periodic dark stripes) ↔ (simultaneous existence of dense arrangement of dark DSs together with periodic dark stripes) ↔ (simultaneous existence of dense arrangement of dark DSs together with dark periodic stripes) ↔ (dense arrangement of dark DSs) ↔ (isolated dark DSs) ↔ (stat hom bright)

Pu131: **Purwins (2007)**

summary - stressing that the formation of DSs in planar low temperature dc and ac gas-discharge systems is a generic phenomenon - stressing that in many respect DSs behave like particles - illustration of the formation of DSs in planar electrical transport systems on the basis of the FHN equation with 2 components - considered experimentally observed phenomena as there are: isolated DS, snaking, bifurcation from stationary to travelling DSs, mutual interaction of DSs with scattering, "molecule" formation, generation and annihilation as well as "many body systems" in the form of crystal-, liquid-, gas-like arrangements, domain structures and chains and nets - pointing out that the 3-component FHN system seems to present a kind of normal form for the qualitative description of self-organized patterns in the discussed gas-discharge systems - listing of potential applications - see also: DC Gas-Discharge Systems: Experiment, AC Gas-Discharge Systems: Experiment, Gas-Discharge: Theory