# 6.
# Gas-Discharge: Theory

**(references refer to the list of publications given in chapter 12)**

## 6.1 General remarks

**The modeling of pattern formation in the investigated experimental dc (see chapter DC Gas-Discharge Systems: Experiment) and ac systems ( see chapter AC Gas-Discharge Systems: Experiment) is done in two ways. Qualitatively all important patterns can be described in terms of the 3-component reaction-diffusion equation that has been derived on the basis of an electronic equivalent in the chapter A Model for Pattern Formation. For this equation numerical and analytical solutions are reported in the chapters Electrical Networks: Experiment and Theory and Reaction-Diffusion Equations. A modified version of this approach is presented in [Pu017]. The success of the reaction-diffusion model approach is amazing in particular for the ac systems because in chapter A Model for Pattern Formation only a dc driver has been considered.**

**The quantitative modeling has been performed within the scope of the drift diffusion approach plus the Poisson equation assuming 2 charge carriers, and among other things neglecting gas heating, plasma chemistry, field emission and surface effects. The work has been performed in close collaboration with P. Boeuf and his research group at the CNRS at Toulouse. Hereby one follows the former work on modeling low temperature gas-discharge. [C. Punset, S. Cany, and J. P. Boeuf, J. Appl. Phys. 86, 124 (1999)]. The model equations read as**

$$\partial_t n_e + div\,\vec{\Gamma}_e = S_e, \quad (1)$$
$$\partial_t n_p + div\,\vec{\Gamma}_p = S_p, \quad (2)$$
$$\Delta\varphi = \frac{-|e|}{\varepsilon_0}(n_p - n_e), \quad (3)$$

**with**

$$\vec{\Gamma}_e = -\mu_e(E)\, n_e \vec{E} - D_e(E)\, \Delta n_e,$$
$$\vec{\Gamma}_p = \mu_p(E)\, n_p \vec{E} - D_p(E)\, \Delta n_p,$$
$$S_e = S_p = \alpha(E)\, j_e / |e|,$$
$$\vec{E} = -\vec{\nabla} \varphi$$

**and**

$$D_{e,p} = k T_{e,p} \mu_{e,p}(E)/|e|, \quad j_e = -|e| \vec{\Gamma}_e.$$

**In these equations the symbols have the following meaning**

| | |
|---|---|
| $n_{e,p}$ | **electron/ion density** |
| $\vec{\Gamma}_{e,p}$ | **electron/ion particle current density** |
| $S_{e,p}$ | **electron/ion source term** |
| $\varphi$ | **electrical potential** |
| $e$ | **elementary charge** |
| $\varepsilon_0$ | **dielectric constant** |
| $\mu_{e,p}$ | **electron/ion mobility** |
| $\alpha$ | **electron impact ionization coefficient** |
| $\vec{E}$ | **electrical field** |
| $D_{e,p}$ | **electron/ion diffusion constant** |
| $T_{e,p}$ | **absolute temperature of electrons/ions** |
| $j_e$ | **local electron current density** |

**We note that the ordinary electrical current density can be written as**

$$\vec{j} = |e|(\vec{\Gamma}_p - \vec{\Gamma}_e).$$

**The equations (1-3)  have been solved numerically in 2- (3-) dimensional space with 1coordinate being oriented vertical to the electrodes and 1 (2) coordinate laying in the discharge plane, respectively.**

# 6.2 Graphical representation of selected results

**The following is a series of figures reflecting main numerical solutions of equation (1-3) that have been obtained in relation to the investigation of planar ac gas-discharge systems discussed in the chapter AC Gas-Discharge Systems: Experiment.**

**We also refer to the results fig. 9.10 of the chapter Reaction-Diffusion Equations that have been obtained by solving the FHN equation (1-3) of that chapter and that to large extent reflects qualitatively the experimental observations 5.18 of the chapter AC Gas-Discharge Systems: Experiment**

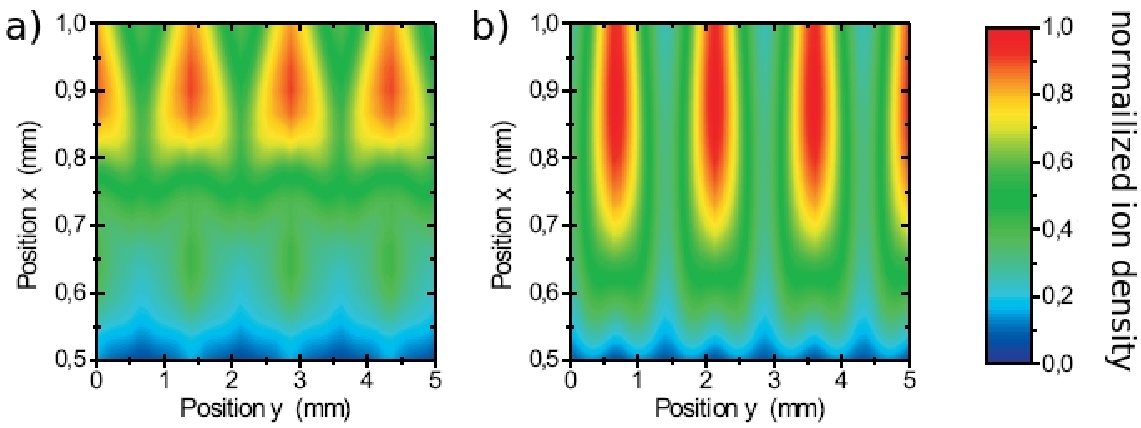


**Fig. 6.1**

**Numerical results for stable stationary filaments obtained from the solutions of equations (1-3) in $R^2$ for the ac system fig. 5.1 of chapter AC Gas-Discharge Systems: Experiment. Two filamentary patterns (a) and (b) are generated in the course of a double breakdown within one half period. The filaments are represented via the ion density distribution in the discharge slit where x and y are the coordinates vertical and parallel to the electrodes, respectively. The parameters are taken from a typical experimental situation with filament formation. [Pu067; Pu068; I. Brauer, “Experimentelle und numerische Untersuchungen zur Strukturbildung in dielektrischen Barrierenentladungen”, Thesis, University of Münster (2000)] - compare to experiment fig. 5.2, 5.8**

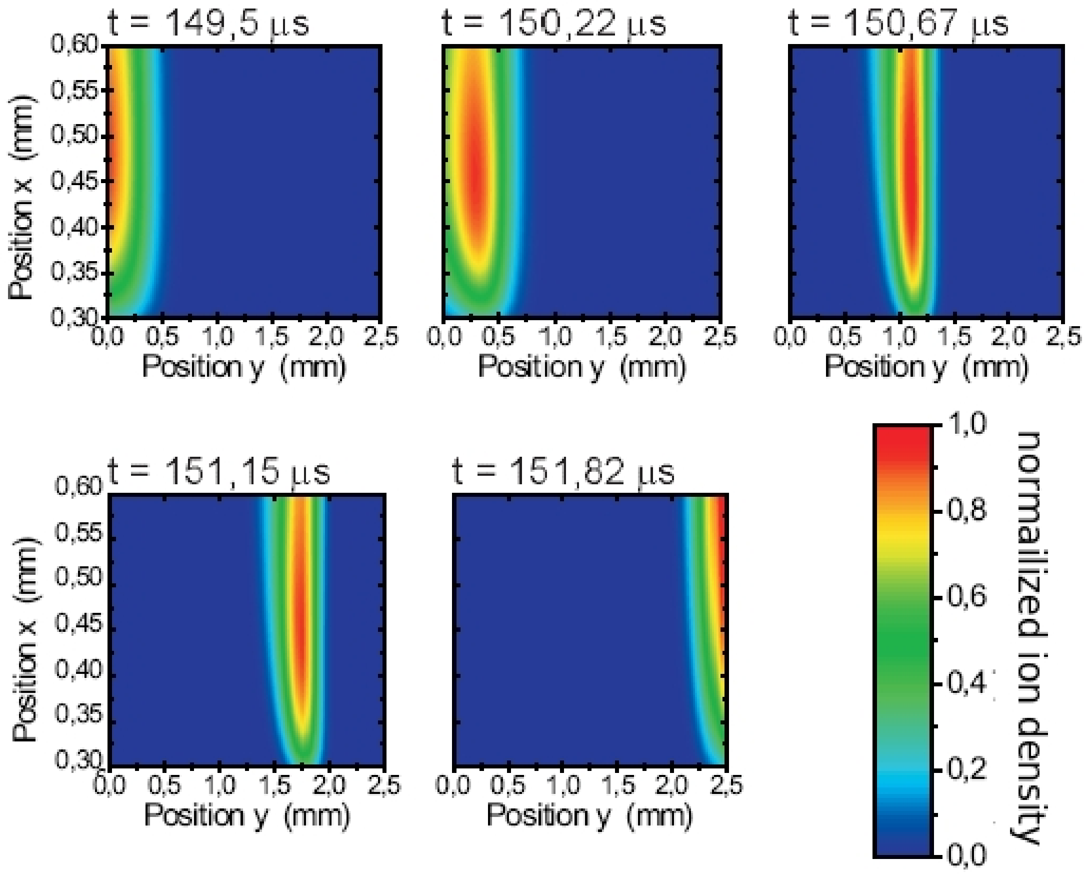


**Fig. 6.2**

**Numerical results for a stable travelling filament obtained from the solutions of equations (1-3) in $R^2$ for the ac system fig. 5.1 of chapter AC Gas-Discharge Systems: Experiment. From left to right the series of pictures represents a travelling filament propagating from left to right in the course of time. The filaments are represented in the ion density distribution in the discharge slit where x and y are the coordinates vertical and parallel to the electrodes, respectively. The parameters are taken from a typical experimental situation with filament formation. [Pu067; Pu068; I. Brauer, Thesis (2000)] - compare to: experiment figs. 5.2; 5.9, 5,13, 5.14)**

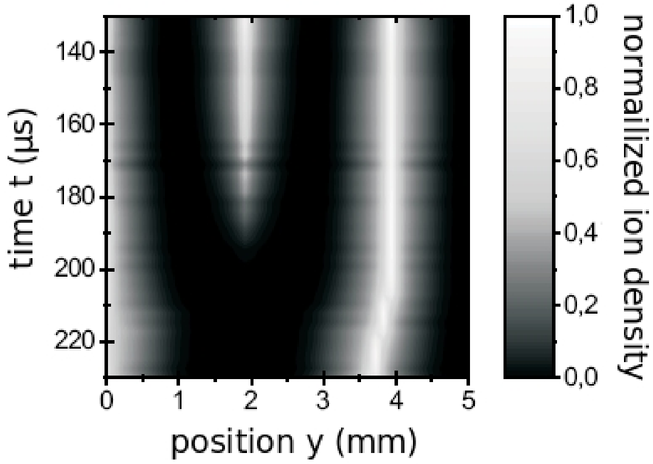


**Fig. 6.3**

**Numerical result for the evolution of filamentary solutions obtained from the equations (1-3) in $R^2$ for the ac system fig. 5.1 of chapter AC Gas-Discharge Systems: Experiment. The filaments are represented in the ion density distribution in the discharge slit where the abscissa is the space coordinate parallel to the edges of the electrodes and the ordinate represents the time. Initially, the pattern consists of 2.5 filaments. After the abrupt decrease of the amplitude $\hat{U}$ of the driving voltage one filament extinguishes in the course of order of 20 breakdowns. The parameters are taken from a typical experimental situation with filament formation. [Pu067; Pu068; I. Brauer, Thesis (2000)] - compare to: experiment figs. 5.7, 5.18)**

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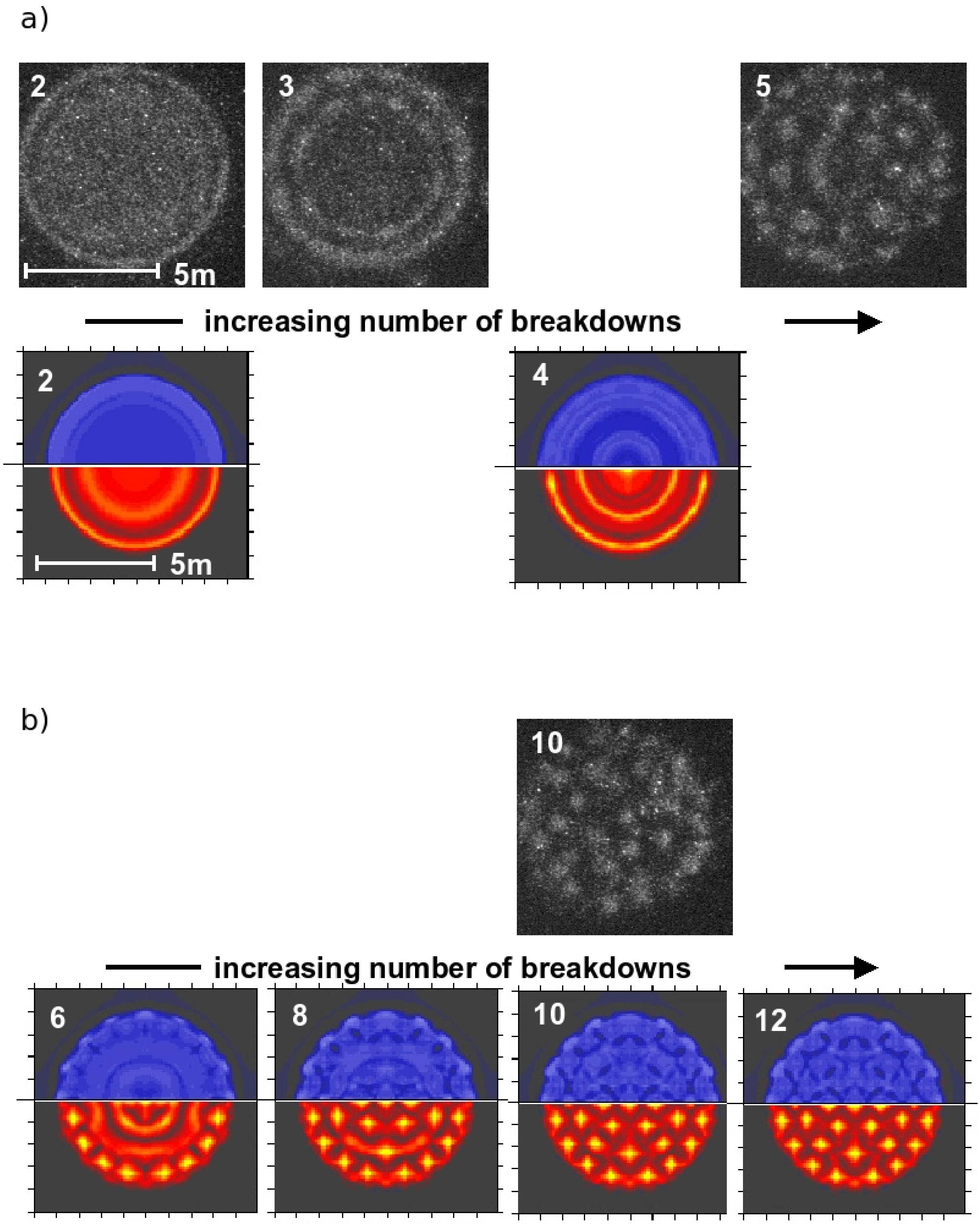
a)
2
5m
3
5
increasing number of breakdowns
2
5m
4
b)
10
increasing number of breakdowns
6
8
10
12

**Fig. 6.4a,b**

**Fig. 6.4a,b**

**Experimental results for the luminescence radiation density distribution (upper series of pictures of a. and b. and numerical simulations (lower series) obtained from the solutions of the equations (1-3) in $R^3$ for the ac system fig. 5.3 of chapter AC Gas-Discharge Systems: Experiment after switching on the driving voltage. The numerical results are depicted for the charge density on top of the electrodes directly after the breakdown at the anode (upper half of each individual image) and the cathode (lower half). The numbers in the figures label the selected break down. Time proceed from left to right and from (a) to (b). The parameters are taken from the experiment. In the calculations no fitting has been applied. Since the radiation density should scale with the transferred charge one obtains good agreement between theory and experiment. [Pu123]**

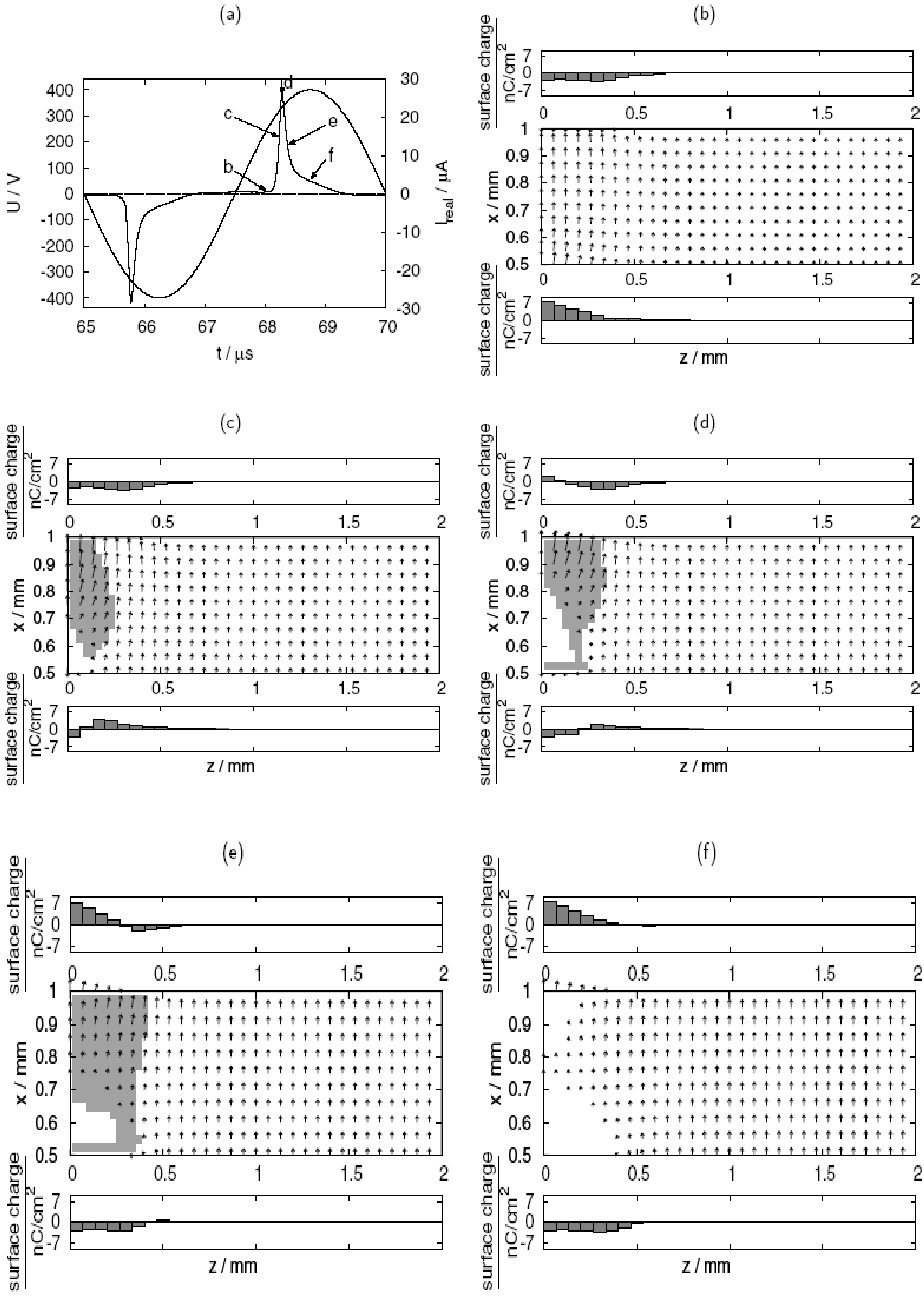


**Fig. 6.5a-f**

**Fig. 6.5a-f**

**Numerical result for a stable stationary single filament obtained from the solutions of the equations (1-3) in $R^3$ for the ac system fig. 5.3 of chapter AC Gas-Discharge Systems: Experiment. The series of pictures represents snapshots of the surface charge density at the stages b – f of a single break down as displayed in (a) . The images (b – f) represent cross sections with the coordinates x and z being oriented vertical and parallel to the electrodes, respectively. The parameters are taken from a typical experimental situation with filament formation. Width of the filament, duration of break down, transferred charge of the filament and break down voltage are in rather good agreement with experiment. [Pu129] - compare to: experiment fig. 5.7, 5.8, 5.18; theory figs. 6.1, 6.3**

# 6.3 Listing of main results

**With respect to the abbreviations used in the following listing of observed phenomena we refer to the Introduction.**

**Pu017:** **Radehaus, Willebrand, Dohmen, Niedernostheide, Bengel, Purwins (1992)**

**periodic pattern in $R^1$**

theo: modified 2-k + gc , $R^1$- period

**bifurcation: Turing**

exp: 1d-dc-GDS, $R_0 \neq 0$ - (stat hom) ↔ (stat periodic), supercritical and subcritical Turing bif, the former with correct scaling law

theo: modified 2-k + gc , $R^1$ - anal: (stat hom) ↔ (stat periodic), supercritical and subcritical Turing bif

**experimentally observed supercritical and subcritical Turing bifurcation treated in terms of a modified 2-component reaction diffusion equation using the centre manifold theory - see also: Reaction-Diffusion Equations**

**Pu067:** **Müller, Punset, Ammelt, Purwins, Boeuf (1999)**

**many f systems**

theo: 2d-ac-GDS - using a gas-discharge specific drift-diffusion model in $R^2$ (one coordinate vertical to the electrodes and one parallel) with parameters taken from experiment - period pattern in the direction parallel to the electrode consisting of filaments and related DSs, correct order of magnitude for the diameter and distance of filaments

**first qualitative theoretical reproduction of the experimentally observed hexagonal pattern in a gas-discharge system using a gas-discharge specific model approach - two drift diffusion equations with 2 charge carriers and the Poisson equation are used while gas heating, plasma chemistry, field emission and surface effects are neglected - 2 space coordinates: one vertical to the electrodes and one parallel - see also: AC Gas-Discharge Systems: Experiment**

**Pu068:** **Brauer, Punset, Purwins, Boeuf (1999)**

**essentially an extended presentation of [Pu067]: more detailed description of the theoretical and the numerical procedure as well as the numerical results**

**Pu119:** **Amiranshvili, Gurevich, Purwins (2005)**
**modelling of the quasi 2-dimensional dc gas-discharge system consisting of a linear high ohmic and a gas layer: use of two drift diffusion equations and the Poisson equation by assuming 2 charge carriers and neglecting gas heating, plasma chemistry, field emission and surface effects - first analytical derivation of a 2-component reaction diffusion equation in $R^2$ from the set of drift diffusion equations in $R^3$ - first analytical indication that reaction diffusion equations may be appropriate for the description of a certain class of planar gas-discharge systems -discussion of various front solutions**

**Pu123:** **Stollenwerk, Amiranshvili, Boeuf, Purwins (2006)**
**hexagons in $R^2$**
exp: 2d-ac-GDS - evolution (transient behaviour) after switching on the driving ac voltage: (stat hom) → (concentric ring near to the boundary) → (washed out sport) → (hex pattern made of fs)
theo: 3d-ac-GDS, using a gas-discharge specific drift-diffusion model in $R^3$ with parameters taken from experiment: evolution (transient) after switching on the driving ac voltage: (stat hom) → (concentric ring near to the boundary) → (washed out sport) → (hex pattern made of fs), (quant)
**after switching on the discharge: experimental observation of the transients to an ensemble of hexagonally arranged filaments and related DSs with neighbouring distance of order of their diameter - first quantitative theoretical reproduction of the experimentally observed pattern using two drift diffusion equation $R^3$ and the Poisson equation with 2 charge carriers, neglect of gas heating, plasma chemistry, field emission and surface effects - no parameter fitting - 3 space coordinates: one vertical to the electrodes and two parallel - see also AC Gas-Discharge Systems: Experiment**

**Pu127:** **Purwins, Amiranashvili (2007)**
**summary - simple patterns: e.g. isolated filaments and related DSs, stripes, hexagons and rotating spirals - patterns of higher complexity with filaments and related DSs as elementary building blocks: e.g. "molecules" and "many body systems" in the form of crystal-, liquid-, gas-like arrangements, chains and nets - universal experimental behaviour for a certain class of systems including: planar ac and dc gas-discharge systems, electrical networks, semiconductor layer systems, chemical solutions and biological systems - theoretical definition of the corresponding universality class: writing down a 3-component reaction-diffusion system serving as a kind of normal form for the qualitative description of the experimentally observed self-organized patterns - illustration of the formation of filaments and related DSs in planar electrical transport systems on the basis of the 2-component reaction diffusion equation - see also: Electrical Networks: Experiment and Theory, DC Gas-Discharge Systems: Experiment, AC Gas-Discharge Systems: Experiment, Semiconductors: Experiment, Semiconductors: Theory, Reaction-Diffusion Equations**

**Pu129:** **Stollenwerk, Amiranshvili, Purwins, Boeuf, Purwins (2007)**

**isolated stationary and travelling fs**

exp: 2d-ac-GDS - stat, slowly drifting fs

theo: 2d-ac-GDS - using a gas-discharge specific drift-diffusion model in $R^3$ (one coordinate vertical to the electrodes andtwo parallel) with parameters taken from experiment: - period pattern in the direction parallel to the electrode consisting of fs, semi-quantitative description of an isolate f

**many body f systems**

exp: 2d-ac-GDS - dense arrangement of fs

**bifurcation of fs**

exp: 2d-ac-GDS - (stat hom dark) → (by increasing the driver amplitude ) → (dense arrangement of fs) → [by decreasing the driver amplitude] → (gradually decreasing number of fs)

**first semi-quantitative theoretical reproduction of a single stationary filament and related DS in a quasi 2-dimensional acgas-discharge system using two drift diffusion equations and the Poisson equation with 2 charge carriers, neglect of gas heating, plasma chemistry, field emission and surface effects - continuation of [Pu067, Pu123] - see also: AC Gas-Discharge Systems: Experiment**

**Pu131:** **Purwins (2007)**

**summary - the formation of DSs in planar low temperature dc and ac gas-discharge systems is a generic phenomenon - in many respect filaments and related DSs behave like particles - illustration of the formation of filaments and related DSs in planar electrical transport systems on the basis of the 2-component reaction diffusion equation - experimentally observed phenomena.: e.g. isolated filaments and related DSs, snaking, bifurcation from stationary to travelling filaments and related DSs, mutual interaction of filaments and related DSs with scattering, "molecule" formation, generation and annihilation as well as "manybody systems" in the form of crystal-, liquid-, gas-like arrangements, domain structures, chains and nets - pointing out that the 3-component reaction-diffusion system seems to present a kind of normal form for the qualitative description of self-organized patterns in the discussed gas-discharge systems - listing of potential applications - see also: DC Gas-Discharge Systems: Experiment, AC Gas-Discharge Systems: Experiment, Reaction-Diffusion Equations**