

3.

Electrical Networks: Experiment and Theory

(references refer to the list of publications given in chapter 12)

3.1 General remarks

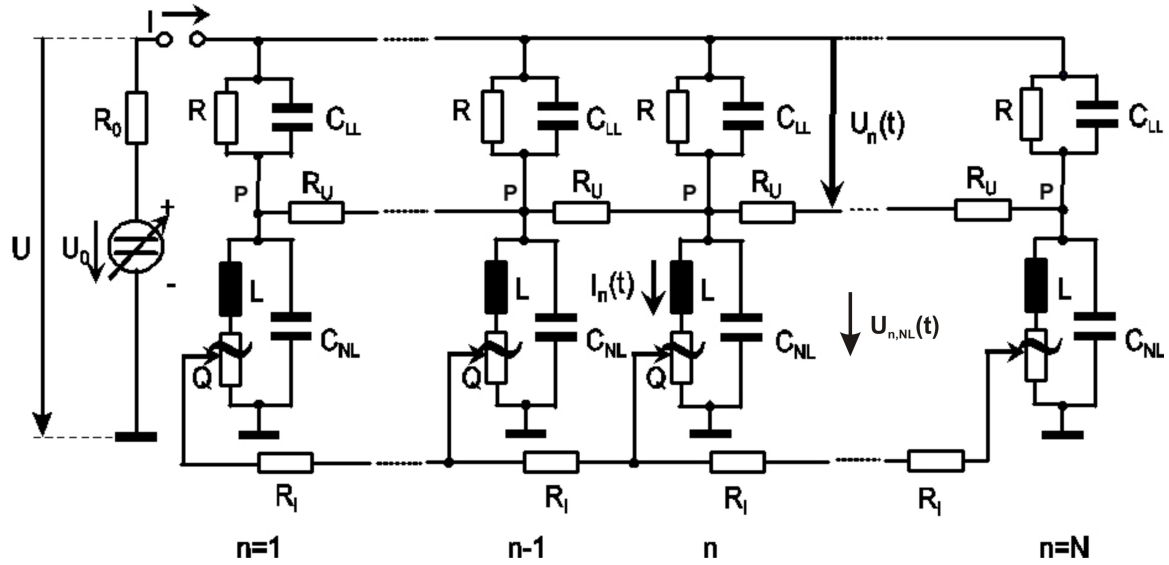


Fig. 3.1

This chapter deals with the experimental and theoretical investigation of a certain class of electrical networks. The circuit diagram for the quasi 1-dimensional device being investigated experimentally with respect to the formation of discrete self-organized patterns in the current distribution $I_n(t)$ and the voltage distributions $U_n(t)$ or $U_{n,NL}(t)$ is represented in fig. 3.1. The quasi 2-dimensional version is generated by putting many of the depicted chains into parallel and connecting them at the points P by R_U and at Q by R_I . The only nonlinearity is the resistor with control via Q having an S-shaped current voltage characteristic of the form fig. 2.2 of the chapter [A Model for Pattern Formation](#). When interpreting the network as an equivalent circuit for the continuum device fig. 2.1 of the chapter [A Model for Pattern Formation](#) the upper part including the elements R , C_{LL} and R_U

corresponds to the linear or monotonous layer L in fig. 2.1 and the lower part with L, C_{NL} and R_I to the nonlinear layer NL.

The discrete dynamical equations for $I_n(t)$ and $U_n(t)$ are obtained from Kirchhoff's rules and for the case $C_U = 0$ they read as

$$L \frac{dI_n}{dt} = \gamma \frac{I_{n+1} - 2I_n + I_{n-1}}{R_I} - S(I_n) + U_n, \quad (1)$$

$$C_{NL} \frac{dU_n}{dt} = \frac{U_{n+1} - 2U_n + U_{n-1}}{R_U} + I_n - \frac{U_n}{R} + \frac{RU_0 + R_0 \sum_{j=0}^N U_j}{R(R + NR_0)}. \quad (2)$$

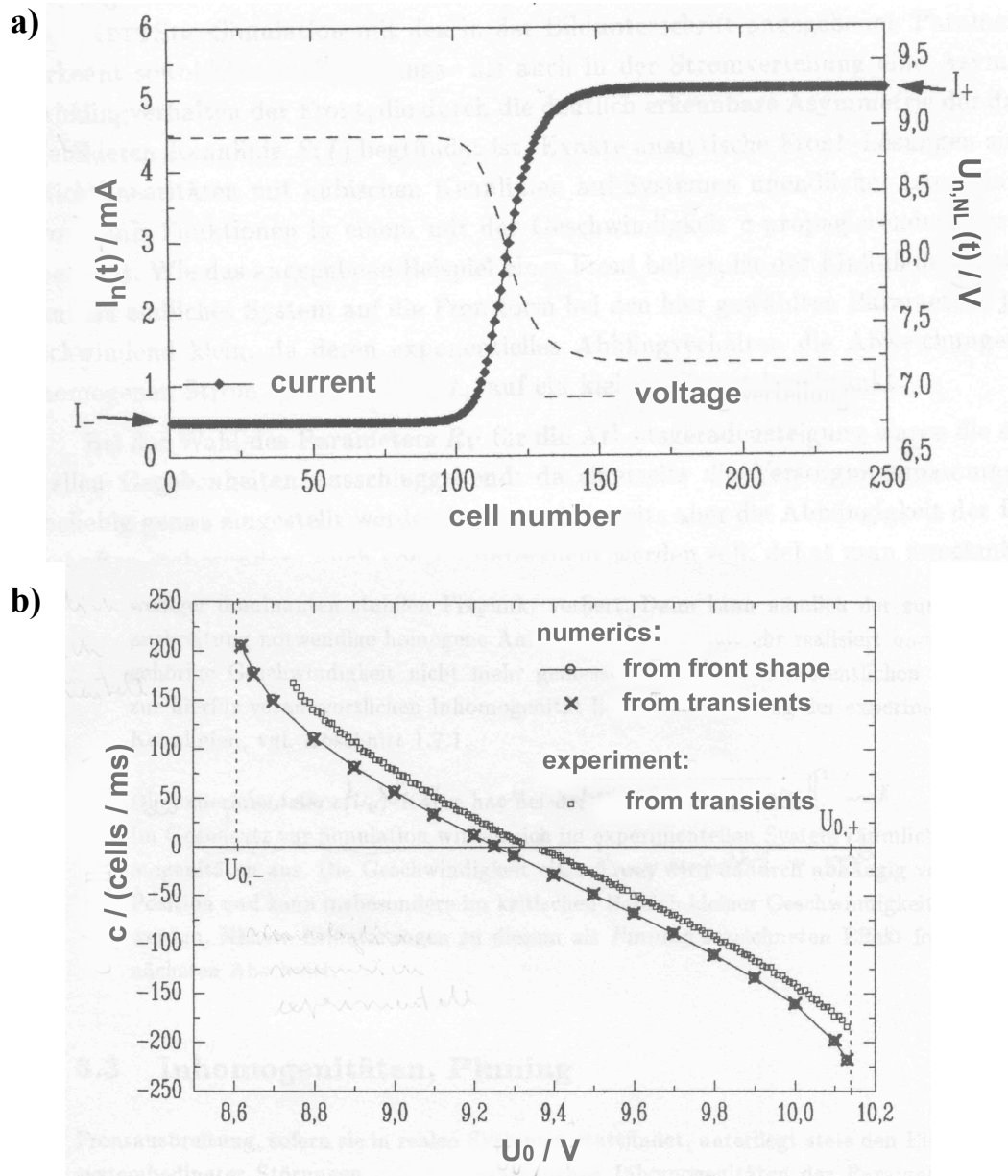
Here γ is a known constant and $S(I_n)$ denotes the experimentally determined (voltage)-(current) characteristic of the nonlinearity in fig. 3.1. The transition to the continuum of the latter results in a 2-component reaction-diffusion equation of the type being discussed in the chapter [A Model for Pattern Formation](#).

We note that in the investigated electrical network fig. 3.1 often a global coupling is present because of $R_0 \neq 0$. - Since the current voltage characteristic $I_n = S^{-1}(U_n)$ of the nonlinearity is similar to the realistic form of fig. 2.2. of chapter [A Model for Pattern Formation](#) the background change due to the number of bright solitary LSs can be neglected in large parameter ranges. Therefore, in what follows often we refer to the observed bright LSs as DS though we have $R_0 \neq 0$.

In addition to the experimental findings the following figures represent also interesting analytical and numerical solutions for the equations (1, 2) which can also be considered as a discretized version of a special cases of the general FHN equation (1-3) of the chapter [Reaction-Diffusion Equations](#).

3.2 Graphical representation of selected results

The following is a series of figures reflecting main results that have been obtained experimentally and theoretically in relation to the investigation of electrical networks. - see also chapter [Reaction-Diffusion Equations](#)



Figs. 3.2a, b

Figs. 3.2a, b

Self-organized fronts and their propagation in the 1-component electronic network fig. 3.1 in R^1 without global coupling. (a) represents a typical numerical snapshot obtained for the distributions of $I_n(t)$ (full line) and $U_{n,NL}(t)$ (dashed line) for a travelling front. In (b) the front speed c is depicted in dependence of the driving voltage U_0 obtained from experiment and from the numerical solutions of equation (1, 2) using the experimental parameters. For the evaluation of the speed the boundaries have no significant influence. Due to the loss of bistability of the uncoupled system the fronts depicted in (a) do not exist outside the interval $[U_{0,-}, U_{0,+}]$. [M. Bode, „Beschreibung strukturbildender Prozesse in eindimensionalen Reaktions-Diffusions-Systemen durch Reduktion auf Amplitudengleichungen und Elementarstrukturen“, Thesis, Institut für Angewandte Physik, University of Münster (1992); R. Schmeling, „Experimentelle und numerische Untersuchung von Strukturen in einem Reaktions-Diffusions-System anhand eines elektrischen Netzwerkes“, Thesis, Institut für Angewandte Physik, University of Münster (1994); Pu045]

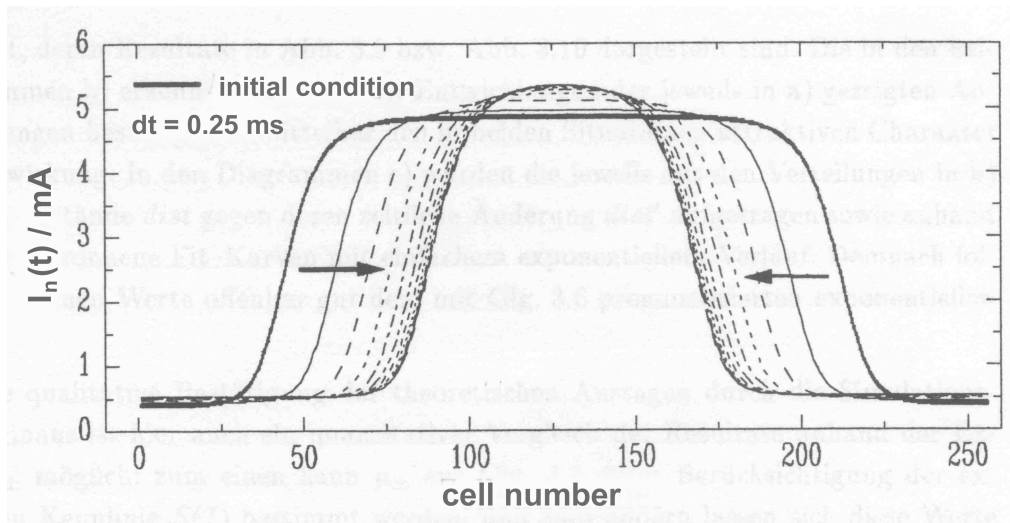


Fig. 3.3

Evolution to a self-organized stable front-antifront LS in the 1-component electronic network fig. 3.1 in R^1 with global coupling by solving numerically the discrete equation (1, 2). In the course of time the front-antifront LS becomes narrower and approaches a stable constant finite width. [R. Schmeling, Thesis (1994)]

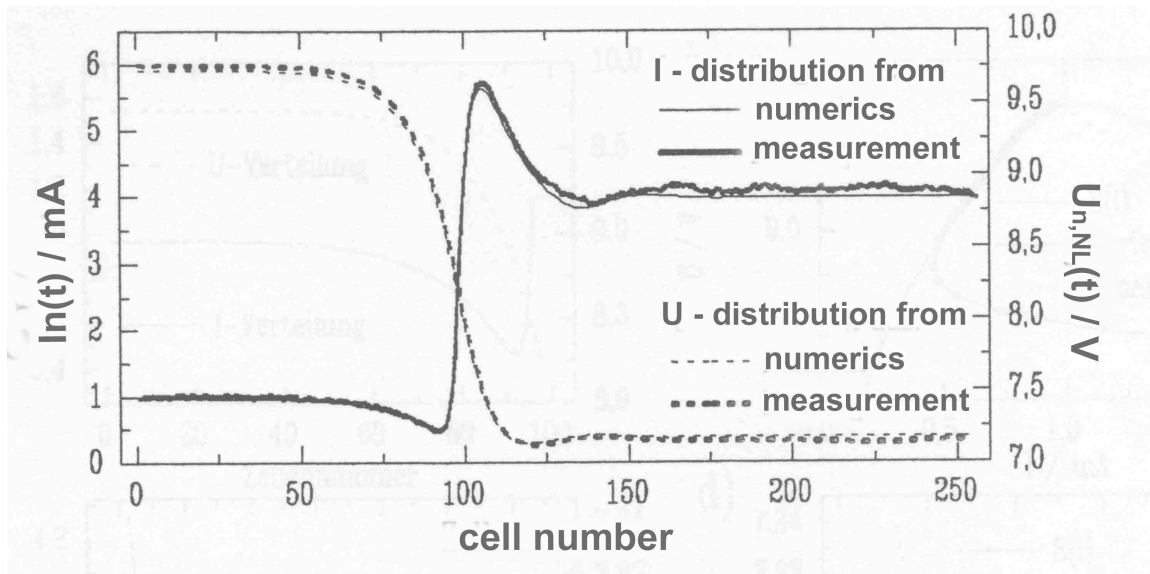
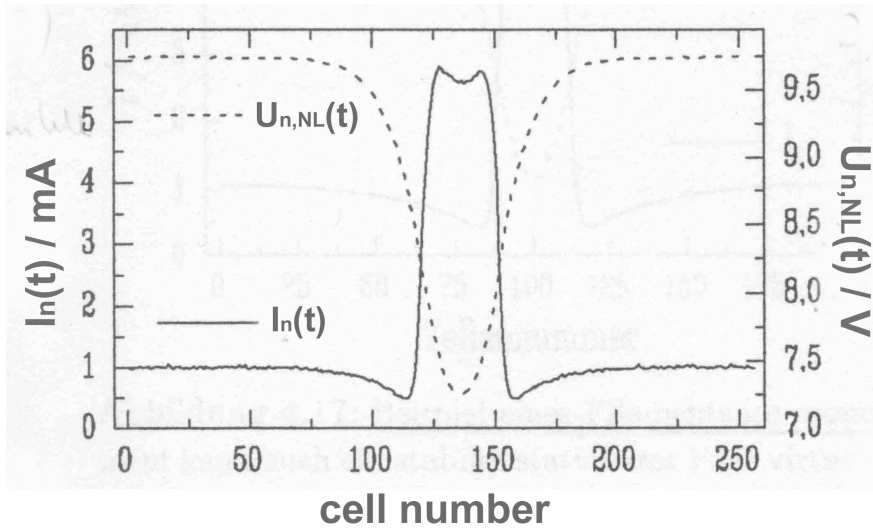


Fig. 3.4

Self-organized stable stationary front in the 2-component electronic network fig. 3.1 in R^1 without global coupling obtained from experiment and by solving numerically the corresponding discrete equation (1, 2) using the experimental parameters. The second component in (1, 2) allows for a non-monotonous transition to the homogeneous part of the solution thereby giving rise to nontrivial interaction phenomena. [M. Bode, Thesis (1992); R. Schmeling, Thesis (1994)]

a)



b)

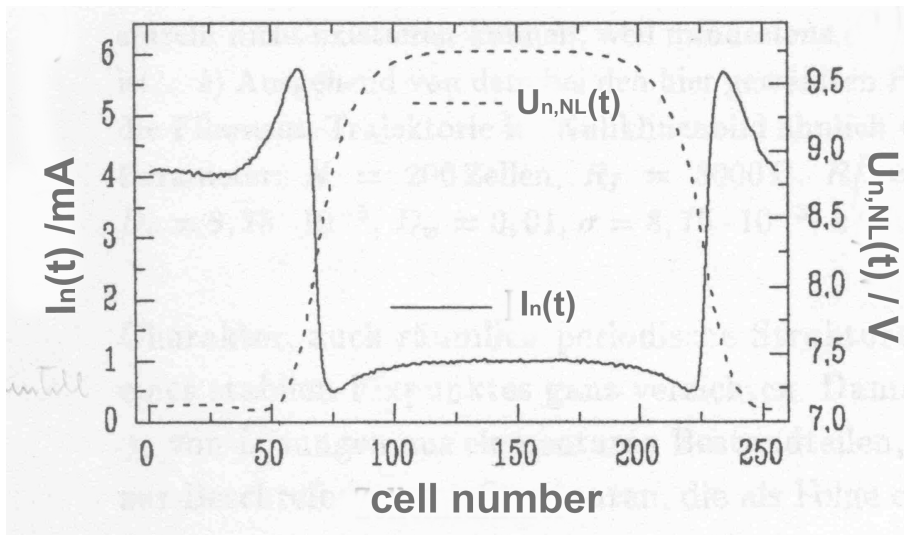
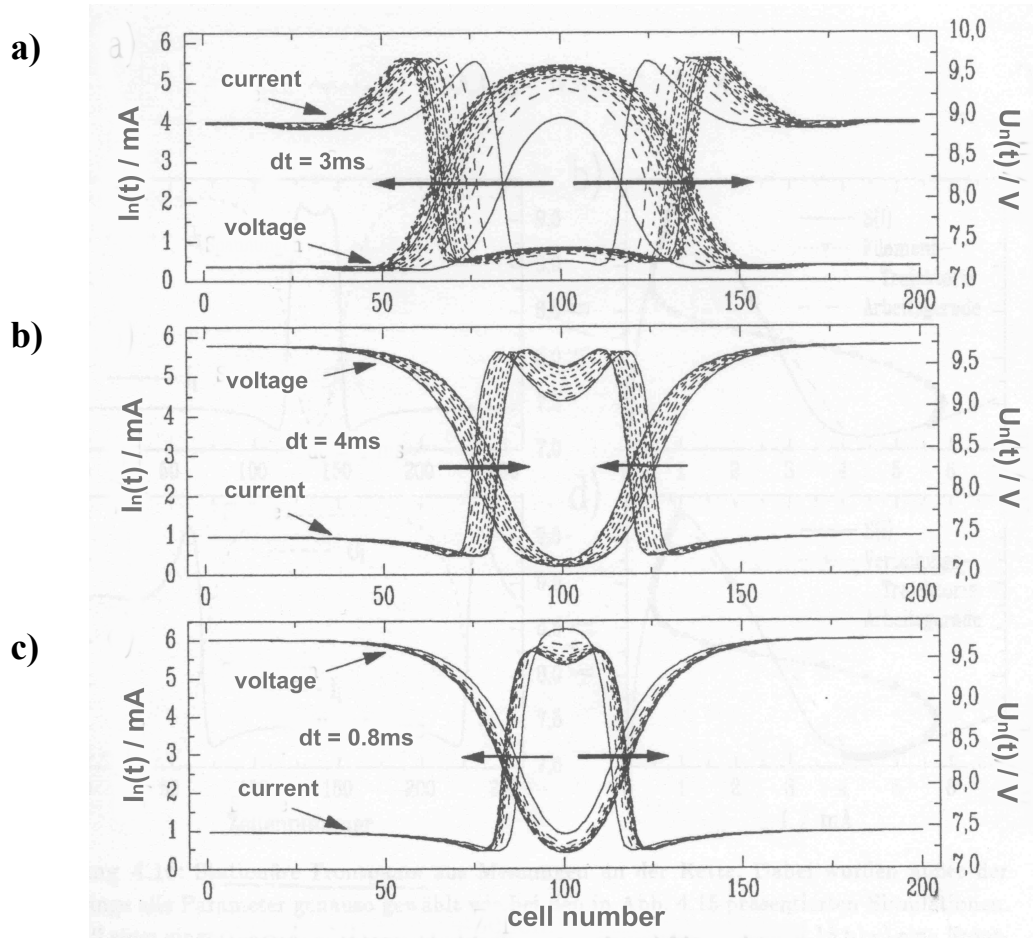


Fig. 3.5a,b

Experimentally observed front-antifront patterns on the 2-component electronic network fig. 3.1 in R^1 without global coupling. For the set of parameters in (a) a stable stationary DS is observed. In contrast for the parameter set of (b) repulsive interaction leads to front separation in the course of time and a DS cannot exist. [R. Schmeling, Thesis (1994)] - compare to: theory figs. 3.6, 9.3, 9.4, 9.5



Figs. 3.6a-c

Transients of front anti-front patterns obtained by solving numerically the discrete 2-component equation (1, 2) in R^1 without global coupling. In (a) and (b) the parameters of the experimental results depicted in fig. 3.5 have been chosen. The transients clearly converge to a stable stationary DS. In (c) the parameters are such that repulsion takes place making impossible the formation of a front anti-front DS. [R. Schmeling, Thesis, (1994)] - compare to: experiment fig. 3.5; theory figs. 9.3, 9.4, 9.5

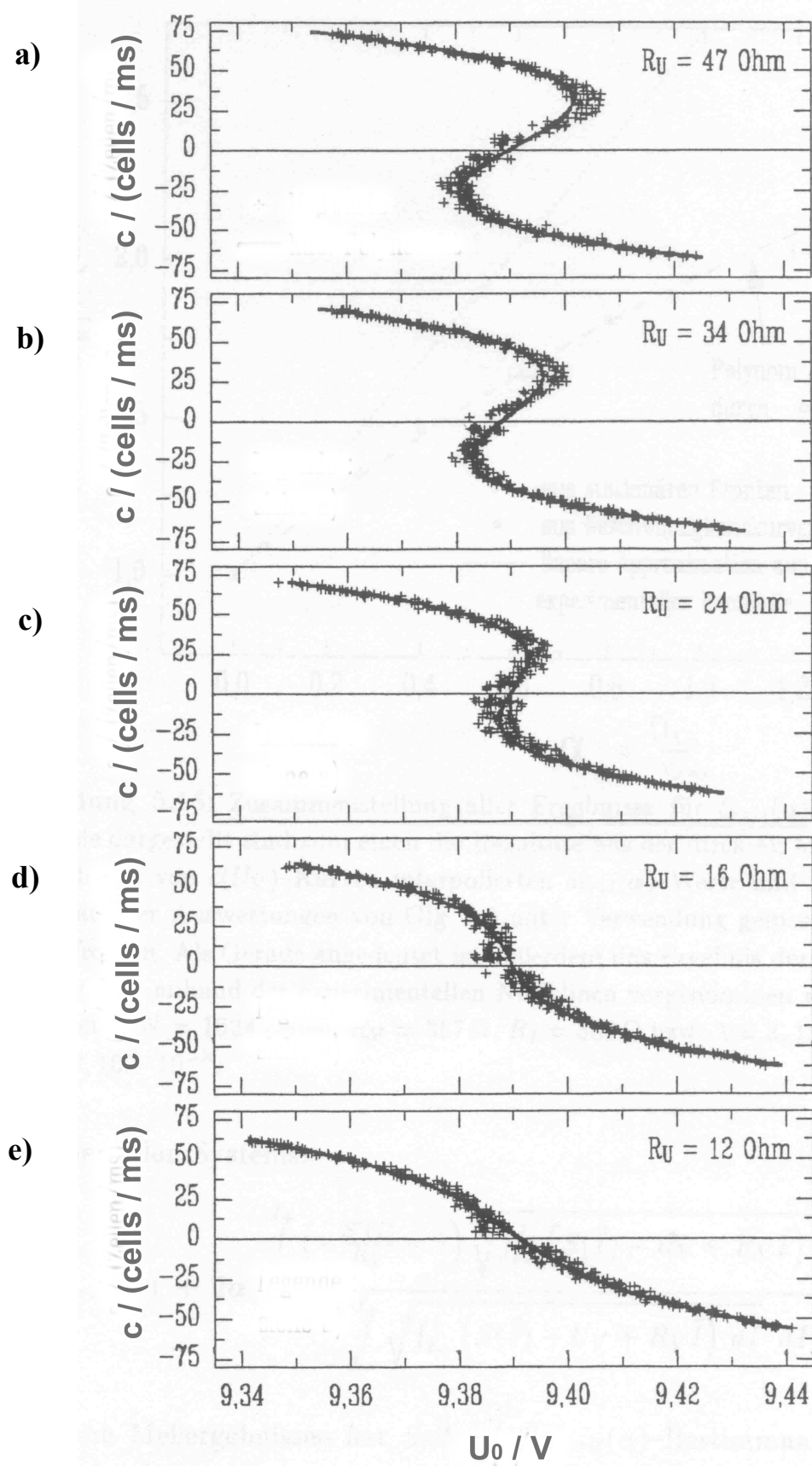


Fig. 3.7a-e

Fig. 3.7a-e

Experimentally determined speed c (crosses) and fit with a cubic polynomial (continuous curve) of isolated fronts on the 2-component electronic network fig. 3.1 in R^1 without global coupling in dependence of the externally applied voltage U_0 for various values of the coupling resistor R_U . From (a) to (e) one observes a transition from a single valued to a multi valued relation corresponding to a transition from monostability to bistability with respect to propagating fronts, as expected from theory. - In the presence of a global coupling the mentioned bistability may give rise to pendulum-like dynamic behaviour of an isolated front. [Pu032, R. Schmeling, Thesis, (1994)] - see also [M. Bode, Thesis (1992); Pu041]

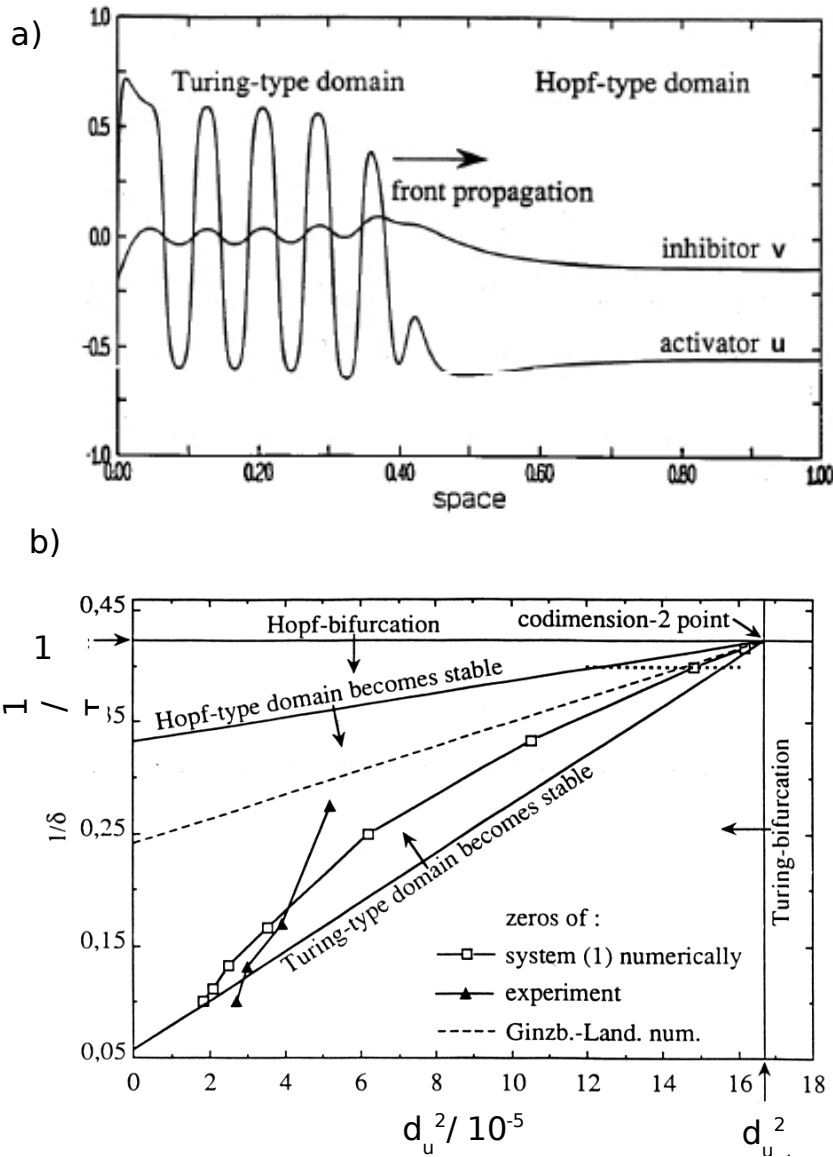
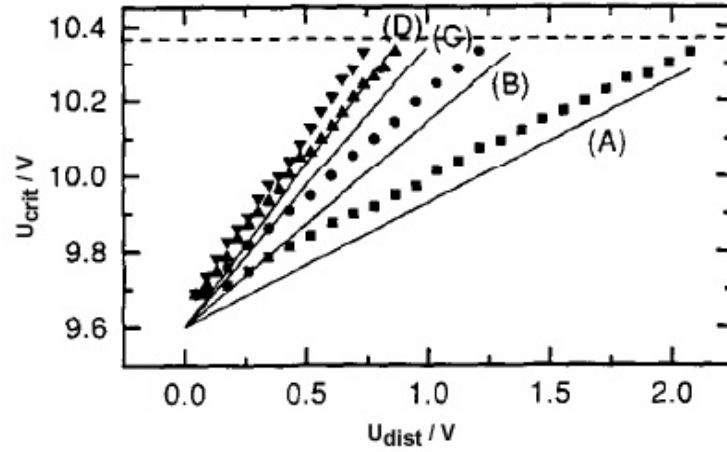


Fig. 3.8a,b

Spatial patterns in u and v obtained for the 2-component electrical network in R^1 without global coupling of the kind fig. 3.1. (a) depicts numerical results for the corresponding 2-component FHN equation (1–3) of the chapter [Reaction-Diffusion Equations](#) in the case of a propagating Turing front (leaving behind a stationary periodic pattern) into a Hopf domain (periodic oscillations of the homogeneous state). The related bifurcation diagrams is determined from experiment as well as from solving numerically the FHN and the related Ginzburg-Landau equation (b). [Pu027]

a)



b)

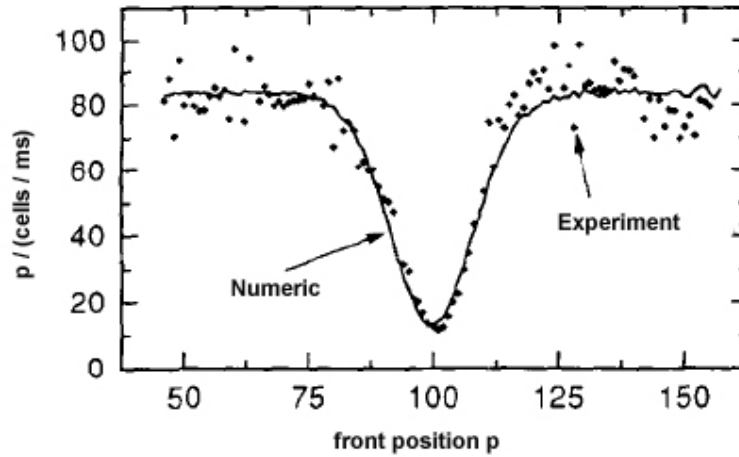
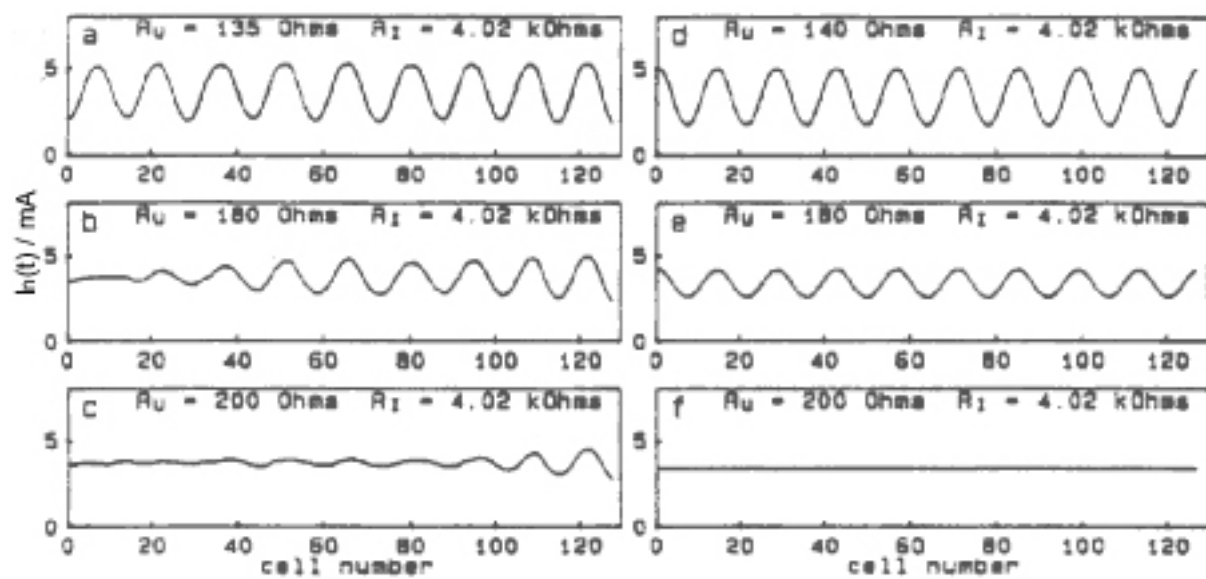


Fig. 3.9a,b

Experimental results on the interaction of a front with a local impurity in a 1-component electrical network in R^1 of the kind fig. 3.1 without global coupling and comparison with analytical and numerical results from the corresponding 2-component FHN (1-3) equation of the chapter [Reaction-Diffusion Equations](#). (a) depicts the experimental points and the continuous lines being obtained analytically for the critical applied voltage U_{crit} at which a front escapes from the inhomogeneity. The results are shown in dependence of the strength of the inhomogeneity at otherwise fixed parameters. (b) shows an example for the dependence of the front speed on the position p near to some inhomogeneity. The points represent experimental results and the continuous line numerical simulations. [Pu045]

A)



B)

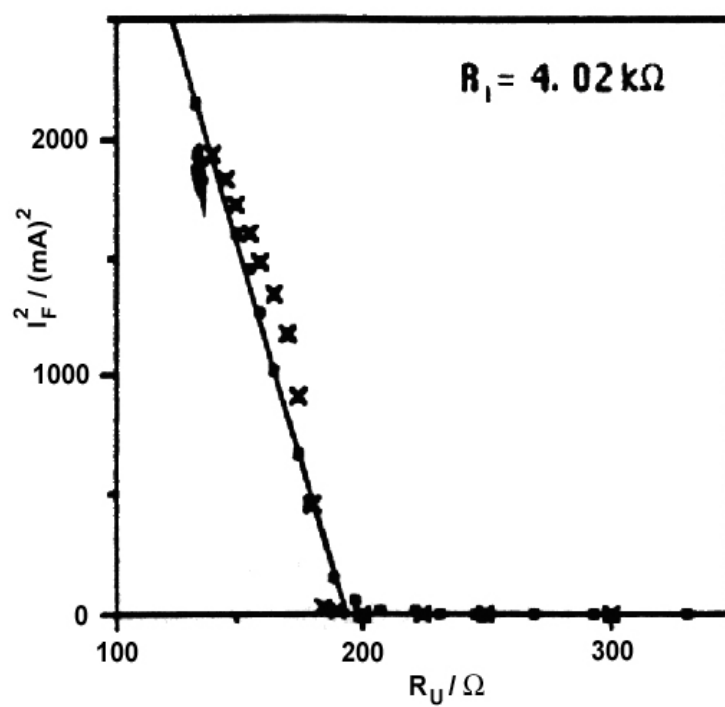
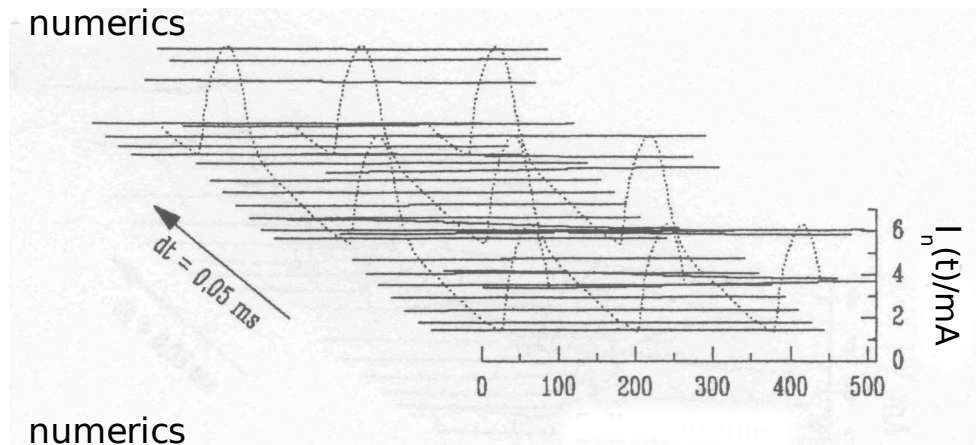


Fig. 3.10A,B

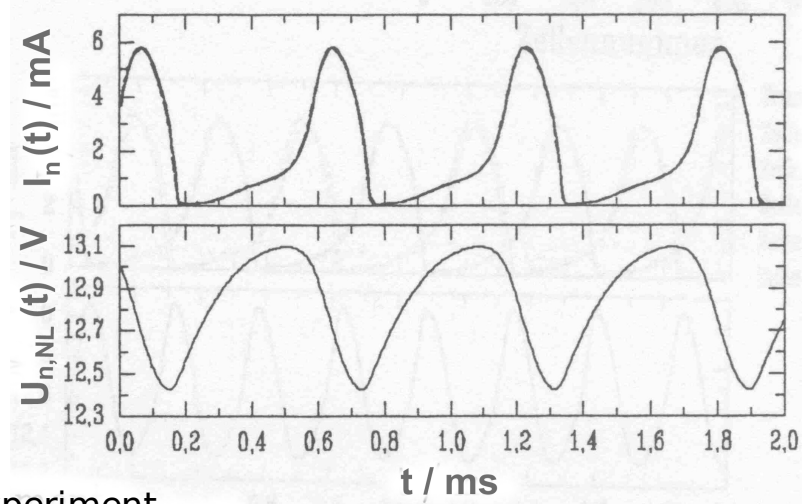
Fig. 3.10A,B

Experimentally observed self-organized stationary periodic patterns on the 2-component electronic network fig. 3.1 in R^1 without global coupling exhibiting a supercritical bifurcation from a stationary homogeneous to a stationary periodic state (Turing bifurcation). In (A) the current is plotted as a function of cell number for R_U increasing from (a) to (f). In (B) the square of the fundamental Fourier mode of the periodic patterns is given as a function of the bifurcation parameter R_U . To good approximation a square law is obtained, as expected from theory. [Pu009; Pu013; R. Schmeling, Thesis (1994)] - compare to: experiment figs. 4.2, 4.3, 4.11, 4.13, 5.4

a) numerics



b) numerics



c) experiment

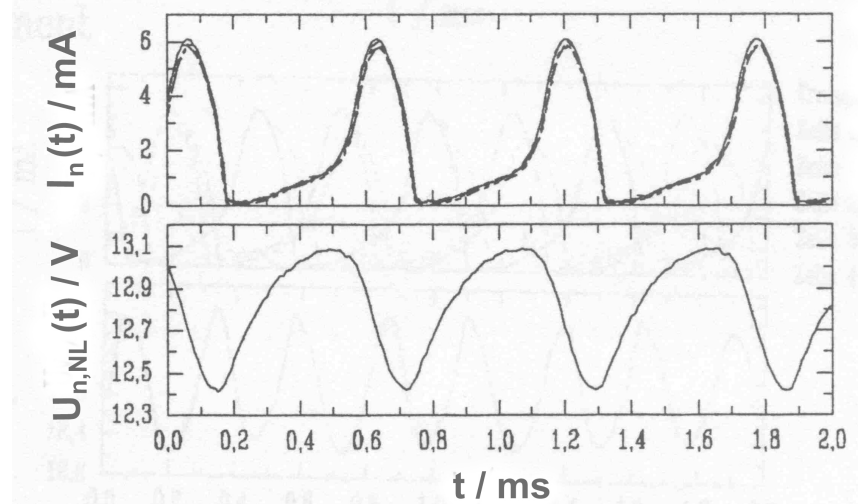


Fig. 3.11a-c

Fig. 3.11a-c

Self-organized periodic spatial pattern undergoing homogeneous periodic oscillations in time on the 1-dimensional electronic network fig. 3.1 in \mathbb{R}^1 with global coupling. In (a) solutions of the discrete 2-component equation (1,2) using the experimental parameters are given for the current distribution $I_n(t)$ in a 3-dimensional representation. In (b) and (c) the numerical and experimental results for $I_n(t)$ and $U_{n,NL}$ for a selected cell are depicted in dependence of time. – For other parameters the superimposed oscillations may be inhomogeneous in space. [R. Schmeling, Thesis (1994)]

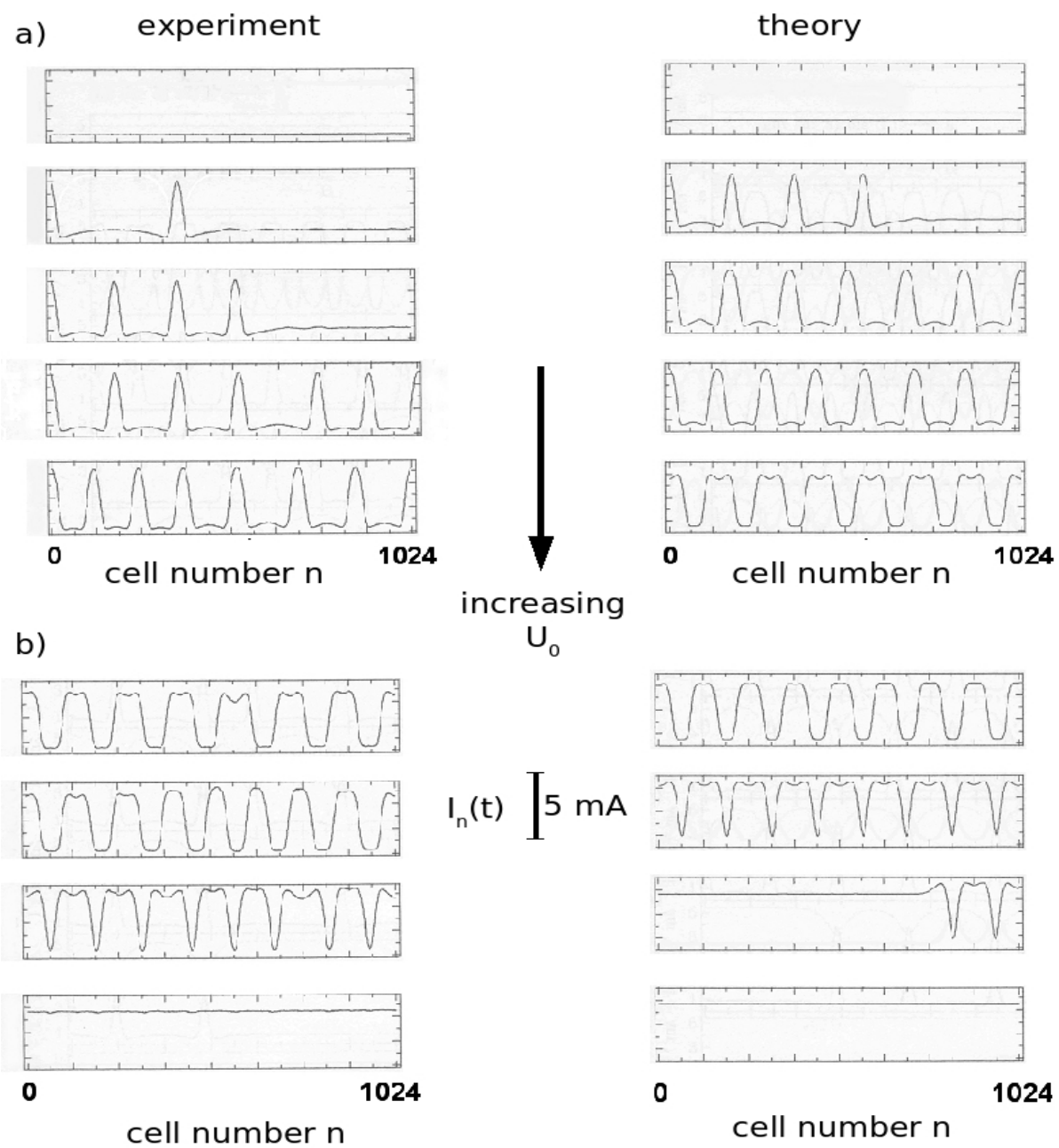


Fig. 3.12

Fig. 3.12

Self-organized patterns in the form of a cascade of stationary LSs observed on the 2-component electronic network fig. 3.1 in R^1 without global coupling. The current $I^n(t)$ is plotted as a function of cell number. The driving voltage U_0 is increased from top to bottom and from (a) to (b). The experimental results (left-hand series of pictures) and the solutions of the corresponding discrete equation (1,2) using the experimental parameters (right-hand series) are in good agreement. We note that we observe a transition from bright to dark LSs. [Pu013; R. Schmeling, Thesis (1994); Purwins, Amiranshveli, Bödeker, to be submitted for publication in 2009)] - compare to: experiment figs. 4.5, 4.6, 4.16, 5.7, 5.18, 7.3, 7.10; theory figs. 9.6, 9.10 - see also e.g. [Pu009; Pu018; R. Dohmen, „Entwicklung von Modellgleichungen zur Beschreibung nichtlinearer elektrischer Systeme und Untersuchung der Lösungsvielfalt mit analytischen und numerischen Methoden“, Thesis, Institut für Angewandte Physik, University of Münster (1991) ; R. Schmeling, Thesis (1994); Pu022; Pu043]

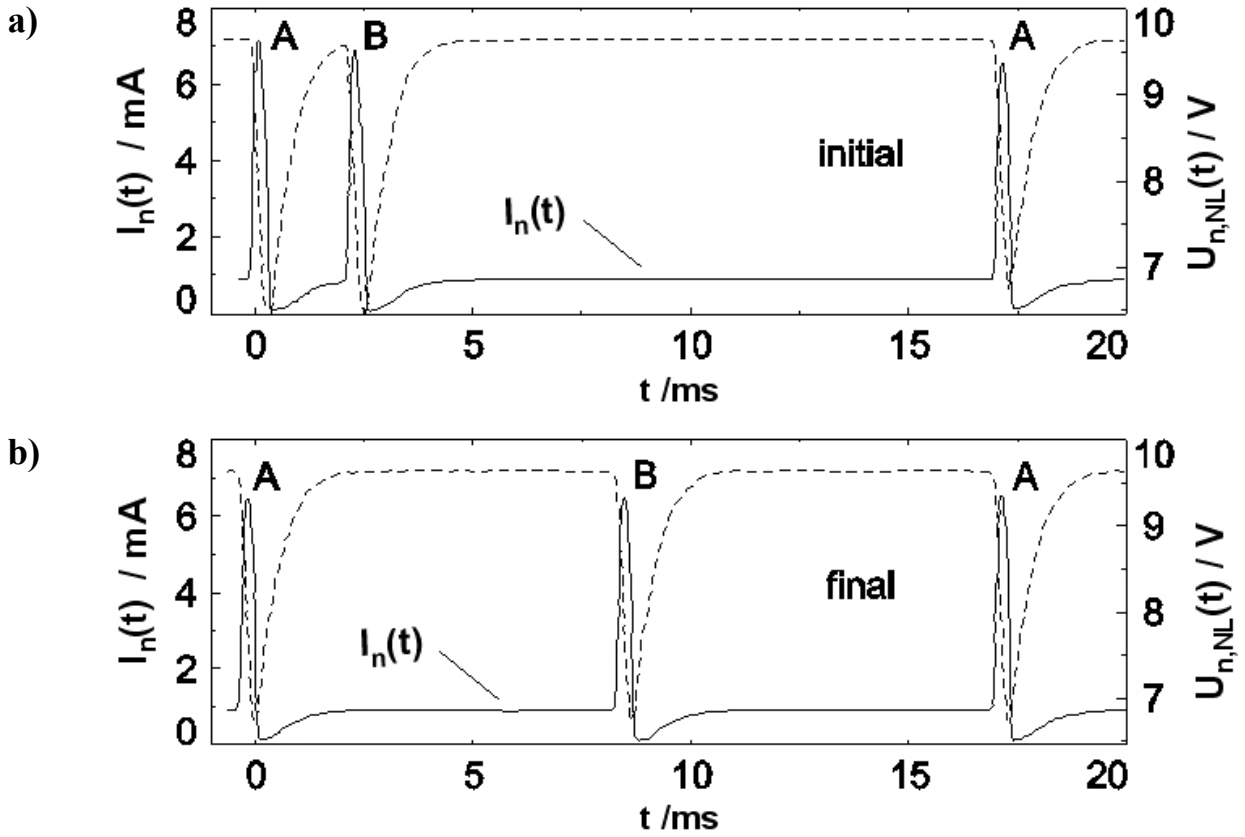


Fig. 3.13

Experimentally observed self-organized patterns in the form of interacting travelling DSs observed on the 2-component electronic network fig. 3.1 in R^1 without global coupling (nerve pulses). The current $I_n(t)$ (continuous curves) and the voltage $U_{n,NL}(t)$ (dashed curve) are plotted as a function of time for fixed cell number. Apparently the DS A reappears after about 17 ms. Initially A and B are close to each other (a) while after some time they arrange at approximately 180° on the ring (b) revealing repulsive interaction. [R. Schmeling, Thesis (1994); Purwins, Amiranshvili, Bodeker (to be published in 2009)] - compare to: experiment figs. 3.14, 4.18, 4.19, 5.9, 7.2; theory figs. 8.1, 9.8, 9.13, 9.14, 9.15

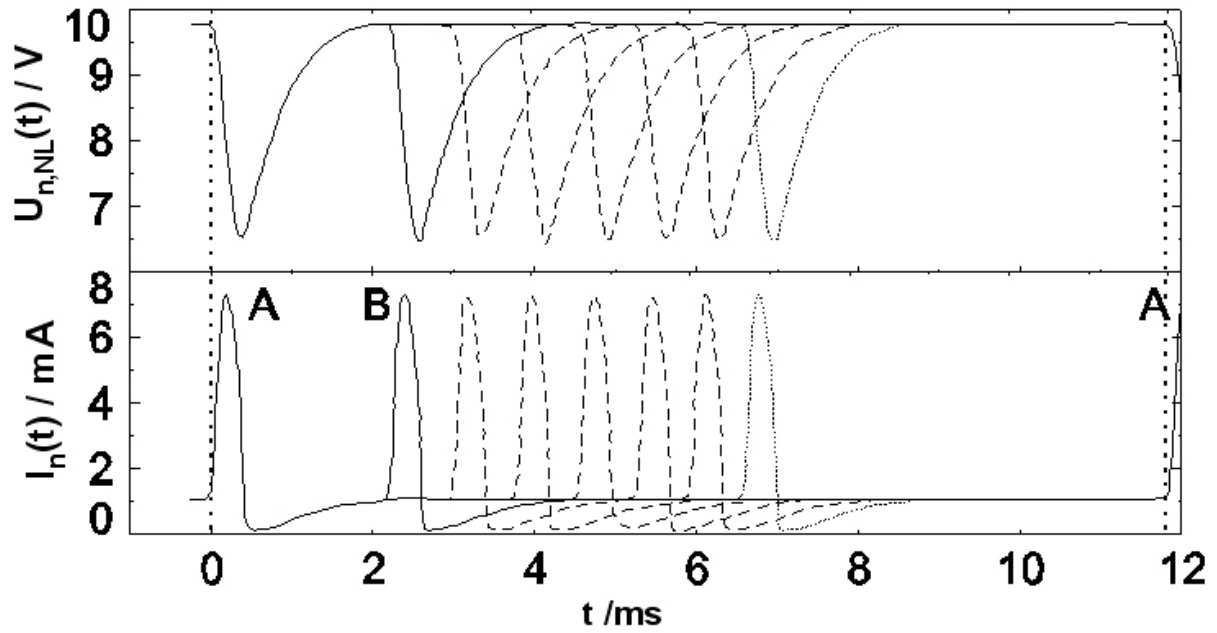


Fig. 3.14

Experimentally observed self-organized patterns in the form of travelling DSs on the 2-component electronic network fig. 3.1 in R^1 without global coupling: “molecule” formation. The voltage $U_{n,NL}(t)$ (upper picture) and the current $I_n(t)$ (lower one) are plotted as a function of time for fixed cell number. Apparently the DS B can lock-in at various discrete distances with respect to A thereby forming a molecule. This is in good agreement with theoretical considerations telling that at the given set of parameters one is near to a Turing bifurcation with DSs exhibiting oscillatory tails. [R. Schmeling, Thesis (1994); Purwins, Amiranshvili, Bödeker (to be published in 2009)] - compare to: experiment figs. 3.13, 4.18, 4.19, 5.9, 7.2; theory figs. 8.1, p.8, 9.13, 9.14, 9.15 - see also e.g. [Pu060; Pu062; Pu084; Pu105, Pu109, Pu118]

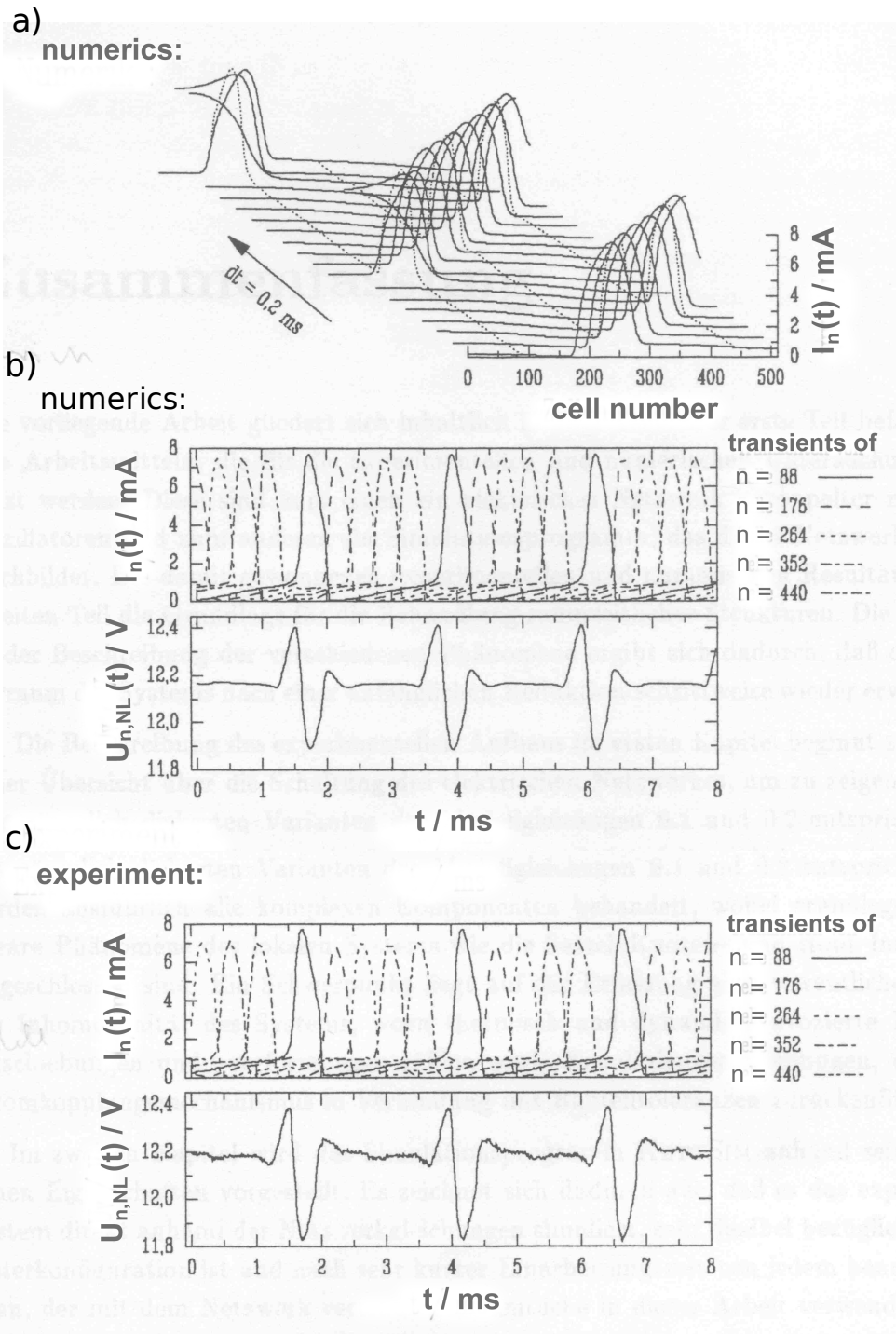
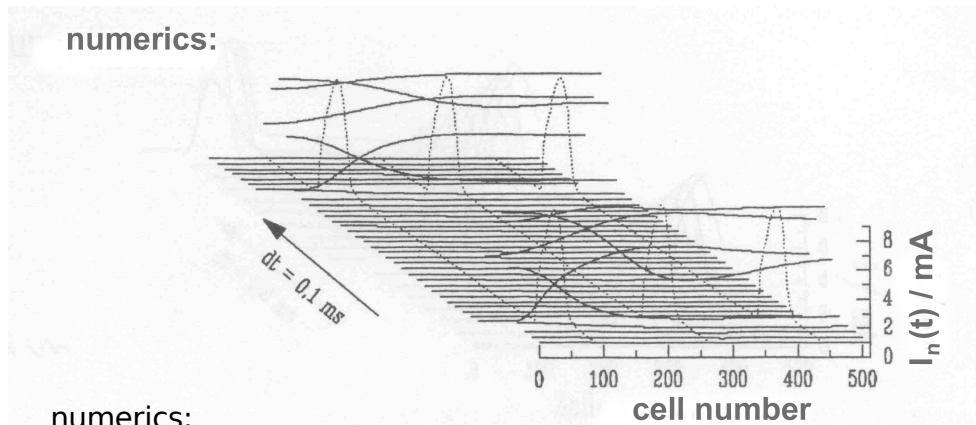


Fig. 3.15a-c

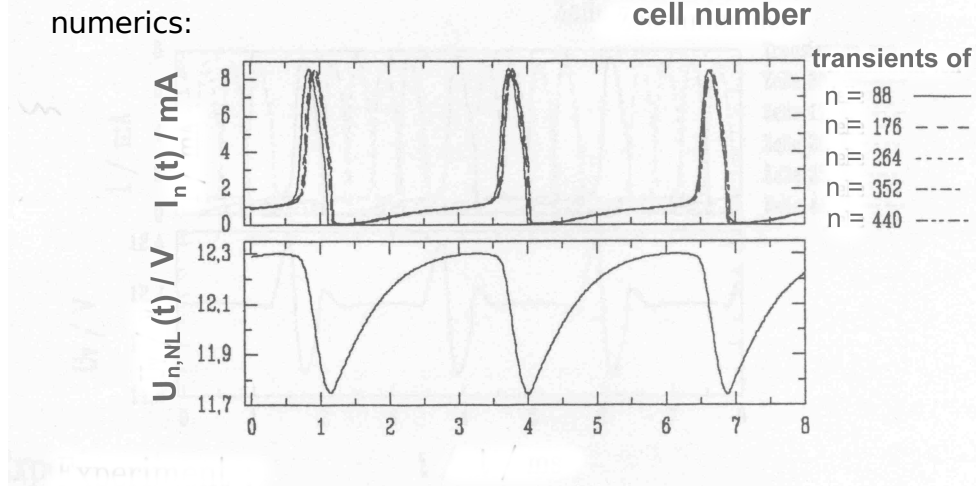
Fig. 3.15a-c

Self-organized travelling LSs on the 2-component electronic network fig. 3.1 in R^1 with global coupling. The LS is generated at one boundary and travels to the opposite boundary where it vanishes while simultaneously at the original boundary another LS is generated. This phenomena is repeated periodically. In (a) the numerically obtained solutions of the discrete 2-component equation (1,2) are given for the current distribution $I_n(t)$ in a 3-dimensional representation. In (b) and (c) the numerical and experimental results for $I_n(t)$ and $U_{n,NL}$ for a selected cell are depicted in dependence of the time. For the calculations the experimental parameters are used. [R. Schmeling, Thesis (1994)]

a)



b)



c)

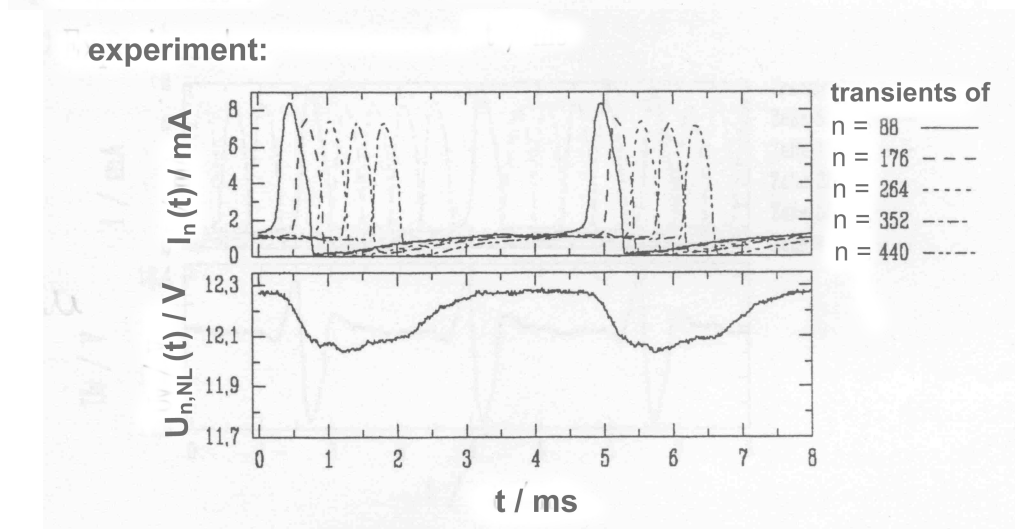


Fig. 3.16a-c

Fig. 3.16a-c

Self-organized patterns in the form of LSs on the 2-component electronic network fig. 3.1 in R^1 with global coupling. In (a) the numerically obtained solutions of the discrete 2-component equation (1,2) are displayed for the current distribution $I_n(t)$ in a 3-dimensional representation using the experimental parameters. In (b) and (c) the numerical and experimental results for $I_n(t)$ and $U_{n,NL}(t)$ for a selected cell are depicted in dependence of time. Apparently the arrangement of LSs undergoes relaxation oscillations incorporating the periodic disappearance and reappearance of the LS. - In other conditions the superimposed oscillations may be inhomogeneous in time. [R. Schmeling, Thesis (1994)]

3.3 Listing of main results

With respect to the abbreviations used in the following listing of observed phenomena we refer to the [Introduction](#).

- Pu002: Berkemeier, Dirksmeyer, Klempt, Purwins (1986)**
periodic patterns
 exp : 1d-ENW – hom periodic oscillations; stat nearly periodic pattern
 theo: 2-k , R^1 , discrete – num: hom periodic oscillations (quant); stat nearly periodic pattern (quant)
[proof that analogue electronic circuits can be described quantitatively by r-d equations with respect to the formation of self-organized patterns](#) - see also: [Reaction-Diffusion Equations](#)
- Pu007: Purwins, Klempt, Berkemeier (1987)**
periodic pattern in R^1
 exp : 1d-ENW - stat. nearly periodic; stat periodic
 theo: 2-k, R^1 , discrete - num: stat. nearly periodic (quant)
stripes in R^2
 exp: 2d-ENW - stat nearly periodic
miscellaneous patterns
 exp: 1d-ENW - periodic hom oscillations; stat nonperiodic inhom; temporally oscillating disconnected localized areas with almost same size and slightly varying frequency being separated by non-oscillating areas
 exp: 2d-ENW - hom periodic oscillations; approximately centre symmetric stat circles
 theo: 2-k , R^2 - num: hom periodic oscillations (quant); stat approximately centre symmetric stat circles (quant)
[interpretation of the electronic network as an equivalent circuit for a system consisting of a high ohmic and a strongly nonlinear layer with monotonous and S-shaped local current-voltage characteristic respectively - derivation of the corresponding 2-component reaction-diffusion equation - basic model for the universal behaviour of a relatively large class of pattern forming systems including biological, chemical and electrical transport systems](#) - see also: [DC Gas-Discharge Systems: Experiment, Reaction-Diffusion Equations](#)
- Pu008: Purwins, Radehaus C. Radehaus, and J. Berkemeier (1987)**
periodic pattern in R^1
 exp : 1d-ENW - stat periodic; stat nearly periodic; dependence of the pattern on the discretization
 theo: 2-k, R^1 , discrete - num: stat. nearly periodic (quant)
 see [Pu007] - detailed derivation of the 2-component reaction-diffusion equation for the electronic network and the corresponding generalized 2-layer model - see also: [DC Gas-Discharge Systems: Experiment”, Reaction-Diffusion Equations](#)

Pu009: Purwins, Radehaus (1987)

isolated stationary and travelling DSs

1d-ENW, $R_0 \neq 0$ - stat DSs, trav DSs

theo: $2-k + gc$, R^1 - num: stat DSs (quant)

periodic pattern in R^1

exp : 1d-ENW - stat periodic

theo: $2-k$, R^1 - anal: stat periodic (quant)

bifurcation: Turing

exp : 1d-ENW - (stat hom) \leftrightarrow (stat periodic), supercritical Turing bif

theo: $2-k$, R^1 - anal: (stat hom) \leftrightarrow (stat periodic), supercritical

Turing bif (quant)

miscellaneous patterns

exp: 1d-ENW - hom periodic oscillations; disconnected oscillating localized areas with almost same size and slightly varying frequency being separated by non-oscillating areas; intermittency for a sinusoidally driven system

exp: 2d-ENW - stat non-period inhom depending on initial conditions (due to network inhomogeneities); stat approximately centre symmetric circles; hom periodic oscillations

theo: $2-k$, R^2 - num: stat approximately centre symmetric circles (quant)

the quantitative theoretical description of the experimentally observed snaking in 1-dimensional electrical networks is reported without any fitting in [R. Schmeling, „Experimentelle und numerische Untersuchung von Strukturen in einem Reaktions-Diffusions-System anhand eines elektrischen Netzwerkes“, Thesis, University of Münster (1994); Purwins, Amiranshvili, Bödeker (to be submitted for publication in 2009)] - detection of a supercritical Turing bifurcation on an electronic network with correct scaling law and its quantitative theoretical description without any fitting - see also: [Reaction-Diffusion Equations: Analytical and Numerical Investigation](#)

Pu011: Purwins, Radehaus, Dirksmeyer, Dohmen, Schmeling, Willebrand (1989)

isolated stationary DSs

exp : 1d-ENW, $R_0 \neq 0$ - stat (bright) DSs; stat (dark ,invers) LSs

theo: $2-k + gc$, R^1 - num: stat (bright) DSs; stat (dark ,invers) LSs

bifurcation: snaking

exp : 1d-dc-GDS - snaking

theo: $2-k$, $\mu \neq 0 + gc$, R^1 - num: snaking

bifurcation: Turing

exp : 1d-ENW - stat (hom) \leftrightarrow (stat periodic), supercritical

Turing bif

theo: $2-k$, R^1 - anal: stat (hom) \leftrightarrow (stat periodic), supercritical

Turing bif (quant)

periodic pattern in R^1

exp: 1d-ENW - stat periodic

theo: $2-k$, R^1 - anal: stat periodic (quant)

focus on the importance of the activator-inhibitor principle for the investigated reaction-diffusion equations and the formation of localized

structures in physical systems - see also: **DC Gas-Discharge Systems: Experiment, Reaction-Diffusion Equations**

Pu013: Dirksmeyer, Schmeling, Berkemeier, Purwins (1990)

isolated stationary LSs

exp : 1d-ENW, $R_0 \neq 0$ - stat (bright) DSs; stat (dark ,invers) LSs

theo: 2-k + gc, R^1 - num: stat (bright) DSs; stat (dark ,invers) LSs

bifurcation: snaking

exp : 1d-ENW, $R_0 \neq 0$ - snaking

theo: 2-k + gc, R^1 - num: snaking

bifurcation: Turing

exp : 1d-ENW - stat (hom) \leftrightarrow (stat periodic), supercritical

Turing bif

theo: 2-k, R^1 - anal: stat (hom) \leftrightarrow (stat periodic), supercritical

Turing bif (quant)

periodic pattern in R^1

exp : 1d-ENW - stat periodic; front travelling through a stat hom state leaving behind a stat periodic pattern, "Turing fronts"

theo: 2-k, R^1 - num: stat periodic (quant); front travelling through a stat hom state leaving behind a stat periodic pattern, "Turing fronts" (quant)

discussion of the electrical network with respect to the introduction of "current diffusion" - detection of stationary dark (inverse) DSs - first quantitative description of the experimentally observed stationary dark (invers) DSs for 1-dimensional electrical networks without any fitting in [R. Schmeling, „Experimentelle und numerische Untersuchung von Strukturen in einem Reaktions-Diffusions-System anhand eines elektrischen Netzwerkes“, Thesis, University of Münster (1994); Purwins, Amiranshvil, Bödeker (to be submitted for publication in 2009)] by solving the corresponding 2-component reaction-diffusion equation - see remarks [Pu009] - see also: **Reaction-Diffusion Equations**

Pu027: Heidemann, Bode, Purwins (1993)

fronts

exp : 1d-ENW - front travelling through a hom oscillating state leaving behind a stat periodic pattern ("Turing fronts") and vice versa (Hopf-front)

theo: 2-k, R^1 - anal: front travelling through a hom oscillating state leaving behind a stat periodic pattern ("Turing fronts") and vice versa (Hopf-front)(semi-quant)

experimental detection of propagating "Turing-" and "Hopf-fronts" and semi-quantitative theoretical description - see also: **Reaction-Diffusion Equations**

- Pu032: Bode, Reuter, Schmeling, Purwins (1994)**
fronts
 exp : 2d-ENW - fronts with non-monotonic transition regions;
 bistability of moving fronts: uni- and bi- directional front
 propagating and corresponding bif
 theo: 2-k, R^1 - anal, bif: (uni-directional front propagating) \leftrightarrow
 (bi-directional front propagation) (quant)
experimental detection of bi-directional front propagating and quantitative
 analytical description of the corresponding bifurcation behaviour - see
 also: [Reaction-Diffusion Equations](#)
- Pu043: Bode, Purwins (1995)**
isolated stationary LSs
 exp: 1d-ENW , $R_0 \neq 0$ - stat (bright) DSs; stat (dark ,invers) LSs
bifurcation of DSs: snaking
 exp: 1d-ENW , $R_0 \neq 0$ - snaking
summary of parts of the works of e.g. [Pu013] - see also: [AC Gas-
 Discharge Systems: Experiment, Reaction-Diffusion Equations](#)
- Pu045: Kulka, Bode, Purwins (1995)**
fronts
 exp : 1d-ENW - front speed near to pinning in the presence of
 inhomogeneities
 theo: 1-k, R^1 - anal: front speed near to pinning in the presence of
 inhomogeneities (quant)
experimental determination of the influence of the inhomogeneity on the
 speed of front propagation - quantitative agreement between experiment
 and theory - see also: [Reaction-Diffusion Equations](#)
- Pu118: Purwins, Bödeker, Liehr (2005)**
 review of previous works on DSs in reaction diffusion systems -
 mechanisms of pattern formation in 2-k systems: Turing patterns,
 localized structures, activator-inhibitor principle - detailed derivation of
 the 2- and 3-component reaction-diffusion equation by starting from an
 electronic equivalent circuit - numerical solutions of the 3-component
 reaction-diffusion equation for isolated DS: DSs with non-oscillatory and
 oscillatory tails - numerical solutions of the 3-component reaction
 diffusion equation for two interacting DSs: scattering, formation of rotating
 molecules, annihilation, generation - analytical treatment of the 3-
 component reaction-diffusion equation: normal form for the drift
 bifurcation, reduction of the field equation to a particle equation for a single
 particle and for mutually interacting particles, calculation of the particle
 interaction law - numerical and analytical results: rotating molecule in R^3
 propagating along the axis of rotation - experimental results on DSs in dc
 gas-discharge: isolated DSs with non-oscillatory and oscillatory tails, drift
 bifurcation, interaction law, generation and annihilation - see also: [AC
 Gas-Discharge Systems: Experiment, Reaction-Diffusion Equations](#)

Pu127: Purwins, Amiranashvili (2007)

summary - simple patterns: e.g. isolated DS, stripes, hexagons and rotating spirals - patterns of higher complexity with DSs as elementary building blocks: e.g. "molecules" and "many body systems" in the form of crystal-, liquid-, gas-like structures, chains and nets - universal experimental behaviour for a certain class of systems containing planar ac and dc gas-discharge systems, electrical networks, semiconductor layer systems, chemical solutions and biological systems - theoretical definition of the corresponding universality class using the 3-component reaction-diffusion system as a kind of normal form for the qualitative description of the experimentally observed self-organized patterns - illustration of the formation of DSs in planar electrical transport systems on the basis of the 2-component reaction diffusion equation - see also: DC Gas-Discharge Systems: Experiment, AC Gas-Discharge Systems: Experiment, Gas-Discharge: Theory, Semiconductors: Experiment, Semiconductors: Theory, Reaction-Diffusion Equations