8th IAPR - TC-15 Workshop on
Graph-based Representations in Pattern Recognition

Smooth Simultaneous Structural Graph Matching and Point-Set Registration

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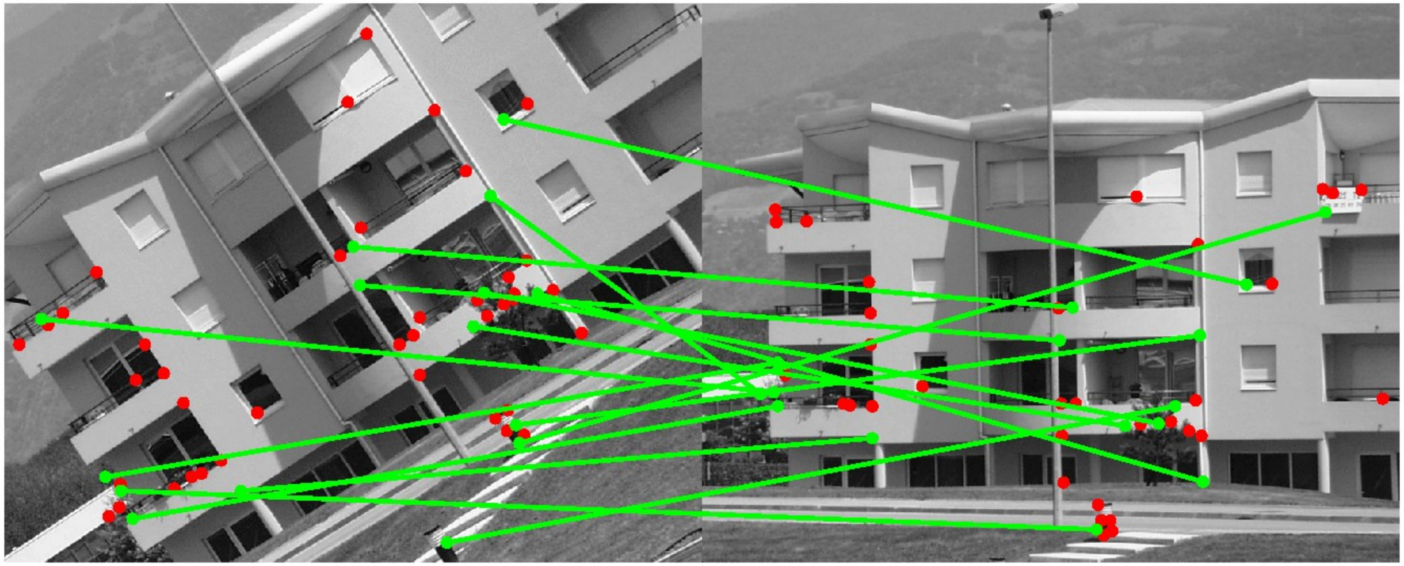
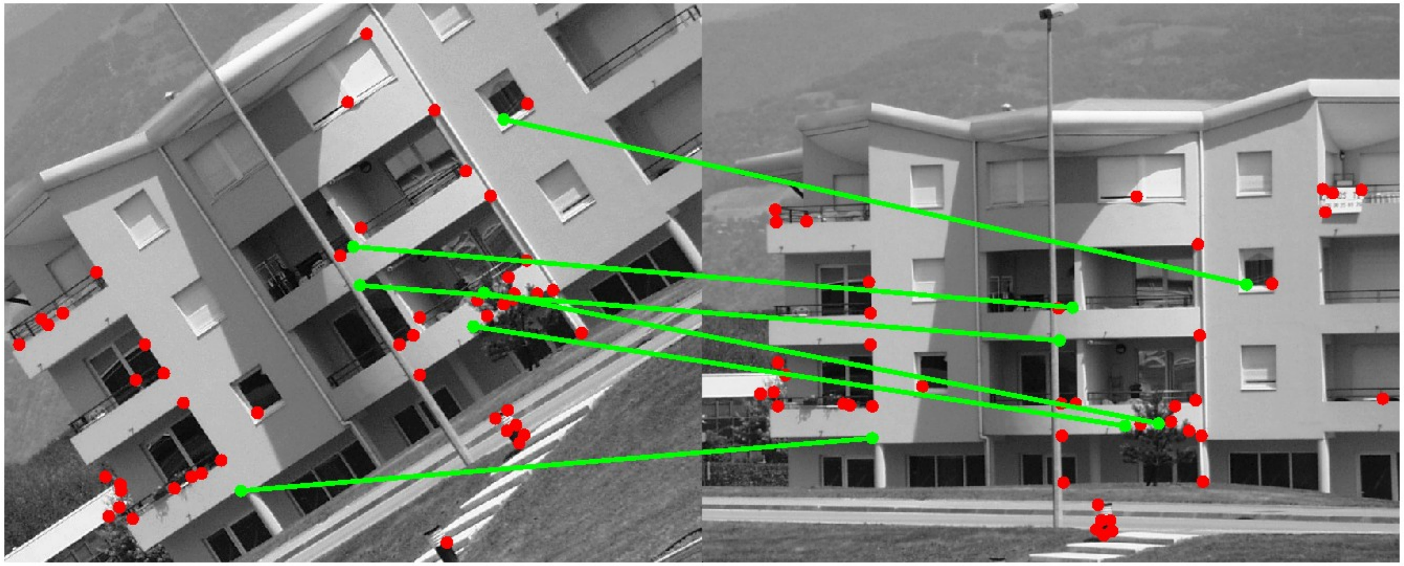
Motivation

- Point-set correspondence/registration is of basic importance in computer vision.
- Tentative correspondences computed using local image contents may fail due to changes in light, viewpoint, ...



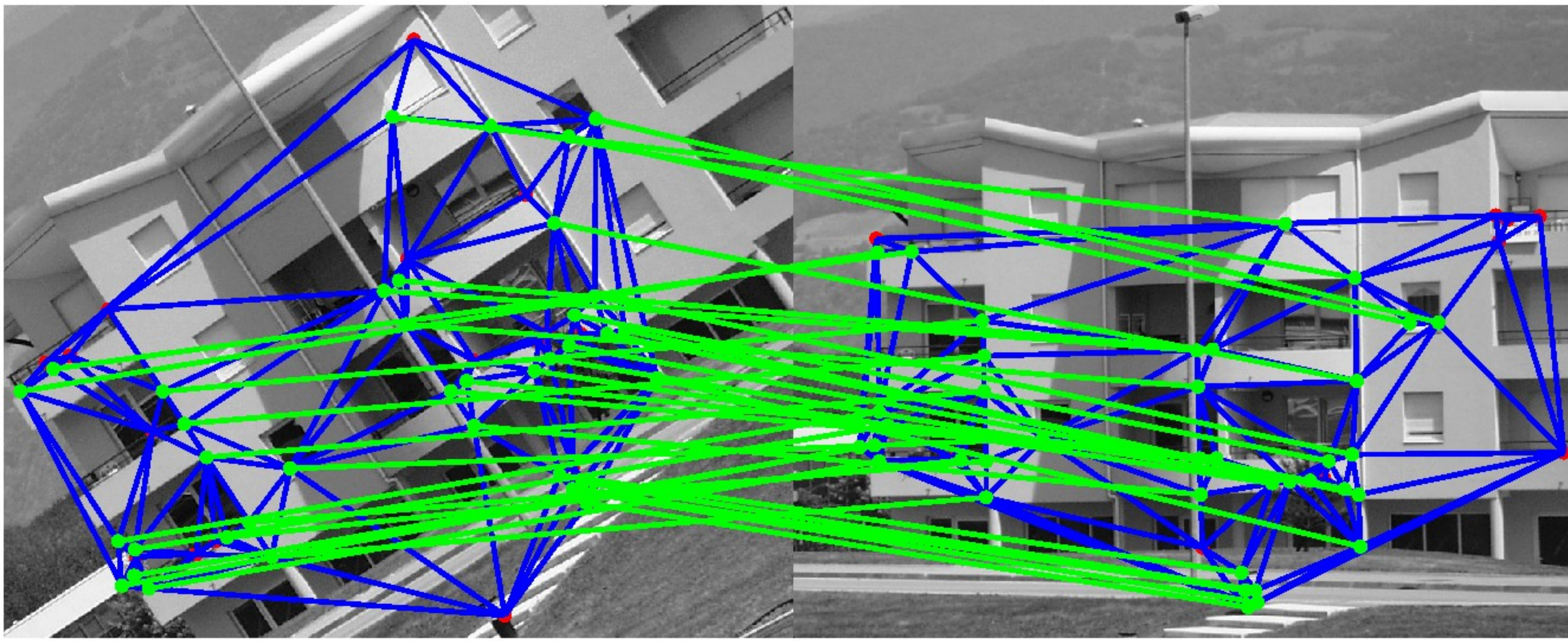
Motivation

- Outlier rejectors may end up with too sparse correspondences if the tentative sets are not accurate enough



Contributions

- We present a model for graph matching that exploits structural relations as well as geometrical arrangement of the points
- We derive the EM algorithm to solve for the registration and correspondence parameters.
- We solve the problem in the domain of continuous (smooth) correspondence indicators using Softassign
- We develop effective mechanisms to reject outliers in both graphs



Continuous Correspondences and Mixture Model

- Correspondences: $S\ ,\ s_{a\alpha} \in S$
  - Subject to the constraint: $s_{a\alpha} \geq 0\ ,\ \sum_{\alpha \in \mathcal{J}} s_{a\alpha} \leq 1\ ,\ \forall a \in \mathcal{I}$

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- Problem formulation:
  - Maximum Likelihood estimation from mixture distribution

Observed graph

Affine registration parameters

$$\arg\max_{S,\Phi} \{ P(G | S, \Phi) \} =$$

Observed graph nodes

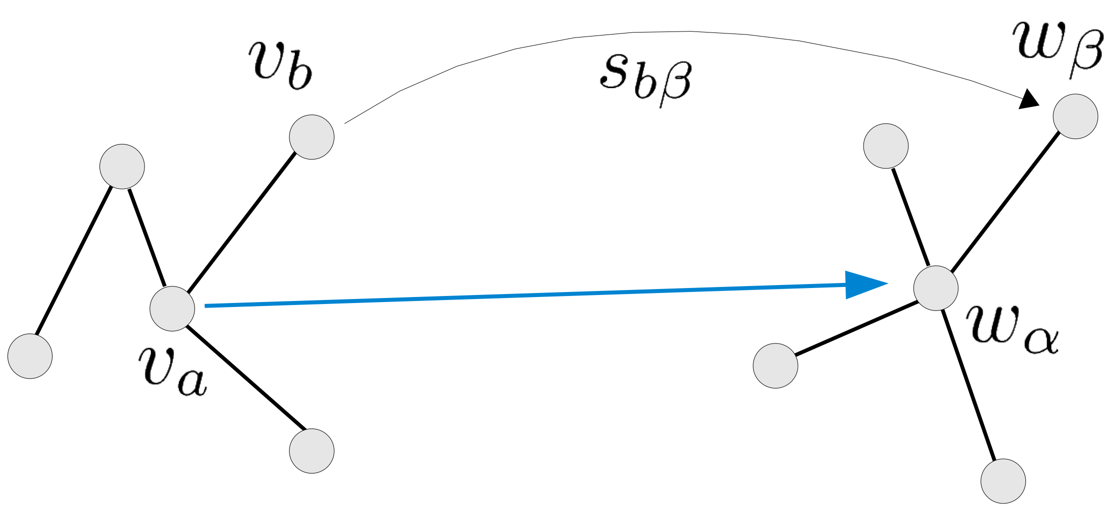
$$\arg\max_{S,\Phi} \left\{ \prod_{a \in \mathcal{I}} \sum_{\alpha \in \mathcal{J}} P(v_a, w_\alpha | S, \Phi) \right\}$$

Hidden graph nodes

Probability Density Function

- We factorize the probability terms using a similar development than Luo and Hancock

$$P\left(v_a, w_\alpha | S, \Phi\right) \propto \prod_{b \in \mathcal{I}} \prod_{\beta \in \mathcal{J}} P\left(v_a, w_\alpha | s_{b\beta}, \Phi\right)$$



Probability Density Function

Adjacency matrices

$$P\left(v_a, w_\alpha \middle| s_{b\beta} \quad\right) =$$

$$\begin{cases} (1 - P_e) & \text{if } s_{b\beta} = 1 \wedge (D_{ab} = 1 \wedge E_{\alpha\beta} = 1) \\ P_e & \text{if } s_{b\beta} = 1 \wedge (D_{ab} = 0 \vee E_{\alpha\beta} = 0) \\ P_e & \text{if } s_{b\beta} = 0 \end{cases}$$

Probability Density Function

$$P\left(v_a, w_\alpha \middle| s_{b\beta}, \Phi\right) =$$

$$\begin{cases} \left(1 - P_e\right) P_{b\beta} & \text{if } s_{b\beta} = 1 \wedge \left(D_{ab} = 1 \wedge E_{\alpha\beta} = 1\right) \\ P_e P_{b\beta} & \text{if } s_{b\beta} = 1 \wedge \left(D_{ab} = 0 \vee E_{\alpha\beta} = 0\right) \\ P_e \rho & \text{if } s_{b\beta} = 0 \end{cases}$$

Probability Density Function

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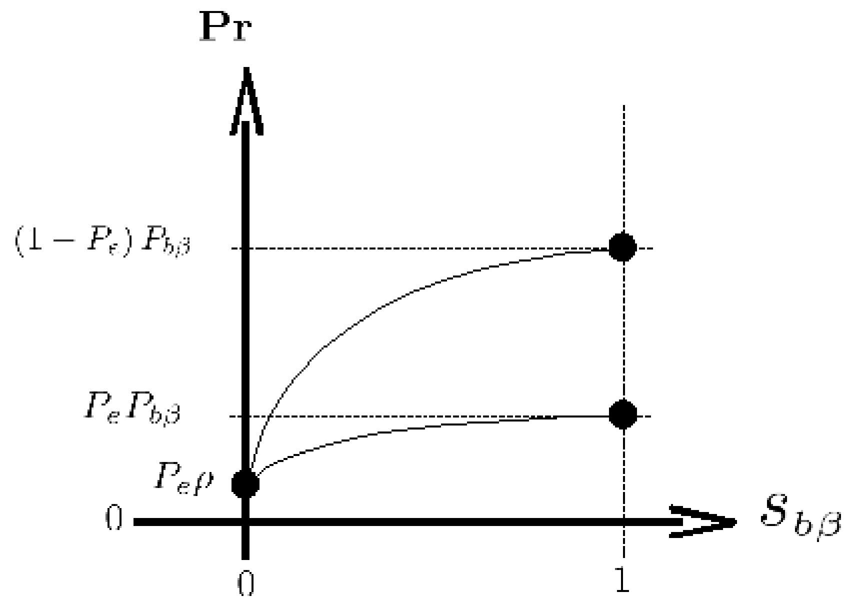
- We extend to continuous correspondence indicators by exploiting each case as an exponential indicator

$$P\left(v_a, w_\alpha \middle| s_{b\beta}, \Phi\right) =$$

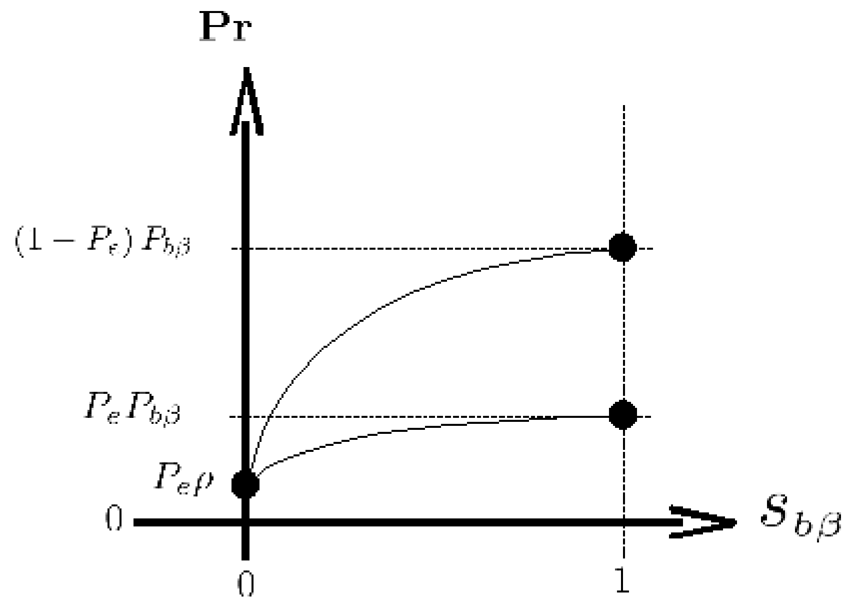
$$\left[\left(1 - P_e\right) P_{b\beta}\right]^{D_{ab} E_{\alpha\beta} s_{b\beta}} \left[P_e P_{b\beta}\right]^{\left(1 - D_{ab} E_{\alpha\beta}\right) s_{b\beta}} \left[P_e \rho\right]^{\left(1 - s_{b\beta}\right)}$$

Probability Density Function



Geometrical term $P_{b\beta}$ above the outlying threshold ρ

Probability Density Function



Geometrical term $P_{b\beta}$ above the outlying threshold ρ

- The final expression using the exponential form is

$$P\left(v_a, w_\alpha | S, \Phi\right) = \exp\left[\sum_{b \in \mathcal{I}} \sum_{\beta \in \mathcal{J}} s_{b\beta} D_{ab} E_{\alpha\beta} \ln\left(\frac{1-P_e}{P_e}\right) + s_{b\beta} \ln\left(\frac{P_{b\beta}}{\rho}\right) + \ln\rho\right]$$

EM Algorithm. Expectation

- The EM algorithm states that maximizing the original incomplete-data likelihood is equivalent at maximizing the expected complete data log-likelihood conditioned by the observed data

$$\Lambda\left(\hat{S},\hat{\Phi}|S^{(n)},\Phi^{(n)}\right)=\sum_{a\in\mathcal{I}}\sum_{\alpha\in\mathcal{J}}P\left(w_\alpha|v_a,S^{(n)},\Phi^{(n)}\right)\ln P\left(v_a,w_\alpha|\hat{S},\hat{\Phi}\right)$$

- The posterior probabilities weigh the contributions of the complete data log-likelihood

$$P\left(w_\alpha|v_a,S^{(n)},\Phi^{(n)}\right)=\frac{P\left(v_a,w_\alpha|S^{(n)},\Phi^{(n)}\right)}{\sum_{\alpha'}P\left(v_a,w_{\alpha'}|S^{(n)},\Phi^{(n)}\right)}\equiv R_{a\alpha}^{(n)}$$

EM Algorithm. Maximization

- Maximization can be done in two steps according to Expectation Conditional Maximization
- Maximum Likelihood (ML) affine registration parameters

$$\Phi^{(n+1)} = \arg\max_{\hat{\Phi}} \left\{ \sum_{a \in \mathcal{I}} \sum_{\alpha \in \mathcal{J}} R_{a\alpha}^{(n)} \sum_{b \in \mathcal{I}} \sum_{\beta \in \mathcal{J}} s_{b\beta}^{(n)} \ln\left(\frac{P_{b\beta}(\hat{\Phi})}{\rho} \right) \right\}$$

$$\Phi^{(n+1)} = \arg\max_{\hat{\Phi}} \left\{ \sum_{b \in \mathcal{I}} \sum_{\beta \in \mathcal{J}} s_{b\beta}^{(n)} \ln P_{b\beta}\left(\hat{\Phi}\right) \right\}$$

ML Correspondence Parameters

- One of the main contributions of our work is to formulate the recovery of the correspondences as a succession of linear assignment problems

- The EM update rule according to our model is

$$S^{(n+1)} =$$

$$\arg\max_{\hat{S}} \left\{ \sum_{a \in \mathcal{I}} \sum_{\alpha \in \mathcal{J}} R_{a\alpha}^{(n)} \sum_{b \in \mathcal{I}} \sum_{\beta \in \mathcal{J}} \hat{s}_{b\beta} \left[D_{ab} E_{\alpha\beta} \ln\left(\frac{1-P_e}{P_e}\right) + \ln\left(\frac{P_{b\beta}^{(n)}}{\rho}\right) \right] \right\}$$

ML Correspondence Parameters

- Which can be expressed as a linear assignment problem

$$S^{(n+1)} = \arg\max_{\hat{S}} \left\{ \sum_{b \in \mathcal{I}} \sum_{\beta \in \mathcal{J}} \hat{s}_{b\beta} Q_{b\beta}^{(n)} \right\}$$

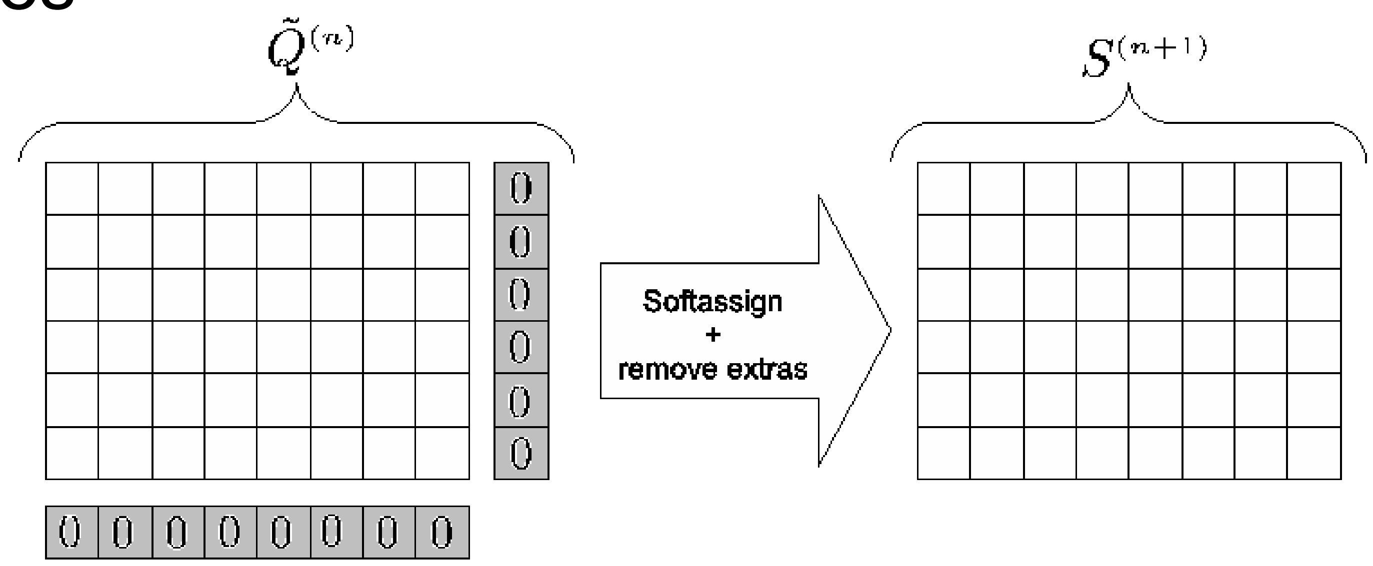
where the benefit coefficients are

$$Q_{b\beta}^{(n)} = \sum_{a \in \mathcal{I}} \sum_{\alpha \in \mathcal{J}} R_{a\alpha}^{(n)} \left[D_{ab} E_{\alpha\beta} \ln \left(\frac{1-P_e}{P_e} \right) + \ln \left(\frac{P_{b\beta}}{\rho} \right) \right]$$

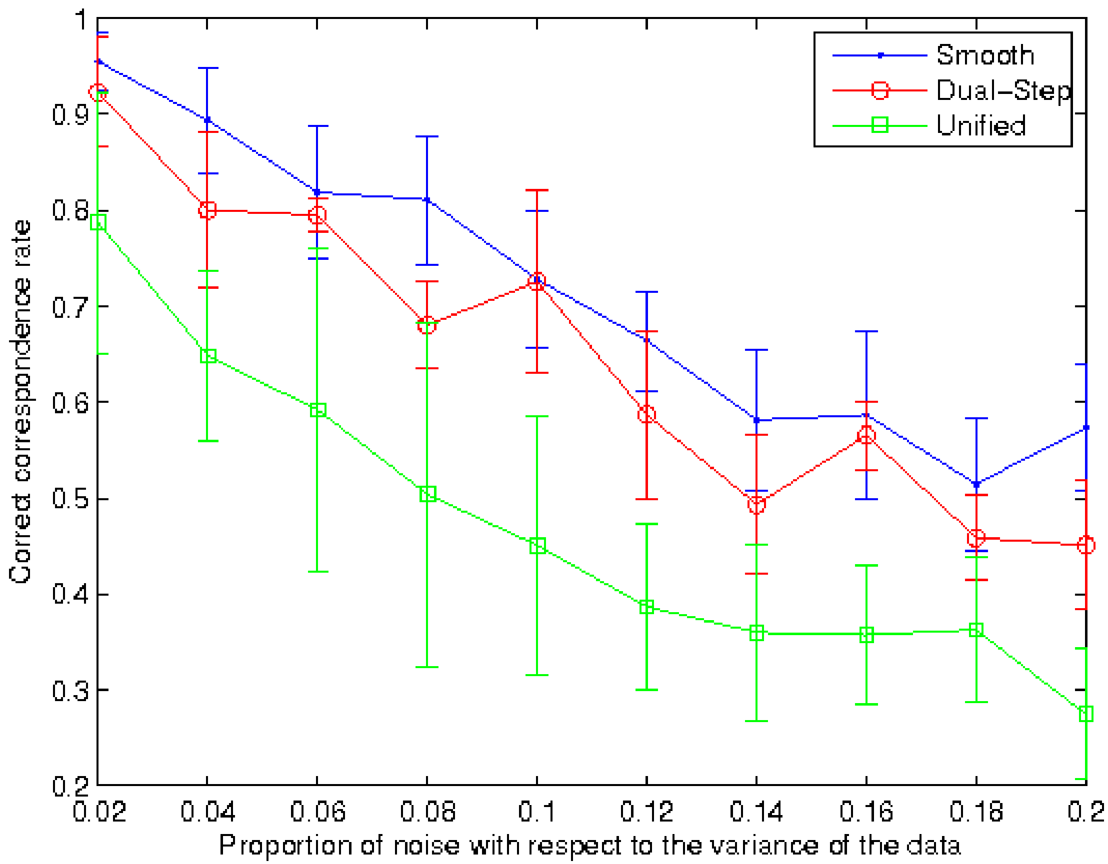
- We use one step of the Softassign each time instead of the whole annealing procedure (Generalized EM).

Outlier Rejection

- Outlier detection is modeled as an assignment to (or from) the NULL node
- We consider that (1) the NULL node has no edges and (2) $P_{b\emptyset} = \rho\,, \forall\, b \in \mathcal{I}$ $\quad P_{\emptyset\beta} = \rho\,, \forall\, \beta \in \mathcal{J}$
  - Therefore NULL-assignments are equivalent to zero benefit values



Results. Synthetic Random Gaussian Noise



Compared Methods

Smooth: Presented method
Dual-Step: Cross and Hancock
Unified: Luo and Hancock

Average Run Time

Smooth: 0.66 sec.
Dual-Step: 14.08 sec.
Unified: 0.91 sec.

Results. Matching of Real Images

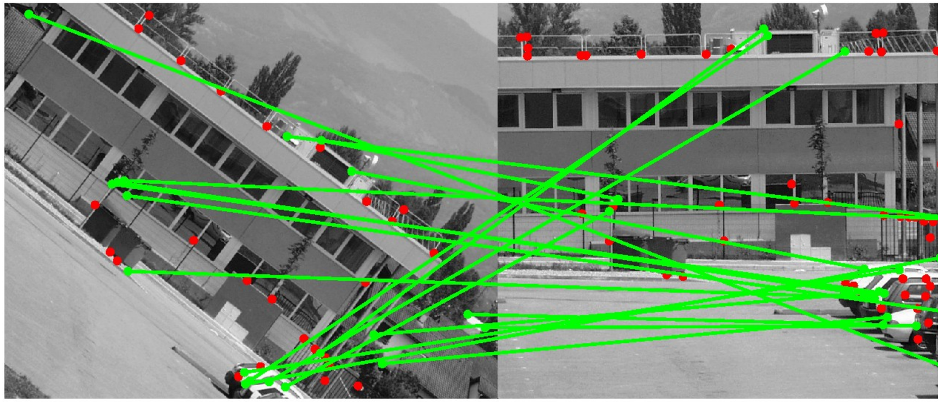
- Oxford Homography Dataset
- Compared Methods:
  - Smooth: Presented method
  - Dual-Step: Cross and Hancock
  - RANSAC: Fischler and Bolles
  - GTM: Aguilar *et al*.

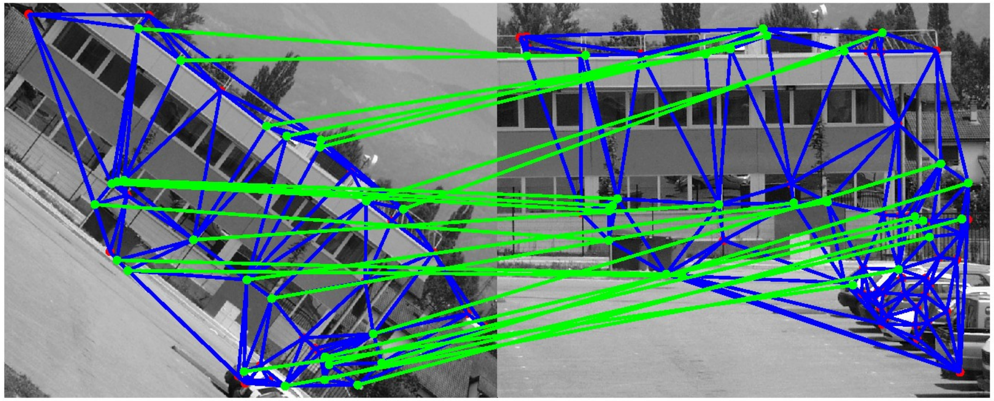
Graph matching

Outlier rejection

- Initialization: Matching by Correlation (Corr)

Results. Matching of Real Images





	Resid		Boat		NewYork		Laptop		Eastpark	
Methods	**MPE**	**time**	**MPE**	**time**	**MPE**	**time**	**MPE**	**time**	**MPE**	**time**
Corr	835.4	1.5	24.48	1.5	31.1	0.98	**0.28**	1.34	463.4	1.62
Smooth	1.5	13.8	**0.72**	18.2	**0.69**	3.6	0.29	8.18	**1.08**	19.7
Dual-Step	**1.33**	3615	1.68	3794	**0.69**	1429	0.3	2693	153.46	3027
RANSAC	20.23	0.2	1.6	0.1	17.11	0.12	**0.28**	0.11	350.13	0.42
GTM	24.15	0.02	0.8	0.1	3.2	0.02	0.34	0.02	359.9	0.04