Dimensionality Reduction for Graph of Words Embedding

GbR'2011 Oral talk

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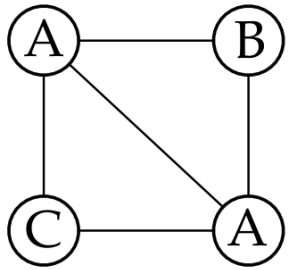
Are these features really non-informative?

Simple features

- Number of nodes, number of edges
- Number of nodes with label A, or label B,...
- Number of edges between label A and label C,...
- ...



$\longrightarrow$ These features might not be informative nor discriminative.

Goals of the paper

- Define a new embedding methodology exploiting features based on statistics of node labels

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- Adapt these features to graphs with continuous labels on nodes

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- Deal with an inherent issue of the methodology: high dimensionality and sparsity

  → Dimensionality reduction

Motivation

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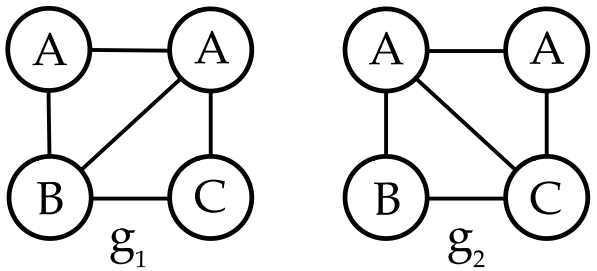
- For instance,



$$\begin{aligned}\mathbf{x}_{g_1} = \mathbf{x}_{g_2} &= (\#(A, g), \#(B, g), \#(C, g)) \\ &= (2, 1, 1)\end{aligned}$$

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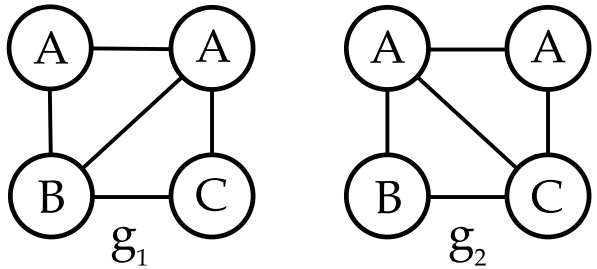

- More features taking topology into account:

$$\#(l_i \leftrightarrow l_j, g),$$

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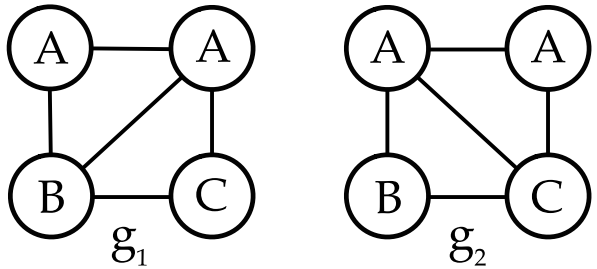

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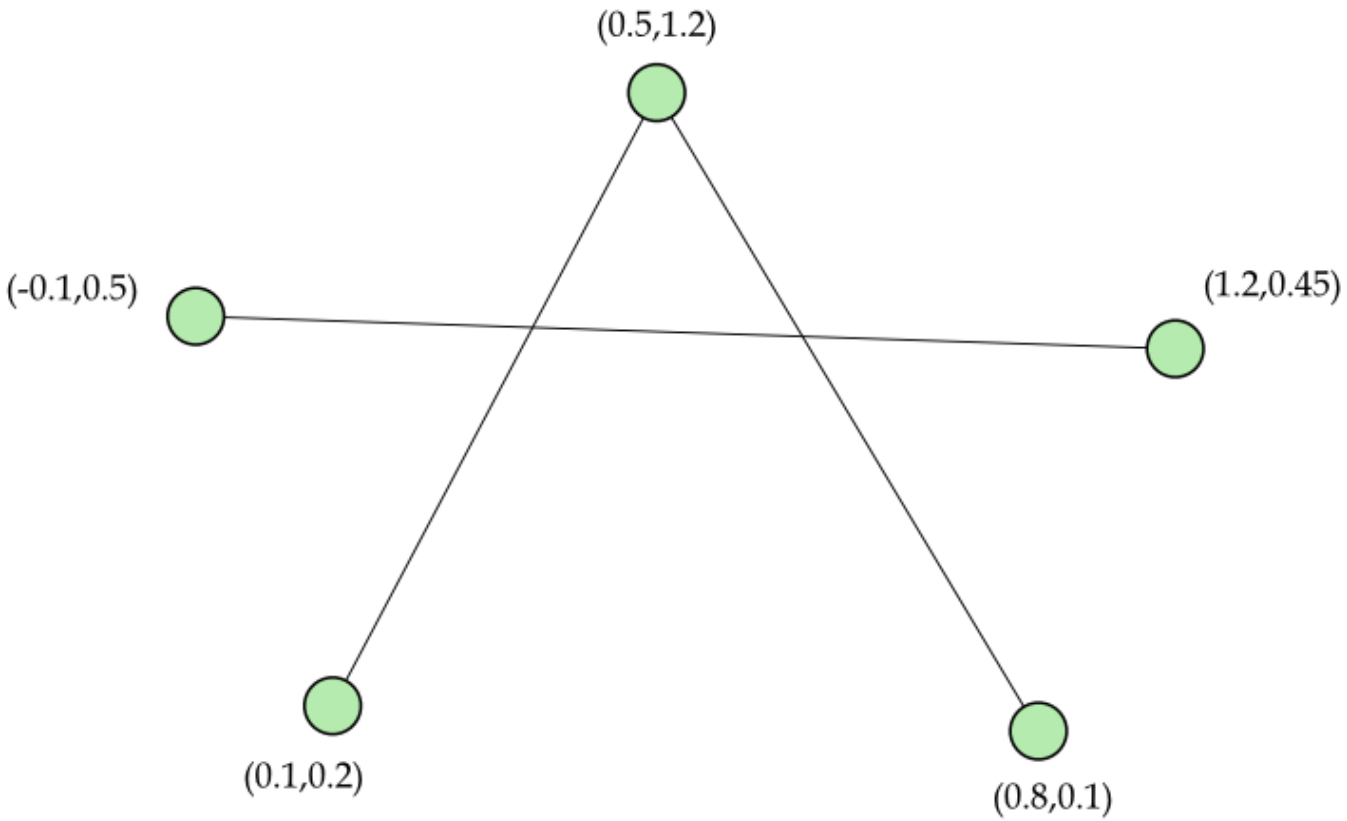
$$\begin{aligned}\mathbf{x}_{g_1} &= (\overbrace{2,1,1}^{\text{nodes}}, \overbrace{1,2,1,0,1,0}^{\text{edges}}),\\ \mathbf{x}_{g_2} &= (2,1,1,1,1,2,0,1,0).\end{aligned}$$

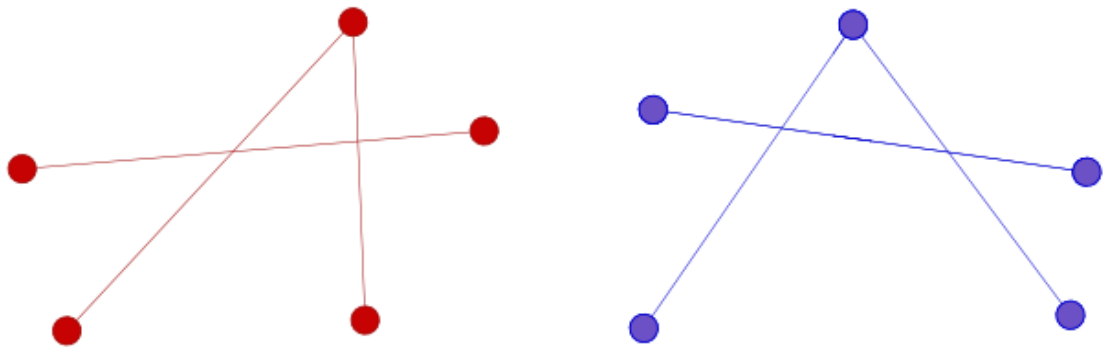
Problem

What if we have graphs with continuous labels?



Idea

→ The more similar two labels are, the more they should count for the same label:



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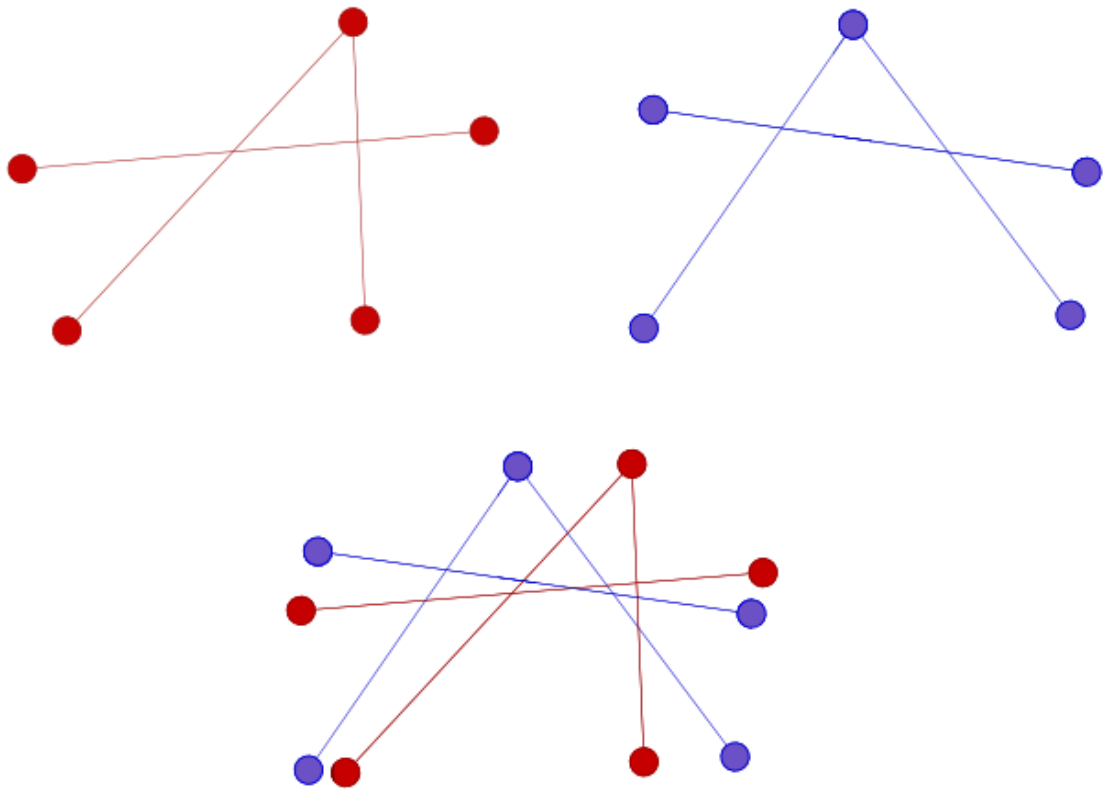

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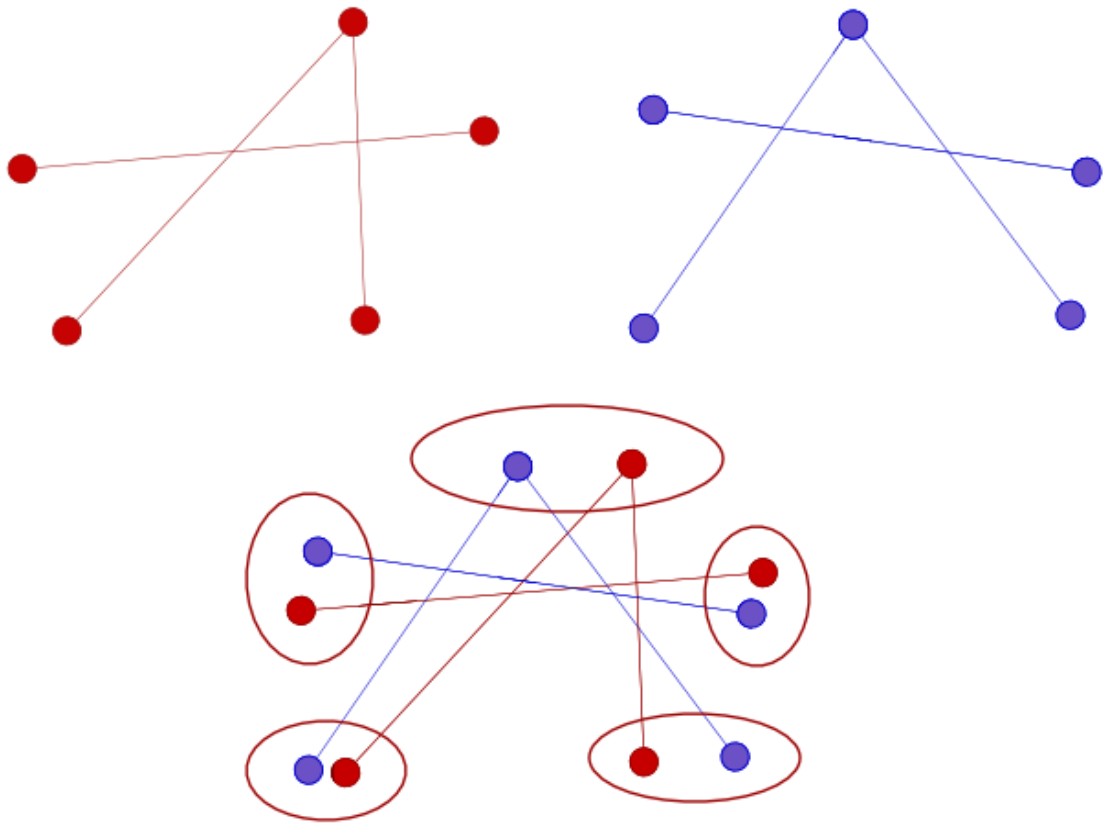

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$$v \longmapsto \lambda_h(v) = \underset{w_i \in \mathcal{W}}{\operatorname{argmin}} \parallel \mu(v) - w_i \parallel_2$$

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- Each edge counts as a relation between the words assigned to its nodes
- Thus we have the features

$$U_i = \#(w_i, g) = |\{v \in V \,|\, w_i = \lambda_h(v)\}|$$

and

$$\begin{aligned} B_{ij} &= \#(w_i \leftrightarrow w_j, g) \\ &= |\{(u, v) \in E \,|\, w_i = \lambda_h(u) \wedge w_j = \lambda_h(v)\}|. \end{aligned}$$

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$\longrightarrow$ Dimensionality Reduction

DR methods

In order to solve the dimensionality problem we have applied two different dimensionality reduction techniques

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- Independent Component Analysis(ICA)
  - Finds linearly uncorrelated components
  - Statistically independent components
  - Maximize the non-Gaussianity of data

Aim of the experimentation

- Discover the effect of the reduction by classification rates on
  - The raw vectors
  - The kPCA reduced vectors
  - The ICA reduced vectors
- We use a kNN classifier and validate the meta-parameters using a validation set

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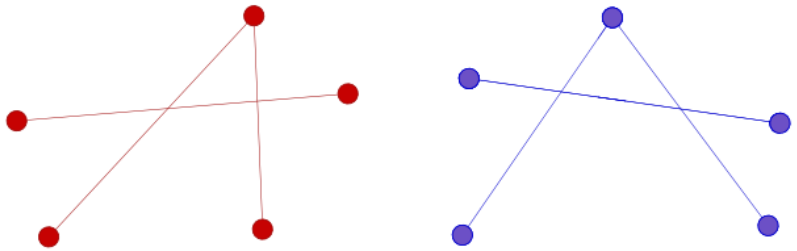

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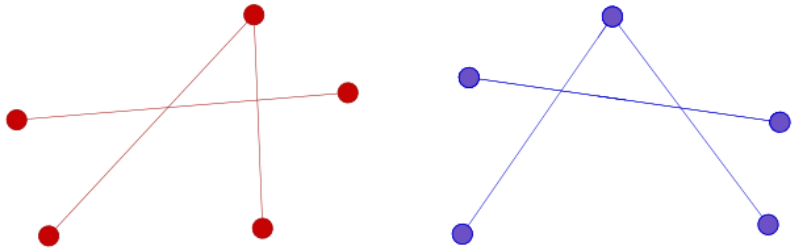
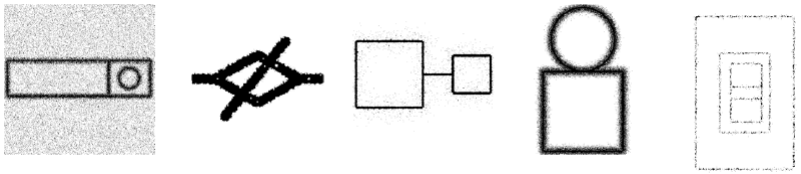


- GREC



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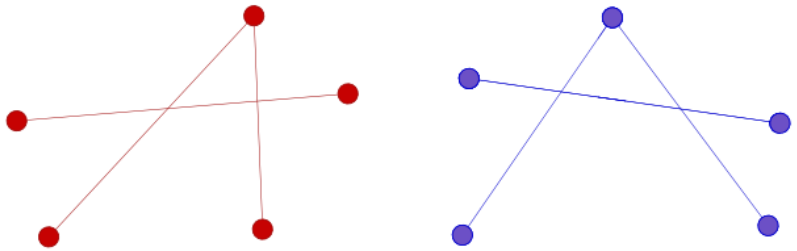

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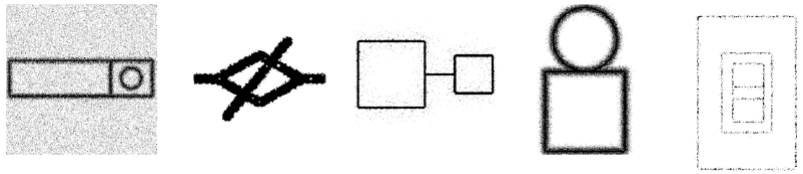

- Fingerprints

Results on validation

Selection of representatives: kMeans

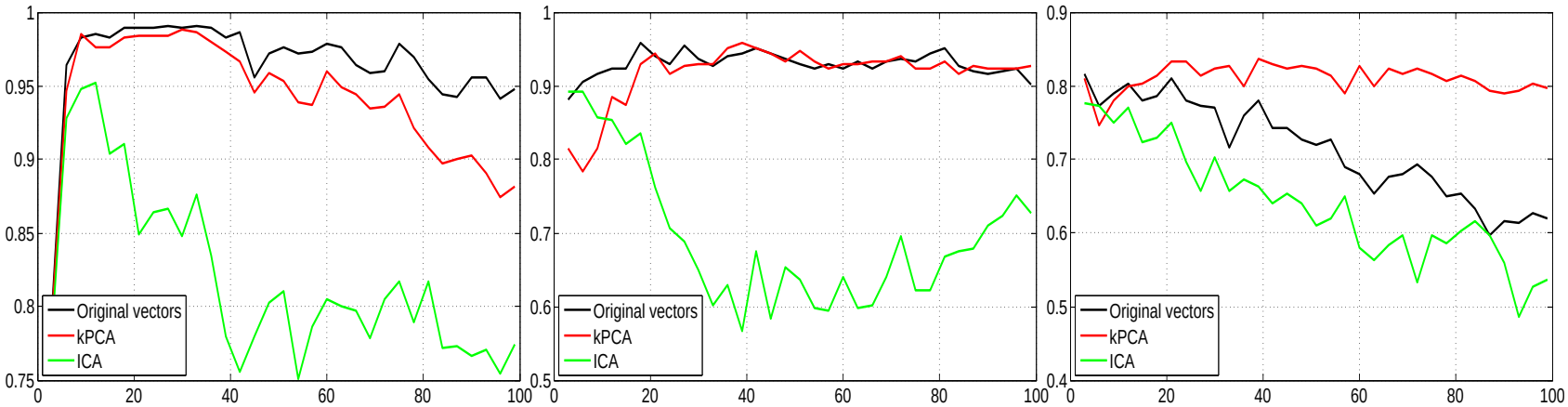

Figure: Accuracy rate on the validation set (vertical axis) for every vocabulary size (horizontal axis). These figures depict the comparison between the classification of the original vectors (without reduction) with the two dimensionality reduction techniques, kPCA and ICA.

Results on test

	Letter		GREC		Fingerprint	
	AR	Dims	AR	Dims	AR	Dims
Original vectors	**98.8**	702	**97.5**	3402	77.7	189
kPCA	97.6	43	97.1	67	**80.6**	108
ICA	82.8	251	58.9	218	63.3	184

- Results with kPCA are as good as with raw vectors while reducing dimensionality
- ICA does not provide good results due to the correlation between features

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- Other selection methods for the set of representatives
- Combination of different sets of representatives

Thanks for the attention!

Questions?