Basic notions of rigid analytic geometry

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The purpose of my lectures at the conference was to introduce the newcomer to the field of rigid analytic geometry. Precise definitions of the key notions and precise statements of the basic facts were given. But, of course, the limited time did not allow to include any proofs. Instead the emphasis was placed on motivating and explaining the shape of the theory. The positive response from the audience encouraged me to write up the following notes which reproduce my lectures in an essentially unchanged way. I hope that they can serve as a means to quickly grasp the basics of the field. Of course, anybody who is seriously interested has to go on and has to dig into the proper literature.

Rigid or non-archimedean analysis takes place over a field K which is complete with respect to a non-archimedean absolute value $|\ |$. The most important examples are the fields of p-adic numbers $\mathbb{Q}_p$ where p is some prime number. For technical purposes we fix throughout an algebraic closure $\overline{K}$ of K and denote by $\widehat{\overline{K}}$ its completion which again is algebraically closed. The absolute value $|\ |$ extends uniquely to an absolute value $|\ |$ of $\widehat{\overline{K}}$.

Fix a natural number $n \in \mathbb{N}$ and let us consider the "n-dimensional polydisk"

$$\mathbb{B}^n := \{(z_1, \ldots, z_n) \in \widehat{\overline{K}}^n : \max |z_i| \leq 1\}$$

in the n-dimensional vector space over $\widehat{\overline{K}}$. We clearly want this polydisk to be a geometric object (something like a "manifold") in our theory. For this we have to decide which functions on $\mathbb{B}^n$ we will call "analytic". The naive answer would be to follow real or complex analysis and to call a function analytic if it has, locally around each point, a convergent Taylor expansion. But we are dealing here with a non-archimedean metric on $\widehat{\overline{K}}^n$ satisfying the strict triangle inequality. This implies that the topology of $\widehat{\overline{K}}^n$ or $\mathbb{B}^n$ is totally disconnected so that there is a huge supply of functions on $\mathbb{B}^n$ which even are locally constant. For this reason that naive definition cannot lead to satisfying geometric properties. (But it has its use and importance in non-archimedean measure theory. Usually it is qualified as "locally analytic" in contrast to "rigid analytic".)

Going somehow to the other extreme let

$$f(T_1, \ldots, T_n) = \sum_{\nu_i \geq 0} a_{\nu_1, \ldots, \nu_n} T_1^{\nu_1} \cdot \ldots \cdot T_n^{\nu_n}$$

be an arbitrary formal power series with coefficients in K. The following two properties are quite immediate:

* The power series f converges on all of $\mathbb{B}^n$ if and only if the coefficients tend to zero, i.e., $|a_{\nu_1, \ldots, \nu_n}| \longrightarrow 0$ if $\nu_1 + \ldots + \nu_n \longrightarrow \infty$.

* If f converges on $\mathbb{B}^n$ then we have $f(\mathbb{B}^n(L)) \subseteq L$ for any intermediate field $K \subseteq L \subseteq \widehat{\overline{K}}$ which is finite over K; here $\mathbb{B}^n(L)$ denotes the set of those vectors

in $\mathbb{B}^n$ with coordinates in L.
The subalgebra

$$T^n := \{f \in K[[T_1, \ldots, T_n]] \ : \ f \text{ converges on } \mathbb{B}^n\}$$

of the algebra $K[[T_1, \ldots, T_n]]$ of formal power series over K is called a **Tate algebra.** We may and will say that any $f \in T^n$ induces an "analytic function on $\mathbb{B}^n$ defined over K". Why is this a good notion? At first glance it does not seem to be local at all! Certainly we do not want to give up completely the possibility of recognizing the analyticity of a function locally. In order to prepare the way out of this apparent trap we first collect a number of properties of the algebra T^n. The two most basic ones are the following:

1) T^n is a K-Banach algebra w.r.t. the multiplicative norm

$$|f| := \max |a_{\nu_1,\ldots,\nu_n}| \ .$$

2) The Maximum Modulus Principle holds:

$$|f| = \max_{z \in \mathbb{B}^n} |f(z)| \ ;$$

in particular: If $f(z) = 0$ for any $z \in \mathbb{B}^n$ then $f = 0$.
The proof of the Maximum Modulus Principle is actually very easy: By scaling we may assume that $|f| = 1$. We then can reduce f modulo the maximal ideal of K obtaining, because of the convergence criterion, a nonzero polynomial $\tilde{f}$ over the residue class field of K. Since the residue field of $\widehat{\overline{K}}$ is infinite we find a point $\tilde{z}$ with coordinates in the latter such that $\tilde{f}(\tilde{z}) \neq 0$. Any lifting $z \in \mathbb{B}^n$ of $\tilde{z}$ then satisfies $|f(z)| = 1$.

Next one shows that the Weierstrass theory (preparation theorem, ...) works for T^n. This eventually leads to many ring theoretic properties of T^n:

3) T^n is noetherian and factorial.

4) T^n is Jacobson, i.e., for any ideal $\mathfrak{a} \subseteq T^n$ its radical ideal $\sqrt{\mathfrak{a}}$ is the intersection of all the maximal ideals containing $\mathfrak{a}$.

5) Any ideal in T^n is closed.

6) For any maximal ideal $\mathfrak{m}$ in T^n, the residue field $T^n/\mathfrak{m}$ is a finite extension of K.
This last property is an analogue of Hilbert's Nullstellensatz. It has the interesting consequence that the map

$$\begin{array}{rcl} \text{Galois orbits in } \mathbb{B}^n(\overline{K}) & \xrightarrow{\sim} & \mathrm{Max}(T^n) \\ z & \longmapsto & \mathfrak{m}_z := \{f : f(z) = 0\} \end{array}$$

is a bijection. The inverse map is obtained as follows: For a maximal ideal $\mathfrak{m}$ let φ denote the composite of the projection $T^n \twoheadrightarrow T^n/\mathfrak{m}$ and some embedding $T^n/\mathfrak{m} \hookrightarrow \overline{K}$ and put $z_i := \varphi(T_i)$.

In this way the maximal ideal spectrum $\mathrm{Max}(T^n)$ appears as an algebraically defined model for the space $\mathbb{B}^n$. This suggests we should proceed as Grothendieck did in algebraic geometry and define a category of analytic spaces as maximal ideal spectra of certain algebras. (The property 4) is the reason that maximal ideals in contrast to arbitrary prime ideals will suffice.)

Definition:

A K-algebra A is called affinoid if $A \cong T^n/\mathfrak{a}$ for some $n \in \mathbb{N}$ and some ideal $\mathfrak{a}$.

For any affinoid algebra A we have:

* A is noetherian and Jacobson (by 3) and 4)).
* A is a K-Banach algebra with respect to any residue norm (by 5)). Moreover:
  - The topology on A is independent of the chosen residue norm;
  - any homomorphism between affinoid K-algebras is automatically continuous.

We put

$$\mathrm{Max}(A) := \text{ set of all maximal ideals of } A \ .$$

By 6), this set depends functorially on A. Also by 6) we may define the so-called supremum or spectral seminorm on A by

$$|f|_{\sup} := \sup_{x \in \mathrm{Max}(A)} |f(x)|$$

where $f(x) := f + \mathfrak{m}_x \in A/\mathfrak{m}_x \hookrightarrow \overline{K}$. It is obviously bounded above by any residue norm. The general **Maximum Modulus Principle** says that

$$|f|_{\sup} = \max_{x \in \mathrm{Max}(A)} |f(x)| \ .$$

If A is reduced then $|\ |_{\sup}$ is a norm which is equivalent to any residue norm.

Using the above description of $\mathrm{Max}(T^n)$ as the Galois orbits in $\mathbb{B}^n(\overline{K})$ we obtain (from the metric topology on $\overline{K}$) a "canonical" Hausdorff topology on $\mathrm{Max}(A)$. Of course it is totally disconnected so that our initial problem persists.

In order to emphasize the geometric intuition we write $X := \mathrm{Max}(A)$ from now on. For any functions $g, f_1, \ldots, f_m \in A$ without common zero (i.e., generating the unit ideal $\langle g, f_i \rangle = A$) we introduce the open subset

$$X(\frac{f.}{g}) := \{x \in X : \max_i |f_i(x)| \leq |g(x)|\}$$

of X called a **rational subdomain.** It is not hard to see that the rational subdomains form a basis for the canonical topology on X.

Very important observation:

The K-algebra $A\langle \frac{f.}{g} \rangle := A\langle T_1, \ldots, T_m \rangle / \langle gT_i - f_i \rangle$ is affinoid and the map

$$\mathrm{Max}(A\langle \frac{f.}{g} \rangle) \longrightarrow \mathrm{Max}(A) = X$$

induced by the obvious algebra homomorphism $A \longrightarrow A\langle \frac{f.}{g} \rangle$ is a homeomorphism onto $X(\frac{f.}{g})$.

Comments: – $A\langle T_1, \ldots, T_m \rangle$ is the algebra of all power series in the variables $T_1, \ldots, T_m$ with coefficients in A tending towards 0.
– The affinoid algebra $A\langle \frac{f.}{g} \rangle$ can be characterized by a universal property which is given solely in terms of the rational subdomain $X(\frac{f.}{g})$.
– In $A\langle \frac{f.}{g} \rangle$ we have $|\frac{f_i}{g}| = |$ residue class of $T_i| \leq 1$.
– The assertion becomes wrong without the assumption that $\langle g, f_i \rangle$ is the unit ideal (look at $A = K\langle T \rangle$, $m = 1$, $g = T$, and $f_1 = 0$).
This observation allows to define a presheaf $\mathcal{O}_X$ at least on the rational subdomains in X by

$$\mathcal{O}_X(X\langle \frac{f.}{g} \rangle) := A\langle \frac{f.}{g} \rangle \ .$$

Main Theorem of Tate:

If $Y, Y_1, \ldots, Y_r \subseteq X$ are rational subdomains such that $Y = Y_1 \cup \ldots \cup Y_r$ then $\mathcal{O}_X$ satisfies the sheaf property for that covering, i.e.,

$$0 \longrightarrow \mathcal{O}_X(Y) \longrightarrow \prod_i \mathcal{O}_X(Y_i) \rightrightarrows \prod_{i,j} \mathcal{O}_X(Y_i \cap Y_j)$$

is exact.

This means that the notion of an analytic function on X, i.e., an element of A, is local as far as finite coverings by rational subdomains are concerned!

This picture can be "enlarged" by a completely formal construction:
$*$ A subset $U \subseteq X$ is called **admissible open** if there are rational subdomains $U_i \subseteq X$ for $i \in I$ such that

i. $U = \bigcup\limits_{i \in I} U_i$ (in particular U is open in the canonical top.) and

ii. for any map $\alpha : Y := \mathrm{Max}(B) \longrightarrow X = \mathrm{Max}(A)$ induced by a homomorphism of affinoid K-algebras $A \longrightarrow B$ with $\mathrm{im}(\alpha) \subseteq U$ the covering $Y = \bigcup\limits_{i \in I} \alpha^{-1}(U_i)$ has a finite subcovering.

$*$ Let V and V_j, for $j \in J$, be admissible open subsets of X such that $V = \bigcup\limits_{j \in J} V_j$; this covering of V is called **admissible** if for any map $\alpha : Y \longrightarrow X$ as above with $\mathrm{im}(\alpha) \subseteq V$ the covering $Y = \bigcup\limits_{j \in J} \alpha^{-1}(V_j)$ can be refined into a finite covering by rational subdomains.

These notions define a Grothendieck topology on X (which is considerably coarser than the canonical topology). The presheaf $\mathcal{O}_X$ extends in a purely formal way (once one knows Tate's theorem) to a sheaf on X with respect to this Grothendieck topology; it is called the structure sheaf of X.

Definition:

The triple $\mathrm{Sp}(A) := (X,$ *Grothendieck topology,* $\mathcal{O}_X)$ *is called an affinoid variety over* K.

Fact:

Any homomorphism of affinoid K*-algebras* $A \longrightarrow B$ *induces a "morphism of affinoid varieties"* $\mathrm{Sp}(B) \longrightarrow \mathrm{Sp}(A)$.

In order to illustrate these concepts let us go back to the 1-dimensional disk $X = \mathrm{Max}(K\langle T\rangle) =$ "$\{z \in \widehat{\overline{K}} : |z| \leq 1\}$" and look at the simplest example. By construction we have $\mathcal{O}_X(X) = K\langle T\rangle$. Clearly the "unit circle" $V := \{x \in X : |T(x)| = 1\} = X(\frac{1}{T}) =$ "$\{z \in \widehat{\overline{K}} : |z| = 1\}$" is a rational subdomain with

$$\begin{aligned} \mathcal{O}_X(V) &= K\langle T\rangle\langle T'\rangle/\langle TT' - 1\rangle \;=\; K\langle T, T^{-1}\rangle \\ &:= \{\textstyle\sum_{\nu \in \mathbb{Z}} a_\nu T^\nu \;:\; |a_\nu| \longrightarrow 0 \text{ if } |\nu| \longrightarrow \infty\}\ . \end{aligned}$$

We now look at the "open unit disk"

$$U := X \setminus V = \{x \in X : |T(x)| < 1\} \ = \ \text{“}\{z \in \widehat{\overline{K}} : |z| < 1\}\text{”}$$

and claim that U is admissible open in X. Choose an $\varepsilon = |\pi| \in |K^\times|$ with $0 < \varepsilon < 1$ and put

$$U_n := X(\frac{T^n}{\pi}) \ = \ \{x \in X : |T(x)| \le \varepsilon^{1/n}\} \ = \ \text{“}\{z \in \widehat{\overline{K}} : |z| \le \varepsilon^{1/n}\}\text{”}$$

for $n \in \mathbb{N}$. These are rational subdomains of X such that $U = \bigcup_{n \in \mathbb{N}} U_n$. Let $\alpha : Y = \mathrm{Max}(B) \longrightarrow X$ be a morphism of affinoid varieties such that $\mathrm{im}(\alpha) \subseteq U$. The Maximum Modulus Principle implies that

$$|\alpha^*(T)|_{\sup} = \max_{y \in Y} |\alpha^*(T)(y)| = \max_{y \in Y} |T(\alpha(y))| < 1 \ \ .$$

It follows that $\alpha^{-1}(U_n) = Y$ for any sufficiently large n.
This fact does not contradict our geometric intuition that the unit disk is a "connected" space. The point is that $X = U \cup V$ is **not** an admissible covering!

Starting from the affinoid varieties as building blocks one constructs general rigid varieties by the usual gluing procedure.

Definition:

A rigid K-analytic variety is a set X equipped with a Grothendieck topology (consisting of subsets) and a sheaf of K-algebras $\mathcal{O}_X$ such that there is an admissible covering $X = \bigcup_{i \in I} U_i$ where each $(U_i, \mathcal{O}_X | U_i)$ is isomorphic to an affinoid variety over K.

A first rather trivial source of examples are admissible open subsets $U \subseteq X = \mathrm{Sp}(A)$; for them $(U, \mathcal{O}_X | U)$ is a rigid K-analytic variety. A much more important source constitute the algebraic varieties over K. There is a natural functor

$$\begin{array}{rcl} K\text{-schemes locally of finite type} & \longrightarrow & \text{rigid } K\text{-analytic varieties} \\ X & \longmapsto & X^{\mathrm{an}} \end{array}$$

together with a natural morphism of locally G-ringed spaces

$$an_X \ : \ X^{\mathrm{an}} \longrightarrow X$$

which has the universal property that any morphism of locally G-ringed spaces $Y \longrightarrow X$ where Y is a rigid K-analytic variety factorizes through X^{an}, i.e., we have a commutative triangle

$$\begin{array}{ccccc} Y & & - - - - \rightarrow & & X^{\mathrm{an}} \\ & \searrow & & \swarrow & an_X \\ & & X & & . \end{array}$$

By a locally G-ringed space we mean a set equipped with a Grothendieck topology consisting of subsets and a sheaf of K-algebras whose stalks are local rings. In order to demonstrate the existence of X^{an} it suffices, by a gluing argument (made possible by the universal property), to consider the case of an affine K-scheme of finite type $X = \mathrm{Spec}(A)$. As a set we put $X^{\mathrm{an}} := \mathrm{Max}(A)$. To define the analytic structure we fix a representation of A as a quotient

$$A \;=\; K[T_1, \ldots, T_d]/\mathfrak{a}$$

of some polynomial algebra and we fix a $c \in K$ with $|c| > 1$. Put

$$U_n := \{x \in X^{\mathrm{an}} \;:\; \max_j |T_j(x)| \;\leq\; |c|^n\} \quad \text{for } n \geq 0$$

so that

$$X^{\mathrm{an}} \;=\; \bigcup_{n \geq 0} U_n \;\; .$$

We define affinoid K-algebras

$$A_n := K\langle T_1, \ldots, T_d\rangle / \langle P(c^{-n}T_j) \;:\; P \in \mathfrak{a}\rangle \;\; .$$

From the commutative diagram

$$\begin{array}{ccccc} \mathrm{Max}(A_n) & \xrightarrow[c^{-n}T_j \leftarrow T_j]{\sim} & U_n & \subseteq & \text{ball of radius } |c|^n \\ \Big\downarrow \begin{array}{c} cT_j \\ \uparrow \\ T_j \end{array} & & & & \Big\downarrow \subseteq \\ \mathrm{Max}(A_{n+1}) & \xrightarrow[c^{-(n+1)}T_j \leftarrow T_j]{\sim} & U_{n+1} & \subseteq & \text{ball of radius } |c|^{n+1} \\ & & & & \Big\downarrow \subseteq \\ & & X^{\mathrm{an}} & \subseteq & \mathbf{A}^d \end{array}$$

it is quite clear that X^{an} has a unique rigid K-analytic structure such that $X^{\mathrm{an}} = \bigcup_n U_n$ is an admissible covering by the affinoid open subsets $U_n \cong \mathrm{Sp}(A_n)$; the morphism $an_X : X^{\mathrm{an}} = \mathrm{Max}(A) \longrightarrow X = \mathrm{Spec}(A)$ is the "inclusion map". It holds quite generally that X^{an} as a set consists of the closed points of the scheme X.

A quite important feature of a K-affinoid variety $X = \mathrm{Sp}(A)$ is its reduction. In order to describe this construction we need to introduce the valuation ring o in K as well as its residue field k. In A we have the o-subalgebra

$$\overset{\circ}{A} \;:=\; \{f \in A \;:\; |f|_{\sup} \leq 1\} \;\;;$$

it contains the ideal

$$\check{A} \;:=\; \{f \in A \;:\; |f|_{\sup} < 1\} \;\;.$$

The k-algebra

$$\tilde{A} \;:=\; \overset{\circ}{A}/\check{A}$$

is finitely generated and reduced (the latter since the spectral seminorm is power-multiplicative).

Definition:

The affine k-scheme $\tilde{X} := \mathrm{Spec}(\tilde{A})$ is called the canonical reduction of the affinoid variety X.

This notion obviously is functorial in A. On the level of sets one has the **reduction map**

$$\begin{aligned} red_X \;:\; X &\longrightarrow \mathrm{Max}(\tilde{A}) \subseteq \tilde{X} \\ x &\longmapsto \ker(\tilde{A} \to (A/\mathfrak{m}_x)^{\sim}) = \{f \in \overset{\circ}{A} : |f(x)| < 1\}/\check{A} \;\;. \end{aligned}$$

It is helpful to realize that $(A/\mathfrak{m}_x)^{\sim}$ is a finite field extension of k.

Fact: *red_X is surjective.*

We claim that red_X is continuous in the sense that the preimage of any Zariski open subset is admissible open. In order to see this let $f \in \overset{\circ}{A}$ with $|f|_{\sup} = 1$ and let $\tilde{f}$ denote its residue class in $\tilde{A}$. Then $\tilde{f} \notin red_X(x)$ if and only if $|f(x)| \geq 1$. Hence we have

$$red_X^{-1}(\mathrm{Max}(\tilde{A}[\tilde{f}^{-1}])) \;=\; X(\frac{1}{f}) \;\;.$$

Note that also the preimage $red_X^{-1}(V(\tilde{f})) = \{x \in X : |f(x)| < 1\}$ of the Zariski closed zero set $V(\tilde{f})$ of $\tilde{f}$ is admissible open (but i.g. not affinoid).

So far I have described the "classical" approach to rigid analytic geometry which was invented by Tate. More details and full proofs for everything which was said can be found in the book [BGR]. Later on Raynaud saw that rigid geometry can be developed entirely within the framework of formal algebraic geometry. Because of the conceptual as well as technical importance of this approach I want to finish by explaining Raynaud's point of view ([R] or [BL]).

For simplicity we assume that $| \ |$ is a discrete valuation on K. As before o denotes the ring of integers in K. We fix a prime element π in o. Let $o\{\{T_1, \ldots, T_n\}\}$ be the ring of restricted formal power series over o; recall that a formal power series over o is restricted if, for any given $m \geq 1$, almost all its coefficients lie in $\pi^m o$. An o-algebra $\mathcal{A}$ of the form $\mathcal{A} = o\{\{T_1, \ldots, T_n\}\}/\mathfrak{a}$ is called **topologically of finite type.** It is a $\pi\mathcal{A}$-adically complete topological ring and gives rise to the affine formal scheme $\mathrm{Spf}(\mathcal{A})$ over o which is the set of all open prime ideals of $\mathcal{A}$ equipped with the Zariski topology and a certain structure sheaf constructed by localization and completion. Roughly speaking one has

$$\mathrm{Spf}(\mathcal{A}) \;=\; \text{“}\varinjlim_m\text{”}\, \mathrm{Spec}(\mathcal{A}/\pi^m\mathcal{A}) \ .$$

The first basic observation is that, for an o-algebra $\mathcal{A}$ topologically of finite type, the tensor product $A := \mathcal{A} \otimes_o K$ is an affinoid K-algebra. The affinoid variety $\mathrm{Sp}(A)$ is called the **general fibre** of the formal scheme $\mathrm{Spf}(\mathcal{A})$. Can we describe the datum $\mathrm{Sp}(A)$ directly in terms of the algebra $\mathcal{A}$?

The set $\mathrm{Max}(A)$:
Consider any prime ideal $\mathfrak{p} \subseteq \mathcal{A}$ such that

1. $\mathfrak{p}$ is not open in $\mathcal{A}$, and
2. $\mathcal{A}/\mathfrak{p}$ is a finitely generated o-module.

We claim that $\mathfrak{p} \otimes K$ is a maximal ideal in A. Condition 1) ensures that the obvious map $o \hookrightarrow \mathcal{A}/\mathfrak{p}$ is injective. Condition 2) implies that $\mathcal{A}/\mathfrak{p}$ is an integral domain finite over o. It follows that $\mathcal{A}/\mathfrak{p}$ is a local ring which is finite and flat over o. Hence $(\mathcal{A}/\mathfrak{p}) \otimes K = A/(\mathfrak{p} \otimes K)$ is a finite field extension of K.
In this way one obtains a bijection

$$\begin{array}{rcl} \{\mathfrak{p} \subseteq \mathcal{A} \text{ prime ideal with 1), 2) } \} & \xrightarrow{\sim} & \mathrm{Max}(A) \\ \mathfrak{p} & \longmapsto & \mathfrak{p} \otimes K \ . \end{array}$$

The rational subdomains (i.e., the Grothendieck topology on $\mathrm{Max}(A)$):
Consider any rational subdomain $X(\frac{f.}{g}) \subseteq X := \mathrm{Max}(A)$. There is no loss of generality in assuming that $g, f_1, \ldots, f_r \in \mathcal{A}$. That $g, f_1, \ldots, f_r$ have no common zero means that the ideal $I := \mathcal{A}g + \mathcal{A}f_1 + \ldots + \mathcal{A}f_r$ is open in $\mathcal{A}$. On the other hand, for any open ideal $I \subseteq \mathcal{A}$ and any element $g \in I$, there is a universal construction called **formal blowing-up** of a homomorphism $\mathcal{A} \longrightarrow \mathcal{A}_{I,g}$ of o-algebras topologically of finite type which is universal with respect to

making I into a principal ideal generated by g. The morphism of formal schemes $\mathrm{Spf}(\mathcal{A}_{I,g}) \longrightarrow \mathrm{Spf}(\mathcal{A})$ induces in the general fibre the inclusion $X(\frac{f.}{g}) \subseteq X$.

The structure sheaf:
As is more or less clear from the above description of the rational subdomains of $X = \mathrm{Sp}(A)$ the structure sheaf $\mathcal{O}_X$ can be reconstructed from the structure sheaves of all the formal blowing-ups of the formal scheme $\mathrm{Spf}(\mathcal{A})$.

Theorem of Raynaud:

The above construction of the "general fibre" induces an equivalence of categories between

> *the category of all formal flat o-schemes topologically of finite type in which all formal blowing-ups are inverted*

and

> *the category of all quasi-compact and quasi-separated rigid K-analytic varieties.*

References

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