IMPRS course

Rational and p-adic homotopy theory

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The course takes place Thursdays 10 am to 12 am in the seminar room at MPIM. The first meeting is at the 14th of April. The lecture is aimed at postdocs and advanced PhD students. The main goal will be to prove Mandell's theorem about an 'algebraic' model for p-adic spaces (see below). We will assume a profound knowledge in homotopy theory (model categories, ∞-categories, Steenrod algebra, Postnikov towers, spectral sequences etc). In case you want to participate and are not sure about the prerequistes do not hesitate to contact me. We plan to roughly cover the following topics:

- Rational homotopy theory. In particular the Sullivan picture and easy applications, see e.g. [Hes07]. This is supposed to be a warm up for the later parts.
- The formal setup for p-adic homotopy theory, in particular p-completion of spaces and spectra [Bou79, Bou75] . We will also discuss profinite spaces. It will be crucial to carefully analyse the Steenrod actions on certain spaces. We will (at least partly) recall the necessary standard Steenrod-Algebra.
- The theorems of Goerss [Goe95] and Mandell [Man01] about the fully faithful embedding from finite p-complete spaces into commutative cosimplicial algebras respectively E_∞-algebras over $\overline{\mathbb{F}}_p$. This embedding takes X to the cochain algebra $C^*(X, \overline{\mathbb{F}}_p)$ considered as a cosimplicial commutative algebra resp. an E_∞-algebra over $\mathrm{H}\overline{\mathbb{F}}_p$.
- Improvements of these theorems for non-algebraically closed fields (partly dicussed in [Lur11] and [Goe95]) and rings taking the Frobenius action correctly into account, in particular an integral version (generalizing ideas from [Man06]). If time permits we discuss the Sullivan conjecture, the Segal conjecture and possible generalizations.

References

[Bou75] A. K. Bousfield. The localization of spaces with respect to homology. *Topology*, 14:133–150, 1975.

[Bou79] A. K. Bousfield. The localization of spectra with respect to homology. *Topology*, 18(4):257–281, 1979.

[Goe95] Paul G. Goerss. Simplicial chains over a field and p-local homotopy theory. *Math. Z.*, 220(4):523–544, 1995.

[Hes07] Kathryn Hess. Rational homotopy theory: a brief introduction. In *Interactions between homotopy theory and algebra*, volume 436 of *Contemp. Math.*, pages 175–202. Amer. Math. Soc., Providence, RI, 2007.

[Lur11] Jacob Lurie. Derived algebraic geometry XIII: Rational and p-adic homotopy theory. 2011.

[Man01] Michael A. Mandell. E_∞ algebras and p-adic homotopy theory. *Topology*, 40(1):43–94, 2001.

[Man06] Michael A. Mandell. Cochains and homotopy type. *Publ. Math. Inst. Hautes Études Sci.*, (103):213–246, 2006.