# Suggestions for talks

March 31, 2017

## Knots and Primes

The main reference for this series of talks will be the book *Knots and Primes: An introduction to arithmetic topology* by M. Morishita.

- **1 talk** Introduce the main players: knots, tubular neighbourhoods, 3-manifolds, number fields, p-adic fields and rings and finite fields. Recall affine schemes (examples of former rings and fields and polynomial rings) and how quotients correspond to closed subschemes and completions to infinitesimal neighbourhoods. Explain how the notion of Krull dimension gives a 1-dimensional picture for Spec($\mathbb{Z}$). Give many examples and say as much as time permits on the structure of the different objects. (What are units, structure of the multiplicative group of p-adic fields, knot diagrams?(are not necessary for the analogy but nice and used a lot)) Roughly lecture 1 of `http://www.math.columbia.edu/~chaoli/tutorial2012/KnotsAndPrimes.html`
- **2 talks** (ramified) coverings and fundamental groups in the topological and arithmetic setting. Introduce the above notions for topological spaces and rings and state main theorems of covering spaces and Galois theory. Explain how étale morphisms are similar to coverings. Give many examples ($S^1$, torus, complement of a knot, finite fields, p-adic numbers $\mathbb{Z}$ (Minkowskis thm)). Introduce the infinite cyclic covering. (chapter 2 Morishita, also check *Galois Groups and Fundamental Groups* by T. Szamuely)
- **1 talk** cohomological dimension. Recall the concept of cohomological dimension. Recall poincaré duality and give an idea of why the cohomological dimension of finite, $p$-adic and number fields (and rings) is 1, 2 and 3 respectively. (chapter 3 Morishita) Maybe talk about class field theory.
- **1 talk** Explain quadratic residue symbols and the linking number. Construct and proof reciprocity/symmetry in similar ways, using the notions from talk 2 and 3. (chapter 4 Morishita)

From here one could go into different directions, depending on interests and knowledge of the participants. Here are some suggestions

- **1 talk** Explain the decomposition of knots and primes in finite coverings and galois-extensions (ch. 5 Morishita)

- **2-4 talks** Alexander Polynomial and Iwasawa theory (chapters 9,11,12 Morishita)
- **2-3 talks** Moduli spaces of representations of knot and link groups, hyperbolic structures and p-adic modular forms (chapters 13,14 in Morishita) Difficult but interesting. Probably only possible to state results.
- **1-2 talks** Milnor invariants and multiple residue symbols. Massey products, fox free differential calculus (embed free groups into noncommutative power series rings).
- **1 talk** Deninger's point of view: Consider a number ring as a 3-dimensional dynamical system, closed orbits (=knots with parametrisation given by the flow) as primes. Lowbrow: proof of F. Kopei of an analytic analogue of the arithmetic product formula for valuations. Highbrow: If conjectural correspondence holds, do index theory to prove formulas in analytic number theory.
- **1 talk** Chebotareff's density theorem (number theory) has an analogue for 3-manifolds (paper by Curtis McMullen)

## Other Analogies

- Harmonic functions on higher-dimensional spaces or manifolds share some properties of holomorphic functions. `https://en.wikipedia.org/wiki/Harmonic_function#Connections_with_complex_function_theory`
- Analogues, versions and generalizations of the Fourier transform: Pontryagin duality – Fourier-Deligne transform – Fourier transform of D-modules – Fourier-Mukai transform – Discrete Fourier transform.
- A discrete analogue of Morse theory. R. Forman: *Morse Theory for Cell Complexes*. Advances in Mathematics **134** (1998), 90-145.
- Thinking about numbers as differentiable functions. A. Buium: *Differential calculus with integers*. `arXiv:1308.5194` | Y. I. Manin: *Numbers as functions*. `arXiv:1312.5160`
- Triviality—Bundle-module-analogies surrounding Serre conjecture and the development of algebraic and topological K-theory. R. G. Swan: review of *Serre's problem on projective modules* by T. Y. Lam. Bull. Amer. Math. Soc. **45**(3) (2008) 451?457. http://www.ams.org/journals/bull/2008-45-03/S0273-0979-08-01171-3/S0273-0979-08-01171-3.pdf | R. G. Swan: *Vector Bundles and Projective Modules*. Trans. Amer. Math. Soc. **105**(2) (1962), 264-277. | J.-P. Serre: *Modules projectifs et espaces fibrés à fibre vectorielle*. Séminaire Dubreil. Algèbre et théorie des nombres 11.2 (1957-1958) 1-18. http://eudml.org/doc/111153.
- Algebraic models of spaces: commutative differential graded algebras. Y. Flix, J. Oprea, D. Tanr: *Algebraic Models in Geometry*. Oxford University Press (2008).

- Analogies between $\mathbb{Z}$ and dynamical systems. C. Deninger: *Arithmetic Geometry and Analysis on Foliated Spaces.* `arXiv:math/0505354`

- Formal analogies between statistical field theory and quantum field theory in mathematical physics. B. McCoy: *The connection between statistical mechanics and quantum field theory.* `arXiv:hep-th/9403084`

- Ideas relating exactly solvable models from statistical physics, the Yang-Baxter equation, quantum groups, and knot theory. M. Jimbo: *Yang-Baxter Equation in Integrable Systems.* World Scientific, Singapore (1990). | C. Kassel, M. Rosso, V. Turaev: *Quantum groups and knot invariants.* Société Mathématique de France, Paris (1997).

- Several proofs of Gromov's systolic inequality for tori, each using a different "metaphor" from another branch of geometry. L. Guth: *Metaphors in Systolic Geometry.* Proceedings of the International Congress of Mathematicians, Hyderabad (2010). `http://math.mit.edu/~lguth/Exposition/systmet.pdf`

- Unformization of Gromov hyperbolic metric spaces similar to the Riemann uniformization theorem in complex analysis. M. Bonk, J. Heinonen, P. Koskela: *Uniformizing Gromov Hyperbolic Spaces.* Astrisque 260 (2001).

- Topological analogues of Mostow rigidity in the work of Farrell and Jones. A.P. Lewis: *Geometric and Topological Rigidity Theorems*, `https://math.berkeley.edu/~alanw/ps/lewis.ps` | F. T. Farrell; L. E. Jones: *A Topological Analogue of Mostow's Rigidity Theorem*, J. Amer. Math. Soc., **2**(2) (1989), 257-370.