

Affine group actions on Banach spaces

Seminar – Wintersemester 2020/2021

Prof. Dr. Tim de Laat

The seminar takes place on **Thursdays, 14:15 – 16:00**, through WWUzoom.

Affine isometric group actions on Banach spaces view Banach spaces as affine spaces. Given a (topological) group G and a Banach space E , it is fundamental to ask whether there exists a proper affine isometric action of G on E , or whether all affine isometric actions have fixed points. The existence of proper actions and fixed point properties are related to structural properties of the groups and the Banach spaces under consideration and have various interesting implications.

We will mainly focus on fixed point properties of affine isometric actions on L^p -spaces. For $1 \leq p < \infty$, a group G is said to have property (FL^p) if every affine isometric action of G on an L^p -space (for this value of p) has a fixed point. Let $\mathcal{P}(G)$ be defined as the set of numbers $p \in [1, \infty)$ for which G has property (FL^p) . One of the main aims of the seminar is to understand the recent breakthrough of Marrakchi and de la Salle [11], who proved the Druţu conjecture, asserting that for every locally compact group G , the set $\mathcal{P}(G)$ is an interval.

Before covering the work of Marrakchi and de la Salle, we will cover the basic theory of affine isometric actions on Banach spaces.

Schedule of talks

1. Introduction to affine isometric group actions on Banach spaces

Speaker: Tim de Laat.

Date: 05.11.2020.

Topics: Affine structure of Banach spaces; Mazur-Ulam theorem; some Banach space geometry; isometric representations of locally compact groups; affine isometric actions; proper actions and fixed points; property (F_E) ; cohomological formulation.

Literature: [14, Sections 1 – 3.2], [2].

2. Invariant vectors, bounded orbits and fixed points

Speaker: René Brüggemann.

Date: 12.11.2020.

Topics: Ryll-Nardzewski fixed point theorem; subspace of G -invariant vectors and its G -invariant complemented subspace; bounded orbits versus fixed points.

Literature: [14, Sections 3.3 and 3.4], [2], [18]

3. L^p -spaces, the Mazur map, and the Banach-Lamperti theorem

This talk will be held in German.

Speaker: Ann-Kathrin Meyer.

Date: 19.11.2020.

Topics: Definition, characterisations and properties of L^p -spaces; the Mazur map; proof of the Banach-Lamperti theorem.

Literature: [5].

4. Property (T) and fixed point properties on L^p -spaces

Speaker: Konrad Krug.

Date: 26.11.2020.

Topics: Delorme-Guichardet theorem (without proof) and [2, Theorem 1.3.(2)–(3)]; relation between the Haagerup property and proper actions (without proofs).

Literature: [2], [3].

5. Property (T) groups with proper actions on an L^p -space for large p

Speaker: Jessica Schukowski.

Date: 03.12.2020.

Topics: Proper affine actions of hyperbolic groups on L^p -spaces.

Literature: [19] (see also [13], [4], [15]).

6. Proof of the Druţu conjecture, I

Speaker: Omar Mohsen, Federico Vigolo.

Date: 10.12.2020.

Topics: Proof of the Druţu conjecture, divided over two talks.

Literature: [11] (see also [1]).

7. Proof of the Druţu conjecture, II

Speaker: Omar Mohsen, Federico Vigolo.

Date: 17.12.2020.

Topics: Proof of the Druţu conjecture, divided over two talks.

Literature: [11].

8. Higher rank Lie/algebraic groups and lattices have property (FL^p)

Speaker: Antje Dabeler.

Date: 07.01.2021.

Topics: Higher rank Lie/algebraic groups and lattices have property (FL^p) for $1 < p < \infty$.

Literature: [2].

9. Computation of $\mathcal{P}(G)$ for semisimple Lie groups and their lattices

Speaker: Kevin Sauer.

Date: 14.01.2021.

Topics: Computation of $\mathcal{P}(G)$ for semisimple Lie groups and their lattices.

Literature: [11].

10. Strong property (T) and fixed point properties

Speaker: Julian Kranz.

Date: 21.01.2021.

Topics: Definition of Lafforgue's strong property (T); relation with fixed point properties.

Literature: [8], [9], [16], [17].

11. Strong property (T) and fixed point properties for $SL(3, \mathbb{R})$ and other groups

Speaker: Jeroen Winkel.

Date: 28.01.2021.

Topics: $SL(3, \mathbb{R})$ has strong property (T) with respect to certain classes of Banach spaces (sketch of proof); other examples of groups with strong property (T).

Literature: [16] (see also [6], [7], [8], [9], [10]).

12. Polish groups without non-trivial representations on reflexive Banach spaces

Speaker: Aristotelis Panagiotopoulos.

Date: 04.02.2021.

Topics: Weakly almost periodic compactification, Grothendieck's limit exchange criterion, Polish groups without non-trivial reflexive representations, connections to model theory.

Literature: [12] and more.

References

- [1] Y. Arano, Y. Isono and A. Marrakchi, *Ergodic theory of affine isometric actions on Hilbert spaces*, preprint (2019), arXiv:1911.04272v3.
- [2] U. Bader, A. Furman, T. Gelander and N. Monod, *Property (T) and rigidity for actions on Banach spaces*, Acta Math. **198** (2007), 57–105.
- [3] B. Bekka, P. de la Harpe and A. Valette, *Kazhdan's property (T)*, Cambridge University Press, Cambridge, 2008.
- [4] Y. De Cornulier, R. Tessera and A. Valette, *Isometric group actions on Banach spaces and representations vanishing at infinity*, Transform. Groups **13** (2008), 125–147.
- [5] R.J. Fleming and J.E. Jamison, *Isometries on Banach spaces: function spaces*, Chapman & Hall/CRC, Boca Raton, FL, 2003.
- [6] T. de Laat, M. Mimura and M. de la Salle, *On strong property (T) and fixed point properties for Lie groups*, Ann. Inst. Fourier (Grenoble) **66** (2016), 1859–1893.
- [7] T. de Laat and M. de la Salle, *Strong property (T) for higher-rank simple Lie groups*, Proc. Lond. Math. Soc. (3) **111** (2015), 936–966.
- [8] V. Lafforgue, *Un renforcement de la propriété (T)*, Duke Math. J. **143** (2008), 559–602.
- [9] V. Lafforgue, *Propriété (T) renforcée banachique et transformation de Fourier rapide*, J. Topol. Anal. **1** (2009), 191–206.
- [10] B. Liao, *Strong Banach property (T) for simple algebraic groups of higher rank*, J. Topol. Anal. **6** (2014), 75–105.
- [11] A. Marrakchi and M. de la Salle, *Isometric actions on L_p -spaces: Dependence on the value of p* , preprint (2020), arXiv:2001.02490v2.
- [12] M.G. Megrelishvili, *Every semitopological semigroup compactification of the group $H_+[0, 1]$ is trivial*, Semigroup Forum **63** (2001), 357–370.
- [13] B. Nica, *Proper isometric actions of hyperbolic groups on L^p -spaces*, Compos. Math. **149** (2013), 773–792.
- [14] P.W. Nowak, *Group actions on Banach spaces*, Handbook of group actions, Vol. II, pp. 121–149, Int. Press, Somerville, MA, 2015.
- [15] P. Pansu, *Dimension conforme et sphère à l'infini des variétés à courbure négative*, Ann. Acad. Sci. Fenn. Ser. A I Math. **14** (1989), 177–212.
- [16] M. de la Salle, *Towards strong Banach property (T) for $SL(3, \mathbb{R})$* , Israel J. Math. **211** (2016), 105–145.

- [17] M. de la Salle, *A local characterization of Kazhdan projections and applications*, Comment. Math. Helv. **94** (2019), 623–660.
- [18] T. Shulman, *On subspaces of invariant vectors*, Studia Math. **236** (2017), 1–11.
- [19] G. Yu, *Hyperbolic groups admit proper affine isometric actions on ℓ^p -spaces*, Geom. Funct. Anal. **15** (2005), 1144–1151.