ON SOME DIRICHLET SERIES ASSOCIATED TO TOTALLY DISCONNECTED, LOCALLY COMPACT GROUPS

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Abstract. It is well known that formal Dirichlet series

$$\zeta_{\underline{a}}(s) = \sum_{n\in\mathbb{N}} a_n \cdot n^{-s} \tag{0.1}$$

provide a sophisticated but in general very complicated tool for many enumeration problems. Provided that $\mathbf{abs}(\zeta_{\underline{a}}) = a < \infty$, $\zeta_{\underline{a}}$ defines a holomorphic function $\zeta_{\underline{a}}(s)$ in some half plane $H_a = \{ z \in \mathbb{C} \mid \Re(z) > a \}$. Although complex analysis does not offer a satisfactory general theory, in many significant cases $\zeta_{\underline{a}}(s)$ has a meromorphic continuation $\hat{\zeta}_{\underline{a}}(s)\colon \bar{\mathbb{C}} \to \bar{\mathbb{C}}$ to the whole complex plane.

For a totally disconnected, locally compact (=t.d.l.c.) group G and a compact open subgroup $\mathcal{O}$, let $\mathcal{R}$ denote a set of coset representatives for $\mathcal{O}\backslash G/\mathcal{O}$. One says that G has *bounded coset growth (with respect to* $\mathcal{O}$*)*, if for all $k \in \mathbb{N}$ one has

$$a_k = |\{ r \in \mathcal{R} \mid \mu(\mathcal{O}r\mathcal{O}) = k\mu(\mathcal{O}) \}| < \infty, \tag{0.2}$$

where μ is some fixed Haar measure on G. In the talk we address several phenomena arising for the formal Dirichlet series $\zeta_{G,\mathcal{O}}(s)$ and ζ-function $\hat{\zeta}_{G,\mathcal{O}}(s)$ for a t.d.l.c. group G of bounded coset growth.

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