General Relativity and Black Holes – Week 13

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1 Exercises

1. Let (M, g) be a globally hyperbolic spacetime. Prove that $J^+(K)$ is closed for K compact.
   HINT: You are free to use that $J^\pm(p)$ is closed as proven on Sheet 6.

2. Show that a compact spacetime (M, g) necessarily contains closed timelike curves.

3. (Topological ingredients in Penrose's theorem.) Prove that

   (a) A closed subset of a compact topological space is compact.

   (b) A compact subset of a Hausdorff topological space is closed.

   (c) For A a compact topological space and B a Hausdorff space, if $f : A \to B$ is a continuous bijection, then it is a homeomorphism.
   HINT: Show that f^{-1} pulls back closed sets to closed sets and use (a) and (b).

2 Problems and Discussion

1. Let $(\mathbb{R}^{1+3}, \eta)$ be 4-dimensional Minkowski space. Do closed trapped surfaces exist in $(\mathbb{R}^{1+3}, \eta)$? Can you construct 2-spheres S in $(\mathbb{R}^{1+3}, \eta)$ which do not satisfy $tr\chi(p) > 0$ and $tr\underline{\chi}(p) < 0$ for *all* $p \in S$?