Higgs Effect for Goldstone Particles with Spin 1/2

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Abstract. We consider the introduction of gauge fields and the Higgs effect for Goldstone particles with spin 1/2

Higgs and others (Higgs (1966), Migdal and Polyakov (1966), Kibble (1967)) have noted that when interactions are turned on between Goldstone particles and gauge fields, the Goldstone particles vanish and those gauge fields whose quantum numbers coincide with the quantum numbers of the Goldstone particles acquire mass (the Higgs effect). We consider in this article a variant of the Higgs effect, in which the "absorption" of the Goldstone particles is effected by a "foreign" gauge field, i.e., by a gauge fields whose quantum numbers differ from those of the Goldstone particles.

We consider as the symmetry group G the direct product of a Poincare group with a certain group of internal symmetry, supplemented by the following transformations:

$$\begin{aligned} &\psi \to \psi' = \psi + \xi, \\ &x_\mu \to x'_\mu = x_\mu - \frac{a}{2\,!}(\xi^+ \sigma_\mu \psi - \psi^+ \sigma_\mu \xi) \ . \end{aligned} \tag{1}$$

We use the definitions of Volkov and Akulov (1973).

The spontaneous violation of the considered symmetry group under the assumption that the direct product of the Poincare group and of the internal–symmetry group leaves the vacuum invariant, leads to the appearance of Goldstone particle with spin 1/2/ (Volkov and Akulov (1972), Volkov and Akulov (1973)).

Let us consider the gauge transformations of the group G with parameters l, u, t, and ξ, which depend on x_μ and gauge fields that are coefficient functions of the differentials $\mathrm{d}x_\mu$ and $d\psi$ of the differential form $A(\mathrm{d})$ with the following transformation law:

$$A'(\mathrm{d}) = G(t, \xi; l, u)\ A(\mathrm{d}) G^{-1}(t, \xi; l, u) + \frac{1}{f} G(t, \xi; l, u) \mathrm{d} G^{-1}(t, \xi; l, u) \ , \tag{2}$$

where $A(\mathrm{d}) = A'(\mathrm{d}) Z_i$ and Z_i are the generators of group G.

The differential forms

$$\begin{aligned}\bar{\omega}(\mathrm{d}) &= H^{-1}(l,u)\omega(\mathrm{d})H(l,u) \\ &= G^{-1}(x,\psi;l,u)\mathrm{d}G(x,\psi;l,u) + fG^{-1}(x,\psi;l,u)A(\mathrm{d})G(x,\psi;l,u)\ ,\end{aligned} \tag{3}$$

which are invariant to the translation (2) and correspond to the transformation generators in the transformations (1), take the following form as functions of the gauge fields:

$$\begin{aligned}\omega_\mu(\mathrm{d}) &= Dx_\mu + \frac{\sigma}{2i}[\psi^+\sigma_\mu(D\psi + 2f\Phi(\mathrm{d})) - \\ &\qquad (D\psi + 2f\Phi(\mathrm{d}))^+\sigma_\mu\psi] + fW_\mu(\mathrm{d})\ , \\ \omega_a^\alpha(\mathrm{d}) &= (D\psi + f\Phi(\mathrm{d}))_a^\alpha\ ,\ \ \omega^{a\dot{\alpha}}(\mathrm{d}) = \omega_a^\alpha(\mathrm{d})^+, \\ Dx_\mu &= \mathrm{d}x_\mu + 2\Omega_{\mu\nu}(\mathrm{d})x^\nu\ , \\ D\psi &= \mathrm{d}\psi + f\Omega^{\mu\nu}(\mathrm{d})I_{\mu\nu}\psi - ifV^a(\mathrm{d})I_a\psi\ ,\end{aligned} \tag{4}$$

where $I_{\mu\nu} = (1/4)(\tilde{\sigma}_\mu\sigma_\nu - \tilde{\sigma}_\nu\sigma_\mu)$, I_a are the generators of the internal–symmetry group, and the gauge fields $\Omega_{\mu\nu}(\mathrm{d}), W(\mathrm{d}), V^a(\mathrm{d})$, and $\Phi_a^\alpha(\mathrm{d})$ are the coefficients of the expansion of the form $A(\mathrm{d})$ in terms of generators of the considered group.

The forms $\omega(\mathrm{d})$, which correspond to the generators of the Lorentz group, as well as the internal–symmetry group, depend on the parameters l and u . There is no such dependence in the second–order differential forms

$$\begin{aligned}R^{\mu\nu}(\delta,\mathrm{d}) &= \delta\Omega^{\mu\nu}(\mathrm{d}) - \mathrm{d}\Omega^{\mu\nu}(\delta) + 2f[\Omega^\mu_\gamma(\delta)\Omega^{\gamma\nu}(\mathrm{d}) - \Omega^\nu_\gamma(\delta)\Omega^{\gamma\mu}(\mathrm{d})]; \\ F^a(\delta,\mathrm{d}) &= \delta V^a(\mathrm{d}) - dV^a(\delta) + fC^a_{bc}V^b(\delta)V^c(\mathrm{d})\ ,\end{aligned} \tag{5}$$

where C^a_{bc} are the structure constants of the internal symmetry group.

The invariant action integral is made up in the form of outer products of fourth order, which are invariant with respect to the Lorentz group and the internal–symmetry group, of the forms (4) and (5) (or in the general case in the form of a homogeneous first–order function of such products (see Volkov and Akulov (1973), Volkov (1973))).

The simplest permissible invariant combinations are

$$i\omega_\alpha^{\dot{\beta}}(\mathrm{d}_0)\Lambda\omega_{\dot{\beta}}{}^\gamma(\mathrm{d}_1)\Lambda\omega_\gamma^{\dot{\delta}}(\mathrm{d}_2)\Lambda\omega_{\dot{\delta}}{}^\alpha(\mathrm{d}_3)\ , \tag{6}$$

$$D_0\Lambda\omega_a^\alpha(\mathrm{d}_1)\Lambda\omega_{\alpha\dot{\beta}}(\mathrm{d}_2)\Lambda\omega^{\alpha\dot{\beta}}(\mathrm{d}_3) - D_0\Lambda\omega^{a\dot{\alpha}}(\mathrm{d}_1)\Lambda\omega_{\dot{\alpha}\beta}(\mathrm{d}_2)\Lambda\omega_a^\beta(\mathrm{d}_3)\ , \tag{7}$$

$$i[R_{\dot{\alpha}}^{\dot{\beta}}(\mathrm{d}_0,\mathrm{d}_1)\Lambda\omega_{\dot{\beta}\gamma}(\mathrm{d}_2)\Lambda\omega^{\gamma\dot{\alpha}}(\mathrm{d}_3) - R_\beta^\alpha(\mathrm{d}_0,\mathrm{d}_1)\Lambda\omega^{\beta\dot{\gamma}}(\mathrm{d}_2)\Lambda\omega_{\dot{\gamma}\alpha}(\mathrm{d}_3)]\ , \tag{8}$$

$$\frac{[F^a(\mathrm{d}_0,\mathrm{d}_1)\Lambda\omega_\alpha^{\dot{\beta}}(\mathrm{d}_2)\Lambda\omega_\gamma^{\dot{\delta}}(\mathrm{d}_3)][F^a(\mathrm{d}_0,\mathrm{d}_1)\Lambda\omega_{\dot{\beta}}^\alpha(\mathrm{d}_2)\Lambda\omega_{\dot{\delta}}^\gamma(\mathrm{d}_3)]}{i\omega_\alpha^{\dot{\beta}}(\mathrm{d}_0)\Lambda\omega_{\dot{\beta}}{}^\gamma(\mathrm{d}_1)\Lambda\omega_\gamma^{\dot{\delta}}(\mathrm{d}_2)\Lambda\omega_{\dot{\delta}}^\alpha(\mathrm{d}_3)}\ , \tag{9}$$

where Λ is the outer–product symbol.

Expressions (7-9) contain kinetic terms for gauge fields with spins $3/2, 2$ and 1, respectively. expression (6) contains a kinetic term for Goldstone particles.

All the field variables in the products (6-9) can be parametrized in the form of functions of x_μ, so that the gauge form can be reduced, by redefining the fields, to a form containing only the differentials x_μ and the redefined gauge fields corresponding to the coefficients of the differentials.

To consider the Higgs effect it suffices to examine the structure of forms (4,5). In the form $\omega_a^\alpha(\mathrm{d})$ the gauge form $\Phi(\mathrm{d})$ for spin 3/2 enters together with $D\psi$, so that we can define as a new gauge form

$$f\Phi'(\mathrm{d}) = D\psi + f\Phi(\mathrm{d}) \ . \tag{10}$$

In this case, however, the Goldstone field ψ in the form $\omega_\mu(\mathrm{d})$ is not eliminated completely, so that there is no Higgs effect for this field. Moreover, by imposing the invariant condition[1]

$$\omega^\alpha(\mathrm{d}) = 0 \tag{11}$$

we can exclude from consideration the gauge field with spin 3/2.

The gauge form $W_\mu(\mathrm{d})$ enters in the form $\omega_\mu(\mathrm{d})$ together with other terms that contain, in particular, kinetic terms of Goldstone particles. A change to a new gauge form

$$fW'(\mathrm{d}) = \omega_\mu(\mathrm{d}) \tag{12}$$

eliminates the variables connected with the Goldstone fields. This circumstance corresponds to the Higgs effect in which the Goldstone particles vanish as a result of a redefinition of the metric tensor.

Owing to redefinition of the gauge field $W_\mu(\mathrm{d})$, the invariants (7-9) correspond to interacting Yang–Mills fields, to a field with spin 3/2, and to gravitational field with a cosmological term (6).

A Goldstone effect field with spin 1/2 can be retained only by violating the gauge group. Therefore the program proposed in [4], of including the weak and electromagnetic interactions in the scheme of Goldstone particles, can be carried only with allowance for this violation.

References

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[1] The possibility of using conditions such as (11) to eliminate some of the considered fields was pointed out to us by V.I.Ogievetskii.