USSR ACADEMY OF SCIENCE

LENIN ORDER

PHYSICAL INSTITUTE

named after P. N. Lebedev

Preprint N 141

INTERNATIONAL SEMINAR

FUNCTIONAL METHODS IN QUANTUM FIELD THEORY AND STATISTICS

(December 7–9, 1971, Moscow)

Extended abstracts of presented talks

Part II

Gauge fields

Moscow - 1971

Phenomenological Lagrangians which are invariant under symmetry groups containing Poincaré group as a subgroup

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November 18, 1971

Recently, the method of phenomenological Lagrangians is widely used for description of Goldstone particles. In the case, when Goldstone particles are connected with arbitrary internal symmetry groups, a general method of phenomenological Lagrangians construction was considered in [1, 2]. For the symmetry groups containing also space-time variables the procedure of phenomenological Lagrangian construction additionally requires taking into account transformation properties of Goldstone field derivatives and four-volume element, which lead to much complexification of the method. Nowadays construction of phenomenological Lagrangians is carried out for one transformation group of space-time variables, that is for conformal transformation group [3].

Here we consider a general scheme of phenomenological Lagrangians construction for an arbitrary symmetry group which contain the Poincaré group as a subgroup.

Let G is such a symmetry group. We present an arbitrary element of G as

$$G = e^{iPx} K\left(a\right) H\left(b\right) L\left(l\right), \tag{1}$$

where P is the energy-momentum operator, x are space-time coordinates, $L\left(l\right)$ is the Lorentz group transformation, $H\left(b\right)$ is the subgroup of internal symmetry transformations which leave the vacuum invariant.

From the presentation (1) it follows that under the left action of the Poincaré group transformation on it, the x coordinates transform in the usual way. Under the left action by an element of an arbitrary group G the x coordinates transform jointly with a parameters which define the fields of Goldstone particles.

To find the phenomenological Lagrangian let us construct Cartan forms defined by the expansion of the expression

$$G^{-1}dG$$

by the generators of group G.

For the generators corresponding to the first two efficients in (1) the Cartan forms have the shape

$$\omega_i = \omega_i\left(x, a, dx, da, b, l\right) \tag{2}$$

and are invariants of G transformation on condition that all arguments of the forms under consideration are transformed. when the parameters b and l are fixed, the forms ω_i are transformed by representations of the subgroup HL. The combinations of the forms which are invariant under HL subgroup transformations do not depend from the parameters b and l, and are invariants of the group G. Such invariant combinations can be used for construction of phenomenological Lagrangians under the additional condition that they are represented as a product of a function and the four-dimensional volume element d^4x. The simplest combination of such kind corresponds to the external product of four forms (2). Indeed, if to place into (2) the fields of Goldstone particles $A\left(x\right)$ instead of the parameters a, then the differentials da are exchanged by $\frac{\partial A}{\partial x}dx$, owing to that the external product of four forms ω_i becomes proportional to d^4x.

More complicated invariants of the group G can be constructed by the following way. Let us denote an arbitrary external product (which is not necessary invariant under the group G) of the four forms (2) as W_k. Each of these forms is proportional to d^4x. Let us compose from the forms W_k an expression which is invariant under the subgroup HL and is homogeneous of degree 1 relatively the element d^4x. Such an expression is invariant under transformations of the group G and is proportional to d^4x, and therefore it can be exploited for construction of phenomenological Lagrangians.

The simplest after the forms W_k expressions of such kind which containing minimal number of field derivatives have the shape

$$\frac{W_i W_i}{W_0}, \tag{3}$$

where W_i is the external product of three forms (2) corresponding to the energy-momentum operators and one form (2) corresponding to any generator b in $K(a)$, and W_0 is the external product of four forms corresponding to energy-momentum generators. In the case, when the group G is a direct product of the Poincaré group and the internal symmetry group, the expression (3) corresponds to the phenomenological Lagrangian which was considered in [1, 2].

The use of the differential forms connected with the generators from H and L allows us to define covariant differentiation for arbitrary representation of the subgroup HL in the standard way, and to include into scheme under consideration the interaction of the Goldstone particles with other fields[1].

The stated above scheme of phenomenological Lagrangians construction on the basis of the presentation (1) can be generalized to the case, when the subgroup HL is exchanged by a subgroup which is not a direct product of the Lorentz group and the internal symmetry group. As an example of such subgroup we point out on the group $SL(6, C)$ and other relativistic generalizations of the group $SU(6)$.

In the scheme under consideration the coordinates x transform together with fields of the Goldstone particles, hence it considerably differs from the standard schemes of $SU(6)$ symmetry relativization and can be free from their difficulties.

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[1]Note, that the Goldstone particles corresponding to the Fermi fields can be included into the scheme under consideration. In this case the expression (1) corresponds to such a parametrization of the group G which contains anticommuting parameters [4].

References

[1] S. Coleman, J. Wess, and B. Zumino, Phys. Rev. **177**, 2239 (1969).

C. G. Callan, S. Coleman, J. Wess, and B. Zumino, Phys. Rev. **177**, 2247 (1977).

[2] D. V. Volkov, "Phenomenological interaction lagrangian of Goldstone particles", Kiev preprint ITF-69-75 (1969).

[3] A. Salam and J. Strathdee, Phys. Rev. **184**, 1750 (1969).

[4] F. A. Berezin and G. I. Kats, Math. Sbornik **11**, 311 (1970) (Mat. Sb. **82**, 343 (1970)).

T-19503, Order N498.
Signed to print November 18, 1971.
Circulation 250 copies.

Printed at rotaprint FIAN USSR.
Moscow, V-312, Lenin prospect, 53.