Regge Poles in Nucleon–Nucleon and Nucleon–Antinucleon Scattering Amplitudes

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Abstract. The spin structure and the character of the kinematic singularities of the contributions of Regge poles with various quantum numbers to nucleon–nucleon and nucleon–antinucleon scattering amlitudes are cinsidered. It is shown that the presence of kinematic singularities at $t = 0$ in the contributions from certain Regge poles together with the requirement of analyticity of the whole amplitude leads to simple relations, at $t = 0$, between the positions of Regge poles belonging to different trajectories. The dependence of the spin structure of the forward scattering amplitude on the character of these relations is discussed.

1 Introduction

Regge poles (Regge (1959, 1960)) in nn scattering amplitudes have been discussed in a number of recent papers (Regge (1959, 1960), Gribov (1961, 1962), Chew, Frautschi (1961), Frautschi, Gell–Mann, Zachariasen (1962), Udgaonkar (1962), Gell–Mann (1962), Gribov, Pomeranchuk (1962), Gribov, Pomeranchuk (1962a)). In particular, a detailed invesrigation has been made in (Gribov, Pomeranchuk (1962a)) of the spin stucture on the nn scatterung amplitude as determined by the dominant vacuum pole – the Pomeranchuk pole.

In the present paper we consider the spin structure of the nn and na scattering amplitudes thaking into account r with other quantum numbers as well. It will be shown that for fixed signature (Frautschi, Gell–Mann, Zachariasen (1962)) and isotopic spin there are, besides the poles whose remaining quantum numbers are those the vacuum, three further kinds of poles which give a contribution to these amplitudes.

It turns out that the analyticity requirement on the amplitude implies that two of the three families of poles must coincide at $t = 0$(t is the momentum transfer), whereas the poles of the third family have angular momenta which differ from the amgular momenta of the first two families by ± 1. The r at $t = 0$ correspond to particles with zero mass. The coincidence of the furst two families implies that there is a degeneracy in parity and angular momentum for zero mass particles not belonging to the vacuum family.

The situation is of a similar nature as the one noted by one of us (Gribov (1962b, 1963)) in discussing the 180^o meson-nucleon scattering amplitudes. In that case the poles of amplitudes with different parity became identical for vanishing mass.

In both cases the degeneracy is due to the fact that there exists an additional symmetry in forwand or backward scattering which is connected with the presence of only one distinguished direction.

The essential difference between the boson r considered in this paper and the fermion poles considered in (Gribov (1962b, 1963)) consists in the fact that in our case poles with different quantum numbers do not become complex conjugated for $t, 0$. As was shown in (Gribov, Pomeranchuk (1962a)), the contribution from the vacuum poles at $t = 0$ to the amplitudes is indeprndent of the spin. The other poles considered in this paper lead to a spin dependence of the forward scattering amplitude. The spin structure is strongly dependent on whether the angular momentum of the dominant pole of the third family differs from that of the dominant pole of the two other families by $+1$ or -1. In the first case the additional contribution to the forward scattering amplitude has axial vector character and leads to a correlation of the longitudinal polarizations of the nucleons. In the second case it has tensor character and leads to a correlation of the transverse polarizations.

2 Classification of the Poles

Let us consider a na system in the channel where t is the energy. We characterize the state of the system by the total angular momentum j and the projections of the particle spins on their directions of motion. Let us call these staes $|j, \pm, \pm >$. The states $|j, \lambda p, \lambda >$ zre related to the states with definite total spin and parity in the following way (Goldberger, Grisaru, MacDowell, Wong (1960)):

$$|j, 0, - >= |j, +, + > - |j, -, - > \ , \tag{1a}$$

$$|j, 1, - >= |j, +, - > - |j, -, + > \ , \tag{1b}$$

$$|j, 0, + >= |j, +, + > - |j, -, - > \ , \tag{1c}$$

$$|j, 1, + >= |j, +, - > - |j, -, + > \ , \tag{1d}$$

where the figures 0 and1 denote the absolute valie of the spin projection of the relative direction of motion and the signs $\pm$ define the symmetry under interchange of the spin projections of the particles.

The states $|j, 0, - >$ are singlets $(s = 0)$ with parity $(-1)^{j+1}$ the state $|j, 0, - >$ is a triplet with parity $(-1)^{j+1}$, and the states $|j, 0, + >$ are triplet states with parity $(-1)^j$.

The first two states are conserved. The amplitudes in these states will be denoted by f_0^j and f_1^j. The last two stated are not conserved. The amplitudes in these two states will be denoted by f_{00}^j and f_{11}^j, and the amplitude for transition between them by f_{01}^j . In what follows we shall consider the amplitudes $f_1^j, f_1^j, f_{00}^j, f_{11}^j$, and f_{01}^j for complex j and their moving poles.
As is known (Gribov (1961a, 1962a), Chew, Frautschi (1961), Frautschi, Gell–Mann, Zachariasen (1962)), it is necessary for this to consider separately the amplitudes withdifferent signature (Frautschi, Gell–Mann, Zachariasen (1962)), i.e., coinciding with the physical partial waves for even and odd j. In order to classify the different Regge trajectories it is convenient to introduce the following quantum numbers: the signature of parity of $j(P_j)$, the parity P, the G parity, and the isospin T. The relation between the trajectories with quantum numbers $P-j, P, G$, and T and the states $|j,0,\pm>$ is given in the table.

P_j	P	G	T	*States*
$+$	$+$	$+$	0	$\vert j,0,+>$,
		$-$	1	$\vert j,1,->$
$+$	$+$	$-$	0	*none*
		$+$	1	
$+$	$-$	$+$	0	$\vert j,0,->$
		$-$	1	
$+$	$-$	$-$	0	$\vert j,1,+>$
		$+$	1	

The simultaneous change of the signes of P_j, P, and G with fixed T does not change the states.

The states $|j,0,+>$ and $|j,1,+>$ are not conserved and have a common family of poles. However, it is convenient for the following to divide this system of poles into two families according to whether they contribute to one or the other eigenvalue of the matrix

$$\begin{pmatrix} f_{00}^j & f_{01}^j \\ f_{10}^j & f_{11}^j \end{pmatrix} \tag{2}$$

for continuous changes of t. It will be shown below that the transition matrix element $f_{01}^j = 0$ for $t=0$. As a consequence, the amplitudes f_{00}^j and f_{11}^j and the corresponding two systems of poles become independent at $t=0$. In the following wee shall call these systems of poles of the type α and the type β.

Since the states of the nucleon–antinucleon system which transform the matrix (2) into diagonal form do not go over into one another duiring the scattering, only one of them can go over into two mesons. Otherwise they would transform into one another via the two–meson state. Therefore the two–meson annihilation and meson–meson scattering amplitudes contain poles of only one type: either α or β. For $t=0$ the state $|j,1,+>$ cannot go into two mesons, since two mesons always have a vanishing spin projection on the relative direction of motion. Thus the meson–meson and meson–nucleon scattering amplitudes have only poles of type α. The vacuum poles usually considered, and in particular, the Pomeranchuk pole, belong to the type α.

3 Partial Wave Expansion of the Amplitude

We write the nucleon–antinucleon scattering amplitude as a sum of the five fermion variants:

$$F = \sum_{i=1}^{5} H_i(t,s)\left(\bar{u}(p_2)O_\alpha^i v(p_2')\right)\left(\bar{v}(p_1')O_\alpha^i u(p_1)\right) \quad , \tag{3}$$

$$\begin{aligned} &Q_\alpha^1 = 1, \;\; Q_\alpha^2 = \gamma_\mu, \;\; O_\alpha^3 = \sigma_{\mu\nu} \;\; (\mu > \nu) \;, \\ &O_\alpha^4 = i\gamma_5\gamma_\mu, \;\; O_\alpha^5 = \gamma_5 \;\; ; \end{aligned} \tag{4}$$

p_1, p_1', p_2, p_2' are the momenta of the nucleons and antinucleons in the initial and final states, respectively;

$$t = (p_1 + p_1')^2 \,, \quad s = (p_1 - p_1')^2 \,.$$

We use the Feynman choice of matrices and the summation convention $p_\mu x_\mu = p_0 x_0 - \mathbf{px}$.

It was shown by Goldberger et al (Goldberger, Grisaru, MacDowell, Wong (1960)) that the invariant functions $H_i(t,s)$ have no kinematic singularities. This will be essential for the following discussion.

In order to calculate the contributions of the different Regge poles to F it is necessary to use the partial wave expansion of the amplitudes. It is convenient to do this in terms of the helicity amplitudes (Jacob, Wick (1959))

$$\varphi(\lambda_2', \lambda_2, \lambda_1', \lambda_1) = <\lambda_2', \lambda_2|F|\lambda_1', \lambda_1>,$$

where λ is the spin projection of the particle on its momentum. Choosing the spinors u and v of the form

$$u_\lambda(p) = \begin{pmatrix} \sqrt{p_0 + m} \\ 2\lambda\sqrt{p_0 - m} \end{pmatrix} \varphi_\lambda, \quad v_\lambda(p') = C\bar{u}_{\lambda'}(p') \;\; , \tag{5}$$

where φ_λ is the spinor of the state with projection λ on the direction of the momentum p, we easily obtain a relation between the amplitudes and the invariant functions $H_i(t,s)$ (Goldberger, Grisaru, MacDowell, Wong (1960)):

$$\begin{aligned} \varphi_1 &= \varphi(\lambda\lambda\lambda\lambda) = 4p^2 H_1 - 4m^2 z H_2 - 4m^2 z H_3 - 4m^2 H_4 - 4p_0^2 H_5, \\ \varphi_2 &= \varphi(\lambda\lambda - \lambda - \lambda) = -4p^2 H_1 + 4m^2 z H_2 - 4(p_0^2 + p^2) z H_3 \\ &\qquad\qquad\qquad\qquad - 4m^2 H_4 - 4p_0^2 H_5, \\ \varphi_3 &= \varphi(\lambda - \lambda\lambda - \lambda) = -4p_0^2(1+z)H_2 - 4m^2(1+z)H_3 + 4p^2(1+z)H_4, \\ \varphi_4 &= \varphi(\lambda - \lambda - \lambda\lambda) = 4p_0^2(1-z)H_2 + 4m^2(1-z)H_3 + 4p^2(1-z)H_4, \\ \varphi_5 &= \varphi(\lambda\lambda\lambda - \lambda) = -2\lambda\sin\theta\,[4p_0 m H_2 + 4p_0 m H_3] \;\; . \end{aligned} \tag{6}$$

Here $p = \frac{1}{2}\sqrt{t - 4m^2}$ and $p_0 = \frac{1}{2}\sqrt{t}$ are the momentum and the energy of the particles in the center of mass system (c.m.s.) and

$$s = -2p^2(1+z), \quad z = \cos\theta, \quad \theta = \theta_{p_1 p_2}.$$

The partial wave expansion of the amplitude $\varphi(\lambda_2'\lambda_2;\ \lambda_1'\lambda_1)$ is of the form (Jacob, Wick (1959))

$$4\lambda_2'\lambda_1'\varphi(\lambda_2'\lambda_2;\lambda_1'\lambda_1) = \sum_j (2j+1) < \lambda_2'\lambda_2|F^j|\lambda_1'\lambda_1 > d^j_{\mu_1\mu_2}(z) \ , \qquad (7)$$

where $d^j_{\mu_1\mu_2}(z)$ is the reduced rotation matrix and $\mu_1 = \lambda_1' - \lambda_1$, $\mu_2 = \lambda_2' - \lambda_2$. The factor $4\lambda_2'\lambda_1'$ is due to our choice of phase for the state with negative energy.

Following (Goldberger, Grisaru, MacDowell, Wong (1960)) it is convenient to introduce instead of the amplitudes $\varphi(\lambda_2'\lambda_2;\ \lambda_1'\lambda_1)$ their linear combinations

$$\begin{aligned} &f_1 = \varphi_1 + \varphi_2, \quad f_2 = \varphi_1 - \varphi_2, \\ &f_3 = \frac{1}{1+z}\varphi_3 + \frac{1}{1-z}\varphi_4, \quad f_4 = \frac{1}{1+z}\varphi_3 - \frac{1}{1-z}\varphi_4, \\ &f_5 = -\frac{m}{\lambda_0 \sin\theta}\varphi_5 \ . \end{aligned} \qquad (8)$$

The partial wave expansion of the amplitudes $f_i(t,s)$ has the form

$$f_1 = \sum_j (2j+1) f_0^j(t) P_j(z), \qquad (9a)$$

$$f_2 = \sum_j (2j+1) f_{00}^j(t) P_j(z), \qquad (9b)$$

$$f_3 = \sum_j \frac{2j+1}{j(j+1)} \left\{ f_1^j(t) \left[P_j'(z) + zP_j''(z)\right] \ - \ f_{11}^j(t) P_j''(z) \right\} \qquad (9c)$$

$$f_4 = \sum_j \frac{2j+1}{j(j+1)} \left\{ -f_1^j(t) P_j''(z) \ + \ f_{11}^j(t) \left[P_j'(z) + zP_j''(z)\right] \right\} \qquad (9d)$$

$$f_5 = \sum_j \frac{2j+1}{\sqrt{j(j+1)}} \frac{m}{p_0} f_{01}^j(t) P_j'(z), \qquad (9e)$$

where the quantities $f^j(t)$ are defined in the Introduction. In deriving (9a) to (9e) we have made use of the explicit form of $d^j_{\mu_1\mu_2}(z)$ (Jacob, Wick (1959)).

The invariant functions $H_i(t,s)$ are related, according to (7) and (8), to the functions $f_j(t,s)$ by

$$H_1 = \frac{1}{8p^2}\left[f_2 + zf_4 + z\frac{p_0^2+m^2}{m^2}f_5\right], \quad H_2 = -\frac{1}{8p^2}[f_4+f_5],$$

$$H_3 = \frac{1}{8p^2}\left[f_4 + \frac{p_0^2}{m^2}f_5\right], \quad H_4 = \frac{1}{8p^2}f_3,$$

$$H_5 = -\frac{1}{8p_0^2}\left[f_1 - zf_4 + \frac{m^2}{p^2}f_3 - z\frac{p_0^2}{m^2}f_5\right] \ . \tag{10}$$

Using (9) and (10), we may now compute the contributions of the different Regge poles to the scattering ampitude. It follows from (10) that the functions $f_i(t,s)$ as well as the functions $H_i(t,s)$ satisfy ordinary dispersion relations in the momentum transfer. This permits us, with the help of (9), to introduce amplitudes $f_0^j(t)$, $f_1^j(t)$, $f_{00}^j(t)$, $f_{11}^j(t)$, and $f_{01}^j(t)$ with complex j in the same manner as in the case of spinless particles (Frautschi, Gell–Mann, Zachariasen (1962), Gribov (1961a, 1962a)). Considering separately the symmetric and antisymmetric parts of the functions $f_i(t,s)$ and changing the sum into an integral, we easily find the contribution from the pole with the largest Rej of the amplitudes f_0^j, f_1^j, f_{00}^j, and f_{01}^j to the amplitudes $f_i(t,s)$ and hence to F.

4 Contributions to the Scattering Amplitude from Poles with Different Quantum Numbers

A. Contribution from the pole with $P_j = \pm 1$, $P = \mp 1$, $G = \pm 1$ for $T = 0$ and $G = \mp 1$ for $T = 1$. This pole occurs only in the amplitude f_0^j. According to (9) and (10), it gives a contribution only to the amplitudes $f_1(t,s)$ and $H_5(t,s)$. Leaving out the spinors, we find that in this case

$$F_0 = \frac{\pi}{4t}\frac{2j_0+1}{\sin \pi j_0}\alpha_{j_0} r_0^{\pm}(t)\left[(-z)^{j_0} \pm z^{j_0}\right]\gamma_5^{(1)} \times \gamma_5^{(2)} \ , \tag{11}$$

where $j_0 = j_0(t)$ is the position of the pole and $r_0^{\pm}(t)$ the residue of $f_0^j(t)$. The signs $\pm$ refer to the signature, and $\alpha_j = \frac{\Gamma(2j+1)}{2^j \Gamma^2(j+1)}$.

B. Contribution from the pole with $P_j = \pm 1$, $P = \mp 1$, $G = \mp 1$ for $T = 0$ and $G = \pm 1$ for $T = 1$. This pole occurs in the amplitude f_1^j and gives a contribution to the functions $f_3(t,s)$ and $f_4(t,s)$. However, f_3 and zf_4 have the same order of magnitude for large z because of the presence of the term $P_j''(z)$ in (9d).

Taking account of the fact that for large $z(s)$ the scalar and pseudoscalar variants give a contribution one power of a smaller than the rest, it can be seen that this pole gives a contribution only to the pseudovector variant for large s. We thus obtain

$$F_1 = -\frac{\pi}{4(t-4m^2)}\,\frac{2j_1+1}{\sin\pi j_1} r_1^{\pm}(t)\frac{j_1}{j_1+1}\alpha_{j_1}\left[-(-s)^{j_1-1} \pm z^{j_1-1}\right]\cdot$$
$$\cdot i\gamma_5^{(1)}\gamma_\mu^{(1)} \times i\gamma_5^{(2)}\gamma_\mu^{(2)} \quad . \tag{12}$$

C. Contibution from tghe poles with $P_j = \pm 1$, $P = \pm 1$, $G = \pm 1$ for $T = 0$ and $G = \mp 1$ for $T = 1$. This family of poles includes, in particular, the vacuum pole considered in (Gribov, Pomeranchuk (1962a)). Poles with these quantum numbers occur in the amplitudes f_{00}^j, f_{11}^j, and f_{01}^j and give contributions to all functions $f_i(t,s)$ except $f_1(t,s)$. However, for large $z(s)$ the function $f_3(t,s)$ can be omitted by the same arguments as were used to leave out the function $f_4(t,s)$ in the case B. The expressions for the remaining functions are of the form

$$f_2 = -\frac{\pi}{2}\,\frac{2j+1}{\sin\pi j}\alpha_j r_{00}^{\pm}(t)\left[(-z)^j \pm z^j\right],$$
$$f_4 = -\frac{\pi}{2}\,\frac{2j+1}{\sin\pi j}\alpha_j \frac{j}{j+1} r_{11}^{\pm}(t)\left[-(-z)^j \pm z^j\right],$$
$$f_5 = -\frac{\pi}{2}\,\frac{2j+1}{\sin\pi j}\,\frac{m}{p_0}\alpha_j\sqrt{\frac{j}{j+1}}\, r_{01}^{\pm}(t)\left[-(-z)^j \pm z^j\right] \quad . \tag{13}$$

At first sight the formulas (13) xontain three independent parametes r_{00}, r_{11}, and r_{01}. But the residues of different amplitudes at one and the same pole factorize by virtue of the unitarity condition (Gell–Mann (1962), Gribov, Pomeranchuk (1962), Charap, Squires (1962), Gribov, Ioffe, Pomeranchuk, Rudik (1962)), so that

$$r_{00}r_{11} = r_{01}^2 \tag{14}$$

This relation implies the factorization of the whole amplitude into a part referring to the initial state and a part referring to the finel state. Substituting (13) in (10), we obtain an expression containing four Fermion variants which do not manifestly factorize even if (14) is taken into account. In order to obtain an explicitly, factorized expression, we proceed in the following fashion. Instead of the tensor variant we introduce a variant of the form

$$\Delta = \gamma_\mu^{(1)}(p_2 - p_2')_\mu + \gamma_\mu^{(2)}(p_1 - p_1')_\mu \quad . \tag{15}$$

It is easy to show that the following relation is satisfied:

$$tT = 4m^2 V + (s-u)(S+P) - 2m\Delta \quad , \tag{16}$$

where

$$V = \gamma_\mu^{(1)} \times \gamma_\mu^{(2)}, \quad S = 1 \times 1, \quad P = \gamma_5^{(1)} \times \gamma_5^{(2)},$$
$$T = \frac{1}{2}\sigma_{\mu\nu}^{(1)} \times \sigma_{\mu\nu}^{(2)}, \quad u = (p_1 - p_2)^2, s - u = -4p'^2 z \quad .$$

Using this relation, we can write the contibution from the poles in the form

$$F_{\text{vac}} = H_1'(p_1 - p_1')_\mu (p_2 - p_2')_\mu S + H_2' V + H_3' \Delta \ , \tag{17}$$

where

$$H_1' = -\frac{1}{32p^4 z}\left(f_2 + z f_4 \frac{m^2}{p_0^2} + 2z f_5\right),$$
$$H_2' = -\frac{1}{8p_0^2} f_4, \quad H_3' = -\frac{m}{16p^2 p_0^2}\left(f_4 + \frac{p_0^2}{m^2} f_5\right) \ . \tag{18}$$

We easily see with the help of (18) and (14) that

$$H_1' H_2' = H_3'^2 \ . \tag{19}$$

This implies that the amplitude F_{vac} factorizes and can ve written in the form

$$\begin{aligned} F_{\text{vac}} = &\frac{\pi}{4(t-4m^2)^2} \frac{2j+1}{\sin \pi j} \alpha_j \left[-(-z)^{j-1} \pm z^{j-1}\right] \cdot \\ &\cdot \left[(p_1 - p_1')_\mu (\rho_0 + \rho_1) + \frac{t-4m^2}{2m} \rho_1 \gamma_\mu^{(1)}\right] \cdot \\ &\cdot \left[(p_2 - p_2')_\mu (\rho_0 + \rho_1) + \frac{t-4m^2}{2m} \rho_1 \gamma_\mu^{(2)}\right] \end{aligned} \tag{20}$$

where

$$\rho_0^2 = r_{00}, \quad \rho_1^2 = \frac{4m^2}{t} \frac{j}{j+1} r_{11} \ . \tag{21}$$

The form (20) of the contribution of the vacuum pole to the scattering amplitude found in (Gribov, Pomeranchuk (1962a)) was proposed by L.B.Okun'. It provided the basis for the derivation given above.

In concluding this section, we note that the structure of (20) does not depend, for $t \neq 0$, on whether we are dealing with a pole of type α or type β. The difference between the two cases for $t = 0$ shows up in the following way.

It is clear from (9e) and the fact that $f_5(t,s)$ has no singularity at $t = 0$ ($p_0 = 0$) that $f_{01}^j = 0$ for $t = 0$. This means that r_{01} for $t = 0$. Then it follows from (14) that at $t = 0$ either $r_{11} = 0$ or $r_{00} = 0$. In the first case we are dealing with a pole of type α. Here the quantities ρ_0 and ρ_1 remain finite of $t = 0$ [see (21)] and both appear in the expression for F_{vac} for $t = 0$. In the second case we have a pole of type β. It then follows from (21) that $\rho_0 = 0$ and $\rho_1 \to \infty$. This case will be discussed in detail in the netxt section.

Up to this point we have discussed the nucleon–antinucleon scattering amplitude in the channel in which t is the energy and (11), (12), and (20) refer to the region of large unphysical momentum transfers. It is clear that for $t \leq 0$ these formulas give the asymptotic form for $s \to \pm\infty$ of the nucleon–nucleon and nucleon–antinucleon scattering amplitudes, respectively, where s or u are the energy.

5 Asymptotic Form of the Forward Scattering Amplitude

It is well known that the forward scattering amplitude has an additional symmety, as there is only one distinguished direction. Because of this the forward scattering amplitude is defined by three invariant functions instead of five. The lower number of independent functions is connected with the circumstance that in forward scattering there is , first, no polarization (the expression for the amplitude in the c.m.s. does not contain $\sigma_1 \cdot n + \sigma_2 \cdot n$; n is the normal to the scattering plane) and, second, the spin correlations can be defined by expressions of the type $\sigma_{1x}\sigma_{2x} + \sigma_{1y}\sigma_{2y}$ and $\sigma_{1z}\sigma_{2z}$ alone (z the relative direction of motion). The vanishing of the polarization in the relativistic treatment is guaranteed by the fact that the invariant functions H_i have no root singularity at $t = 0$. The necessary form of the correlation terms is guaranteed by the vanishing of the pseudoscalar variant (H_5 has no pole at $t = 0$).

It is obvious that the number of independent amplitudes in the nucleon–antinucleon scattering channel must be the same for $t = 0$. This implies that there must be two relations between the five partial wave amplitudes $f_0^j, f_1^j, f_{00}^j, f_{11}^j$, and f_{01}^j.

As already noted, the partial wave amplitude f_{01}^j is equal to zero for $t = 0$ because of the absence of a $\frac{1}{\sqrt{t}}$ type singularity in the amplitude f_5. The second relation is obtained with the help of the last of Eqs. (10), by requiring that $H_5(t, s)$ does not become infinite at $t = 0$ ($P_0 = 0$). This requirement leads to the condition

$$f_1 - z f_4 - f_3 = 0 \ . \tag{22}$$

The relation (22) can easily be transformed into a relation between the partial wave amplitues, using (9a) and (9b):

$$\begin{aligned} &f_0^{j-1} - f_0^{j+1} - \frac{j-1}{j} f_{11}^{j-1} + \\ &\frac{j+2}{j+1} f_{11}^{j+1} - \frac{2j+1}{j(j+1)} f_1 j = 0 \end{aligned} \tag{23}$$

Relations (22) and (23) have been used for different purposes in (Goldberger, Grisaru, MacDowell, Wong (1960)). It is clear that (23), which is valid for integer j, will also hold true for arbitrary complex j. Interpreting the relation (23) in this sense, we find that the positions of the singularities fo different amplitudes are related at $t = 0$.

We thus obtain relations between the positions of the poles of the amplitudes f_0^j and f_1^j and the positions of the poles of type β which enter in the expression for the amplitude f_{11}^j at $t = 0$. We note that the poles of the type α are distinguised in the sense that there are no relations between their positions and the positions of other poles. Since the above–mentioned relation is a finite difference equation for the amplitudes f_0^j and f_{11}^j, there will in general be, for each pole of f_1^j, an infinite number of poles of f_0^j and f_{11}^j which are displaced from one another by $\Delta j = 2$. On the other hand, if we arbitrarily select a pole at $j = j_0$ in the amplitude f_0^j or f_{11}^j, we find from (23) that the amplitude f_1^j will have two poles at $j = j_0 \pm 1$.

According to the physical interpretation of Regge poles as the analytic continuations in the angular momentum of possible states of a dymanical system, both these situations are completely unreasonable. In particular, there is no reason whatsoever that an arbitrary interaction should have an additional symmetry at $t = 0$ which leads to integral numberd intervals between the angular momenta of the poles of one and the same amplitude. The coincidence of poles of different amplitudes, on the othe hand, is ietirely natural, since there is an additional space symmetry for $t = 0$.

If we require that there be no simple integral numbered relations between the positions of the poles of one and the same amplitude, then we have two possibilities of preserving the validity of (23); the poles of the anplitues f_0^j and f_{11}^j coincide and th angular momentum of the pole of f_1^j differs by ± 1. This implies that it is impossible to ascribe a definite spin and parity to particles with vanishing mass which do not belong to family α.

1. If f_0^j and f_{11}^j have a pole at $j = j_0$ and f_1^j has one at $j = j_0 + 1$, we obtain [by setting $j = j_0 \pm 1$ in (23)] the following relations between the residues of the amplitudes at these poles:

$$r_{11} = r_0 \frac{j_0}{j_0 + 1}, \quad r_1 = r_0 \frac{(2j_0 + 1)(j_0 + 2)}{2j_0 + 3)(j_0 + 1)} \ . \tag{24}$$

2. If f_0^j and f_{11}^j have a pole at $j = j_0$ and f_1^j has one at $j = j_0 - 1$, we have

$$r_{11} = r_0 \frac{j_0 + 1}{j_0}, \quad r_1 = r_0 \frac{(2j_0 + 1)(j_0 - 1)}{(2j_0 - 1)j_0} \ . \tag{25}$$

Let us consider, in correspondence with these two possibilities, the forward scattering amplitude in the chammels where s or u are the energy.

In case 1, where the pole of f_1^j is to the right of the poles of f_0^j and f_{11}^j, we can neglect the contribution of the latter for large z. The forward scattering amplitude will be a sum of contributions from all vacuum poles of the type α which do not lead to a spin dependence and of the expression (12).

In two–component form the amplitude can be written as

$$F = A + B\sigma_{1z}\sigma_{2z} \ . \tag{26}$$

In case 2, where the pole of f_1^j is to the left of the poles of f_0^j and f_{11}^j, we can neglect the contribution from the pole of f_1^j. The scattering amplitude has the form

$$F = \sum_i F_{\alpha_i} + F_0 + f_\beta \ , \tag{27}$$

where F_0 is defined by(11) and F_β is easily obtained from (20) by setting $\rho_0 = 0$:

$$F_\beta = \frac{\pi m^2}{(t-4m^2)^2}\,\frac{2j_0+1}{\sin\pi j_0}\,\alpha_{j_0}\left[-(-z)^{j_0-1} \pm z^{j_0-1}\right]\frac{r_{11}}{t}\,\frac{j_0}{j_0+1}\cdot$$
$$\cdot\left\{(s-u)S + \frac{(t-4m^2)^2}{4m^2}V + \frac{t-4m^2}{2m}\Delta\right\} \quad . \tag{28}$$

Keeping the terms linear in t in the curly brackets and using (16) and the first of Eqs. (25) we easily find

$$F_0 + F_\beta = -\,\frac{\pi}{4m^2}\,\frac{2j_0+1}{\sin\pi j_0}\,\alpha_{j_0}\left[-\left(-\frac{s}{4m^2}\right)^{j_0-1} \pm \left(\frac{s}{4m^2}\right)^{j_0-1}\right]\cdot$$
$$\cdot\frac{r_0}{2}\,\sigma^{(1)}_{\mu\nu} \times \sigma^{(2)}_{\mu\nu} \quad . \tag{29}$$

Here the amplitude has the following two–component form:

$$F = A + B(\sigma_{1x}\sigma_{2x} + \sigma_{1y}\sigma_{2y}) \quad . \tag{30}$$

We note that we have not made full use of (24) and (25) in deriving (12), (26), and (29). Therfore, a comparison of these formulas with experiment does not verify our picture of the coincidence of poles at $t = 0$. However, the mere appearance, in (29), of the tensor variant in which each vertex does not have a definite parity, implies already that the "particle" whose exchange gives rise to such an amplitude has no definite parity.

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