# On the Quantization of Half–Integer Spin Fields

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**Abstract.** A method of quantization for half–integer spin fields is considered which is different from the usual one involving anticommutators and is consistent with the principle of relativistic causality, positive definiteness of the energy (for non–interacting fields), the Lagrangian formalism in Schwinger's formulation (Schwinger (1956)), and with invariance under TCP transformations ( Pauli (1955)). The main difference between the proposed method and the usual one is that the maximal occupation number is two.

## 1 Introduction

It is a well–known fact that nonrelativistic quantum mechanics does not explain the connection between spin and statistics. Moreover, the equations of nonrelativistic quantum mechanics admit of solutions which are neither completely summetric nor completely antisymmetric, and which transform according to different irreducible representations of the permutation group if the particles are interchanged. In the relativistic quantum theory, the existing methods of quantization lead to a unique connection between spin and statistics (Pauli(1947)); however, from the very beginning only two alternatives are considered in this case: either we quantize with commutators, or with anticommutators. In this connection it is of interest to investigate whether other possibilities, which are admissible in nonrelativistic quantum mechanics, are consistent with the basic principles of the relativistic theory.

In the present paper we consider, on the simplest example of the Dirac equation, the possibility of constucting an algebra of operator wave functions with the following properties: it leads to a new statistics with the maximal occupation number two for each individual state,[1] and is at the same time consistent with the principle of relativistic causality, the positive definiteness of the enery (for non–interacting fields), and with the Lagrangian formalism. In setting up the Lagrangian formalism we make use of the variational principle of (Schwinger (1956)). We show that this method of quantization

[1] This method of quantization can be generalized for the case of arbitrary maximal occupational numbers.

is a consequence of the variational principle of Schwinger based on a class of admissible variations of the operator wave function which is different from that used in the usual quantization scheme.

## 2 The Condition of Relativistic Causality

The requirement of relativistic causality is usually formulatied as the condition that commutators of physical operators reduce to zero for points which are separated by a space–like interval (outside the light cone). With the condition that operators corresponding to measurable quantities are bilinear combinations of the opoerator wave functions (as in the case of half–integer spin fields), the requirement of relativistic causality will be fulfilled if the commutators or anticommutators of the operator wave functions reduce to zero outside the light cone. These are the only two cased usually considered. It is, however, possible to satisfy the requirement of relativistic causlity using a different algebra for the operator wave functions.

As an example, we consider the field satisfying the free Dirac equation,

$$\left(\gamma_\mu \frac{\partial}{\partial x_\mu} + m\right)\psi = 0 \ , \tag{1}$$

where $\gamma_\mu$ ($\mu$=1,2,3,4) are matrices defined by the relation $\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu = 2\delta_{\mu\nu}$.

We define the operator properties of the wave functions $\psi(c)$ and $\bar{\psi}(x) = \psi^+(x)\gamma_4$ with the help of the commutation relations

$$\begin{aligned}
&\psi_\alpha(x)\psi_\beta(x^{'})\psi_\gamma(x^{''}) + \psi_\gamma(x^{''})\psi_\beta(x^{'})\psi_\alpha(x) = 0,\\
&\psi_\alpha(x)\bar{\psi}_\beta(x^{'})\psi_\gamma(x^{''}) + \psi_\gamma(x^{''})\bar{\psi}_\beta(x^{'})\psi_\alpha(x) =\\
&\qquad = -\mathrm{i}S_{\alpha\beta}(x-x^{'})\psi_\gamma(x^{''}) - \mathrm{i}S_{\gamma\beta}(x^{''}-x^{'})\psi_\alpha(x),\\
&\bar{\psi}_\alpha(x)\bar{\psi}_\beta(x^{'})\psi_\gamma(x^{''}) + \psi_\gamma(x^{''})\bar{\psi}_\beta(x^{'})\psi_\alpha(x) =\\
&\qquad = -\mathrm{i}S_{\gamma\beta}(x^{''}-x^{'})\bar{\psi}_\alpha(x) \ ,
\end{aligned} \tag{2}$$

where $S(x)$ is the known commutator function (Akhiezer and Berestetskii (1953))

$$\begin{aligned}
S_{\alpha\beta}(x) &= -\left(\gamma_\mu \frac{\partial}{\partial x_\mu} - m\right)_{\alpha\beta} \mathit{\Delta}(x),\\
\mathit{\Delta}(x) &= \mathrm{i}(2\pi)^{-3}\int \mathrm{e}^{\mathrm{i}px}\varepsilon(p)\delta\left(p^2+m^2\right)\mathrm{d}^4p \ .
\end{aligned} \tag{3}$$

It follows from the properties of the function $S(x)$ that the relations (2) are consistent with Eq. (1). We verify that all commutators of the form $\left[\psi_\alpha(x)\psi_\beta(x),\ \psi_\gamma(x^{'})\psi_\delta(x^{'})\right]$ are zero outside the light cone. For the proof we write the relations (2) in the following compact form:

$$\psi_\alpha(x)\psi_\beta(x^{'})\psi_\gamma(x^{''}) + \psi_\gamma(x^{''})\psi_\beta(x^{'})\psi_\alpha(x) =$$
$$= \left\{\psi_\alpha(x), \psi_\beta(x^{'})\right\}_F \psi_\gamma(x^{''}) + \left\{\psi_\gamma(x^{''}), \psi_\beta(x^{'})\right\}_F \psi_\alpha(x), \qquad (2^{'})$$

where $\psi$ now stands for the usual as well as the Dirac conjugate spinor, and the brackets with the index F are given by

$$\left\{\psi_\alpha(x), \bar{\psi}_\beta(x^{'})\right\}_F \equiv -\mathrm{i}S_{\alpha\beta}(x - x^{'}),$$
$$\left\{\psi_\alpha(x), \psi_\beta(x^{'})\right\}_F \equiv 0 \ , \qquad (4)$$

i.e., by the usual values for the anticommutators in the quantization according to Fermi–Dirac statistics.[2]

Using the relations $(2^{'})$, we can transform this commutator to the form

$$\psi_\alpha(x)\psi_\beta(x)\psi_\gamma(x^{'})\psi_\delta(x^{'}) - \psi_\gamma(x^{'})\psi_\delta(x^{'}\psi_\alpha(x)\psi_\beta(x) =$$
$$- \left\{\psi_\alpha(x), \psi_\delta(x^{'})\right\}_F \psi_\gamma(x^{'})\psi_\beta(x) + \left\{\psi_\beta(x), \psi_\gamma(x^{'})\right\}_F \psi_\alpha(x)\psi_\delta(x^{'}) \ .$$

It follows from the properties of the function $S(x)$ that this expression is zero outside thew light cone.

It is easily shown that the commutators of physical quantities also reduce to zero outside the light cone for commutation relations of the type

$$\psi_\alpha(x)\psi_\beta(x^{'})\psi_\gamma(x^{''}) + \psi_\gamma(x^{''})\psi_\beta(x^{'})\psi_\alpha(x) =$$
$$= \left\{\psi_\alpha(x), \psi_beta(x^{'})\right\}_F \psi_\gamma(x^{''}) + \left\{\psi_\gamma(x^{''}), \psi_\beta(x^{'})\right\}_F \psi_\alpha(x)$$
$$+ \rho \left\{\psi_\alpha(x), \psi_\gamma(x^{''})\right\}_F \psi_\beta(x^{'}); \qquad (5)$$

$$\psi_\alpha(x)\psi_\beta(x^{'})\psi_\gamma(x^{''}) - \psi_\gamma(x^{''})\psi_\beta(x^{'}\psi_\alpha(x) =$$
$$\left\{\psi_\alpha(x), \psi_\beta(x^{'})\right\}_F \psi_\gamma(x^{''}) - \left\{\psi_\gamma(x^{''}), \psi_\beta(x^{'})\right\}_F \psi_\alpha(x) \ , \qquad (6)$$

where $\rho$ is an arbitrary number.

The commiutation relations (5) have nono-zero solutions only for $\rho$=0 and $\rho$=-1. In the last case the algebra for the operator wave functions correspons to th usual method of quantization using anticommutators.

The commutation relations (6) differ from the commutation relations (2') by a change of sign. This difference (as in the case of commutators and anticommutators) leads to an energy for halfinteger spin fields which is not positive definite.

In principle, the relations (6) can be used for the quantization of fields with integer spin, with the requirement that the interaction Lagrangian contains an even number of field operators.

[2] We note that in this quantization scheme the anticommutator $\left\{\psi_\alpha(x), \psi_\beta(x^{'})\right\} \neq \left\{\psi_\alpha(x), \psi_\beta(x^{'})\right\}_F$.

## 3 Momentum Representation.

The algebra for the field operators is conveniently realized in the momentum representation. We make the transition to the momentum representation by expanding the operator wave functions and the commutator function $S(x)$ into Fourier series (Akhiezer and Berestetskii (1953)):

$$\begin{aligned} \psi_\alpha(x) &= V^{-\frac{1}{2}} \sum_p \sum_{r=1}^{2} \left\{ a_{pr} u_\alpha^r(\mathrm{p}) \mathrm{e}^{\mathrm{i}px} + b_{pr}^+ v_\alpha^r(\mathrm{p}) \mathrm{e}^{-\mathrm{i}px} \right\}, \\ \bar{\psi}_\alpha(x) &= V^{-\frac{1}{2}} \sum_p \sum_{r=1}^{2} \left\{ a_{pr}^+ \bar{u}_\alpha^r(\mathrm{p}) \mathrm{e}^{-\mathrm{i}px} + b_{pr} v_\alpha^r(\mathrm{p}) \mathrm{e}^{\mathrm{i}px} \right\}, \end{aligned} \tag{7}$$

$$-\mathrm{i} S_{\alpha\beta}(x) = \frac{1}{V} \sum_p \sum_r^{2} \left\{ u_\alpha^r(\mathrm{p}) \bar{u}_\beta^r(\mathrm{p}) \mathrm{e}^{\mathrm{i}px} + v_\alpha^r(\mathrm{p}) \bar{v}_\beta^r(\mathrm{p}) \mathrm{e}^{-\mathrm{i}px} \right\} \quad , \tag{8}$$

where $V$ is the normalization volume; the sum over $r$ implies summation over the states with different polarization; $u_\alpha^r$ and $v_\alpha^r$ are constant spinors subject to the orthonormality conditions

$$\sum_\alpha u_\alpha^{r\,*} u_\alpha^s = \delta_{rs}, \quad \sum_\alpha v_\alpha^{r\,*} v_\alpha^s = \delta_{rs} \quad ,$$

Substituting the expansions (7) and (8) in (2), we obtain the following commutation relations for the operators $a$ and $b$:

$$\begin{aligned} & a_k a_l^+ a_m + a_m a_l^+ a_k = \delta_{kl} a_m + \delta_{ml} a_k, \\ & a_k a_l^+ a_m^+ + a_m^+ a_l^+ a_k = \delta_{kl} a_m^+, \\ & b_k b_l^+ b_m + b_m b_l^+ b_k = \delta_{kl} b_m + \delta_{ml} b_k, \\ & b_k b_l^+ b_m^+ + b_m^+ b_l^+ b_k = \delta_{kl} b_m^+, \\ & a_k a_l^+ b_m + b_m a_l^+ a_k = \delta_{kl} b_m, \\ & b_k b_l^+ a_m + a_m b_l^+ b_k = \delta_{kl} a_m \quad ; \end{aligned} \tag{9}$$

the indices $k$, $l$, and $m$ define the momentum and the polarization.

All the remaining commutation relations of the same type, except those which derive from (9) by Hermitian conjugation, are equal to zero.[3]

The operators corresponding to the basic physical quantities can be simply expressed in terms of the operators $N_k = a_k^+ a_k - a_k a_k^+ + 1$ [cf. Eqs. (17') to (19') below]. We show that this operator can be interpreted as the opeeeerator

[3] With $a_k = \alpha_k + \mathrm{i}\beta_k$ and $b_k = \gamma_k + \mathrm{i}\delta_k$, where $\alpha_k$, $\beta_k$, $\gamma_k$ and $\delta_k$ are Hermitian matrices, relations (9) go over into the Duffin–Kemmer relations. The algebra of the Duffin–Kemmer matrices for an arbitrary number of matrices was considered by ( Fujiware (1953)).

corresponding to the number of particles in the state $k$. To determine the eigenvalues of the operator $N_k$, we use the relation

$$\left(a_k^+ a_k - a_k a_k^+\right)^3 = a_k^+ a_k - a_k a_k^+ \ , \tag{10}$$

which is readily proven with the help of the commutation relations (0. It follows from formula (10) that the eigenvalues of the operator $a_k^+ a_k - a_k a_k^+$ are equal to $-1$, 0, or 1, i.e., the corresponding eigenvalues of the operator $N_k$ are equal to 0,1,or 2.

We consider the commutation relations of the operator $N_k$ with the operators $a_l$ and $a_l^+$:

$$\left[a_k^+ a_k - a_k a_k^+, a_l\right] = a_k^+ a_k a_l - a_k a a_k^+ a_l - a_l a_k^+ a_k + a_l a_k a_k^+ \ . \tag{11}$$

From the relations (9) we have

$$\begin{aligned} &a_k^+ a_k a_l + a_l a_k a_k^+ = a_l, \\ &a_k a_k^+ a_l + a_l a_k^+ a_k = a_l + \delta_{lk} a_k \ . \end{aligned} \tag{12}$$

Substituting (12) in (11), we obtain

$$[N_k, a_l] = -\delta_{kl} a_l \ . \tag{13}$$

Similarly, we have for the operator $a_l^+$:

$$\left[N_k, a_l^+\right] = \delta_{kl} a_l^+ \ . \tag{14}$$

The relations (13) and (14) are analogous to the corresponding relations in the usual quantization scheme. In particular, they inmply that the operators $a_l^+$ and $a_l$ can be interpreted as the cr4eation and annihilation operators for particles in the stae $l$.

Indeed, if $\Psi_{n_1,n_2,\ldots}$ is the simultaneous eigenvector of the operators $N_k$ (the operators $N_k$ commute with one another) with the eigenvalues $n_k$, then $a_k \Psi_{n_1,\ldots}$ and $a_k^+ \Psi_{n_1,\ldots}$ are also eigenvectors of these operators, where the eigenvalue $n_k$ is loweered or raised by unity, respectively.

We define the vacuum state as the statew in which all occupation numbers $n_k$ are equal to zero, i.e.,

$$\left(a_k^+ a_k - a_k a_k^+ + 1\right) \Psi_0 = 0 \ . \tag{15}$$

It follows from the definition (115) and the relations (13) and (14) that

$$\begin{aligned} &a_k \Psi_0 = 0, \quad a_k a_l^+ \Psi_0 = 0 \quad (k \neq l); \\ &a_k a_k^+ \Psi_0 = \Psi_0 \ . \end{aligned} \tag{16}$$

We can generate a complete set of basis vectors by successively acting on the vacuum vector with the creation operators $a_k^+$:

$$\Psi_0,\ a_k^+ \Psi_0,\ a_k^+ a_l^+ \Psi_0,\ \text{etc.}$$

We note that, in contrast to the quantization scheme using anticommutators,, we now have $a_k^+ a_k^+ \Psi_0 \neq 0$; futhermore, basis vectors differing in the order of the operators $a_k^+$ can be independent. For example, in the case of two particles the vectors $a_k^+ a_l^+ \Psi_0$ and $a_l^+ a_k^+ \Psi_0$ are independent; for three particles, we have the following independent vectors:

$$a_k^+ a_l^+ a_m^+ \Psi_0, \quad a_k^+ a_m^+ a_l^+ \Psi_0, \quad a_m^+ a_k^+ a_l^+ \Psi_0$$

etc. This difference manifests itself in configuration space through the appearance of partially symmetric wave functions.

The commutation relations (9) together with the relations (16) permit the calculation of the result of operating with the operators $a_k$ and $a_k^+$ on an arbitrary basis vector; thus we can determine the explicit form of these operators in the representation under consideration.

## 4 Operators Corresponding to Physical Quantities

The operators of energy, momentum, and charge for the free Dirac field are given by the following expressions, which are antisymmetric in the operator wave functions:

$$E = \mathrm{i} \int \left( \bar{\psi} \gamma_4 \frac{\partial}{\partial t} \psi - \frac{\partial}{\partial t} \psi \gamma_4^T \bar{\psi} \right) \mathrm{d}V ; \tag{17}$$

$$\mathbf{P} = -\mathrm{i} \int \left( \bar{\psi} \gamma_4 \nabla \psi - \nabla \psi \gamma_4^T \bar{\psi} \right) \mathrm{d}V \tag{18}$$

$$Q = e \int \left( \bar{\psi} \gamma_4 \psi - \psi \gamma_4^T \bar{\psi} \right) \mathrm{d}V \quad . \tag{19}$$

The conventional expression for the energy, momentum, and charge differ from the expressions (17) to (19) by the factor $\frac{1}{2}$. This difference is connected with the normalizations of the commutation relations for the operator wave functions; it can be removed by changing the function $S(x - x^{'})$ in the commutation relations to $2S(x - x^{'})$.

In the momentum representation the operators of energy, momentum, and charge have the form

$$\begin{aligned} E &= \sum_{pr} |p_0| \left( a_{pr}^+ a_{pr} - a_{pr} a_{pr}^+ - b_{pr} b_{pr}^+ + b_{pr}^+ b_{pr} \right) \\ &= \sum_{pr} |p_0| \left( N_{pr}^{(+)} + N_{pr}^{(-)} - 2 \right) ; \end{aligned} \tag{20}$$

$$\mathbf{P} = \sum_{pr} \mathbf{p} \left( a_{pr}^+ a_{pr} - a_{pr} a_{pr}^+ - b_{pr} b^+ pr + b_{pr}^+ b_{pr} \right)$$

$$= \sum_{pr} \mathbf{p} \left( N_{pr}^{(+)} + N_{pr}^{(-)} \right); \tag{21}$$

$$Q = e \sum_{pr} \left( a_{pr}^{+} a_{pr} - a_{pr} a_{pr}^{+} + b_{pr} b_{pr}^{+} - b_{pr}^{+} b_{pr} \right)$$
$$= e \sum_{pr} \left( N_{pr}^{(+)} - N_{pr}^{(-)} \right) \quad , \tag{22}$$

where $N_{pr}^{(+)}$ and $N_{pr}^{(-)}$ are the number operators for particles and antiparicles.

The infinite term $\sum 2|p_0|$ in expression (20) does not contain any operators and can therefore be omitted, as in the usual theory. As a result, the energy becomes a positive definite quantity. The spectum of the operators $E$, $\mathbf{P}$, and $Q$ admits of the usual interpretation in terms of the number of particles occupying the individual states, with the only difference that now the maximal occupation number for each state is equal to two.

We now consider the commutators of operators for physical quantities with the operator wave functions. From the relations (13) and (14) we have

$$\begin{aligned} [E, \psi(x)] &= -\mathrm{i} \frac{\partial \psi(x)}{\partial t}, \\ [\mathbf{P}, \psi(x)] &= \mathrm{i} \nabla \psi(x), \\ [Q, \psi(x)] &= -\psi(x) \ . \end{aligned} \tag{23}$$

The relations (23) give the usual connection between the operators of energy, momentum, and charge and the infinitesimal canonical transformations. Similar relations can also be obtained for other physical operators.

## 5 The Variational Principle of Schwinger TCP Invariance

The variational principle of (Schwinger (1956)) contains the most consistent formulation of the basic postulates of the quantum theory of localized fields. We show that our method of quantization is contained in the variational principle of Schwinger as a special solution.

Since the detailed exposition of Schwinger's variational principle can be found in the literature, we shall deal only with those aspects which change as we make the transition to our method of quantization.

We take the Lagrangian in the form

$$L = -\frac{1}{2} \left[ \bar{\psi}, \gamma_\mu \frac{\partial}{\partial x_\mu} \psi + m\psi \right] - \frac{1}{2} \left[ -\frac{\partial}{\partial x_\mu} \bar{\psi} \gamma_\mu + m\bar{\psi}, \psi \right] + \dots \ , \tag{24}$$

... stands for any arbitrary interaction terms.

We shall assume that the terms describing the interaciton are antisymmetrized with respect to the operator functions $\psi$ and $\bar{\psi}$. The class of admissible variations is restricted by the conditions

$$\psi_\alpha(x)\delta\psi_\beta(x)\psi_\gamma(x) + \psi_\gamma(x)\delta\psi_\beta(x)\psi_\alpha(x) = 0; \tag{25}$$

$$\begin{aligned}&\delta\psi_\alpha(x)\left(\psi_\beta(x)\psi_\gamma(x) - \psi_\gamma(x)\psi_\beta(x)\right)\\ &\qquad + \left(\psi_\gamma(x)\psi_\beta(x) - \psi_\beta(x)\psi_\gamma(x)\right)\delta\psi_\alpha(x) = 0 \ .\end{aligned} \tag{26}$$

where $\psi(x)$ stands for the usual as well as the Dirac conjugate spinor. We shall see later on that we have to supplement the definition of the class of admissible variations to obtain the commutation relations.

Condition (26) is sufficient for the derivation of the equations of motion. Indeed, owing to the antisymmety of the Lagrangian, condition (26) permits us to move the variations either completely to the right or to the left, depending on whether the variation is in an even or an odd position in the formula. Again we see from the antisymmetry of the Lagrangian that the coefficients of the variations standing to the left or to the right in the formula are equal, and can be set equal to zero simultaneously.

To obtain the commutation relations, we consider the operators $G(\psi)$ and $G(\bar{\psi})$ which generate an infinitesimal transformation of the functions $\psi(x)$ and $\bar{\psi}(x)$ (Schwinger (1956));

$$G(\psi) = \mathrm{i}\int \mathrm{d}V \left[\bar{\psi}(x), \gamma_4\delta\psi(x)\right]; \tag{27}$$

$$G(\bar{\psi}) = -\mathrm{i}\int \mathrm{d}V \left[\delta\bar{\psi}(x), \gamma_4\psi(x)\right] \ ; \tag{28}$$

the time $t$ is assumed to be the same in both operators and is not indicated ecplicitly.

The commutators of $G$ with $\psi$ and $\bar{\psi}$ are equal to

$$[\psi(x), G(\psi)] = \mathrm{i}\delta\psi(x); \tag{29}$$

$$\left[\bar{\psi}(x), G(\psi)\right] = \mathrm{i}\delta\psi(x) \ . \tag{30}$$

The other commutators are equal to zero.

Substituting (27) in (29) and using (25), we find

$$\begin{aligned}\int \mathrm{d}V &\Big\{\psi_\mu(x^{'})\bar{\psi}_\nu(x)(\gamma_4)_{\nu\rho}\delta\psi_\rho(x) +\\ &\qquad \delta\psi_\rho(x)(\gamma_4)_{\nu\rho}\bar{\psi}_\nu(x)\psi_\mu(x^{'})\Big\} = \delta\psi_\mu(x^{'}) \ .\end{aligned} \tag{31}$$

Hence

$$\begin{aligned}&\psi_\mu(x^{'})\bar{\psi}_\mu(x)(\gamma_4)_{\nu\rho}\delta\psi_\rho(x) + \delta\psi\rho(x)(\gamma_4)_{\nu\rho}\bar{\psi}_\nu(x)\psi_\mu(x^{'})\\ &\qquad = \delta(x^{'} - x)\delta\psi_\mu(x) \ .\end{aligned} \tag{32}$$

Analogous relations are obtained for the other commutators of $G$ with $\psi$ and $\bar{\psi}$.

The relations (32) further delimits the class of admissible variations. However, this delimitation is not sufficiently comlete to obtain the commutation relations in explicit form.

We note that the more general relations of the type

$$\psi_\mu(x)\bar{\psi}_\nu(x^{'})\delta\psi_\rho(x^{''}) + \delta\psi_\rho(x^{''})\bar{\psi}_n u(x^{'})\psi_\mu(x) = (\gamma_4)_{\mu\nu}\delta(x - x^{'})\delta\psi_\rho(x^{''}); \quad (33)$$

$$\psi_\mu(x)\delta\bar{\psi}_\nu(x^{'})\psi_\rho(x^{''}) + \psi_\rho(x^{''})\delta\bar{\psi}_\nu(x^{'})\psi_\mu(x) = 0 \ , \quad (34)$$

are also valid. They are consistent with (25), (26), and (32).

Applying formula (33) to the relations

$$\left[\bar{\psi}(x)\psi(x^{'}), G(\psi)\right] = \mathrm{i}\bar{\psi}(x)\delta\psi(x^{'}) \quad (35)$$

we obtain the following expression for the commutation relations:

$$\psi_\mu(x)\bar{\psi}_n u(x^{'})\psi_\rho(x^{''}) + \psi_\rho(x^{''})\bar{\psi}_\nu(x^{'})\psi_\mu(x) = (\gamma_4)_{\mu\nu}\delta(x - x^{'})\psi_\rho(x^{''}) + (\gamma_4)_{\rho\nu}\delta(x^{''} - x^{'})\psi_\mu(x) \quad (36)$$

etc., in agreement with (2').

All otherapplications of the variational principle of Schwinger remain practically unaltered in changing the quantization method.

In concluding this section, we note that our method of quantization preserves the TCP invariance ( Pauli (1955)). Indeed, the TCP invariance for the case of spin $\frac{1}{2}$ fields is a consequence of the antisymmetrization of the equations of motion with respect to the operator wave functions. But this antisymmetrization also lies at the basis of our method of quantization.

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