# $N=1$ superconformal invariance

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Possible generalizations of $N=1$ superconformal transformations are studied. Weakening of the requirement that they be invertible leads to transformations belonging to a superconformal subgroup. A matrix realization is obtained. The relationship between the form of the transformations and the structure of the superfield being transformed is investigated.

## 1. INTRODUCTION

Two-dimensional superconformal symmetry[1,2] plays an exceptionally important role in the construction of the superstring theories[3] that aspire to give a unified description of all interactions, and also in the field-theoretical analysis of critical phenomena in two-dimensional statistical models.[4] On the one hand, the classical vacuum of superstring theory corresponds to a certain superconformal field theory.[5] On the other hand, from a geometrical point of view[6] the world sheet of a fermion string is the superanalog of a Riemann surface, a superconformal manifold,[7] or a super-Riemann surface[8,9] as a set of super-regions pasted together by superconformal transformations. In the covariant quantization of a fermion string the calculation of the amplitudes[10] reduces to integration over the set of inequivalent classes of superconformal manifolds, i.e., over the finite-dimensional superconformal space of the moduli.[11]

In similar and analogous investigations an important role can be played at various stages by the explicit form of the superconformal transformations. In this paper we analyze in detail the structure of $N=1$ superconformal transformations, and also point out ways in which various admissible generalizations of them can be made. It is shown that a specific feature for the supersymmetric case is the existence of possible exotic transformations that have a degenerate numerical part and can belong to a superconformal subgroup. The structure of superfields that possess superconformal covariance under superanalytic but not superconformal transformations is also found. The account is given in the language of local coordinates and at a physical level of rigor. The constructions introduced may be considered in the framework of a coordinate-free approach with the aid of Ref. 12.

## 2. $N=1$ SUPERCONFORMAL INVARIANCE

The complex $(1|1)$-dimensional superspace $C^{1,1}$ is defined in the approach of Ref. 13 as the outer product of the even part and the odd part (infinite-dimensional[14]) of a Banach–Grassmann algebra over the field $C$ with natural $Z_2$ grading, and is locally described by the set $Z=(z,\theta)$, where $z$ is the even coordinate and $\theta$ is the odd coordinate. A superanalytic mapping of a region in $C^{1,1}$ has, after expansion in a series in $\theta$, the form

$$z=f(z)+\theta\chi(z), \quad \theta=\psi(z)+\theta g(z). \tag{1}$$

Here and below, we shall use Latin letters to denote even smooth functions $C^{1,0}\to C^{1,0}$, and Greek letters to denote odd functions $C^{1,0}\to C^{0,1}$. The Berezinian[15] of the mapping (1) is equal to

$$J_B=\mathrm{Ber}(Z/Z)=\frac{f'(z)}{g(z)}-\frac{\chi(z)\psi'(z)}{g^2(z)}+\theta\left[\frac{\chi(z)}{g(z)}\right]', \tag{2}$$

and is defined for $m[g(z)]\neq 0$, where $m(x)$ is the numerical part of $x$ (Ref. 11). For invertibility of (1) it is necessary that $m[J_B]\neq 0$.

At every point of $C^{1,1}$ we can distinguish a $(0|1)$-dimensional subspace of the tangent space, which corresponds to a differential operator $D$ (superderivative) that has, in the canonical local coordinate system,[16] the form

$$D=\partial_\theta-\theta\partial_z, \quad D^2=-\partial_z(\partial_\theta=\partial/\partial\theta,\ \partial_z=\partial/\partial z).$$

Under the action of the transformations (1) the operator $D$ goes over into

$$D=(D\theta)\,\tilde{D}-(D\tilde{z}-\theta D\theta)\,\tilde{D}^2. \tag{3}$$

In order that the $(0|1)$-dimensional subspace of the tangent space of $C^{1,1}$ not depend on the choice of the local coordinate system,[17] it is necessary that the transformation of the operator $D$ in (3) be homogeneous. This can be achieved in three ways. For example, the vanishing of the expression in the brackets in (3) leads to a further restriction on the coordinate transformations, which are called superconformal transformations.[18] A zero value of the second factor in the inhomogeneous term in (3) makes it possible to distinguish a class of superfields whose superderivatives transform homogeneously under superanalytic transformations (1). We may also consider the intermediate case when both factors in the inhomogeneous term in (3) are nonzero but their product is equal to zero, e.g., if they are proportional to the same nilpotent quantity whose degree of nilpotence is equal to two, which is a consequence of the existence of divisors of zero among the elements of the Grassmann algebra.[13]

## 3. $N=1$ SUPERCONFORMAL TRANSFORMATIONS

The first way of making the inhomogeneous term in (3) equal to zero leads to the equation

$$D\tilde{z}-\theta D\theta \tag{4}$$

or

$$\partial_z\tilde{z}+\theta\partial_z\tilde{\theta}=(D\tilde{\theta})^2, \tag{5}$$

and this defines superconformal transformations of the coordinates. These smooth and invertible transformations can serve as transition functions on the intersection of the $(1|1)$-dimensional regions (charts) from which the superconformal manifolds[11,17] or $N=1$ super-Riemann surfaces[8,9] are pasted. It follows from (4) that only in superconformal


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