Limits of Optimal Parallel Flow Analysis [Abstract]

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Static program slicing [13] is an established program reduction technique that has applications in program understanding, debugging, and testing [12]. It has also been proposed as a technique for ameliorating the state-explosion problem when formally verifying software or hardware [3, 2, 1, 6]. The goal of program slicing is to identify and remove parts of the program that cannot (potentially) influence given value(s) at given program point(s) of interest.

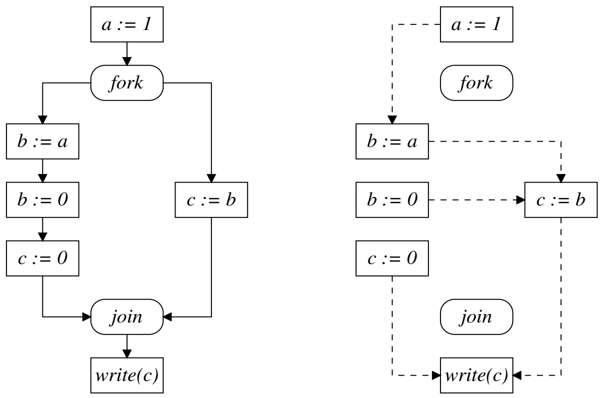
In recent years, the standard slicing technique in sequential program which is based on a backwards reachability analysis in the program dependency graph (PDG) has been extended to certain classes of parallel languages in order to allow slicing of languages like JAVA or VHDL. The resulting slicing algorithms are sound – they never remove statements that can influence the slicing criterion – but unlike the sequential algorithms they are not complete[1]. Consider, e.g., the program

$$a := 1; [(b := 1; b := 0; c := 0) \parallel c := b] \,; \mathbf{write}(c) \,,$$

for which in Fig. 1 (a) a control flow graph-like representation is shown and in (b) the data and interference dependences. We are interested in slicing w.r.t. variable c at the write instruction. Clearly, the instruction $a := 1$ is backwards reachable in the dependency graph from the write-instruction. But it is not hard to see that there is no execution of the program that realizes all dependences in this path and therefore an optimal slicer must remove $a := 1$.

Is just the extension of the sequential techniques too naive or are there fundamental limits? In my talk which was based on [8, 7], I presented new complexity and undecidability results showing the latter to be the case. Specifically, the interprocedural parallel slicing problem is undecidable and the intraprocedural problem is PSPACE-complete. Even if we restrict attention to parallel programs without loops and procedures the problem remains NP-complete. Similar results can be shown for other data-flow problems like detection of copy constants, and detection of truly-life variables in parallel programs. This comes as a surprise, as only slightly simpler analysis questions, namely, *strong*

[1]We call a slicer *complete* if it yields statement-minimal slices after abstraction of conditional branching to non-deterministic branching, an abstraction that is quite common in data-flow analysis in order to ban undecidability.



(a) CFG-like representation (b) Data and interference dependences

Figure 1: An example program.

copy constant propagation and (ordinary) liveness of variables can be optimally solved in polynomial time [5, 4, 10].

Previous complexity and undecidability results for dataflow problems for concurrent languages [11, 9] exploit in an essential way synchronization primitives of the considered languages. In contrast our results hold independently of any synchronization and exploit only interleaving of atomic assignment statements. Therefore, they apply to a much wider class of concurrent languages.

References

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