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Next-Generation Lagrangian Reachability

Sophie Grünbacher, Jacek Cyranka, Md. Ariful Islam, Scott A. Smolka
and Radu Grosu

IFIP WG 2.2 meeting in Vienna

25 September 2019



Der Wissenschaftsfonds.
funded by FWF project W1255-N23



Safety of Cyber-Physical Systems

Risk of Blackout

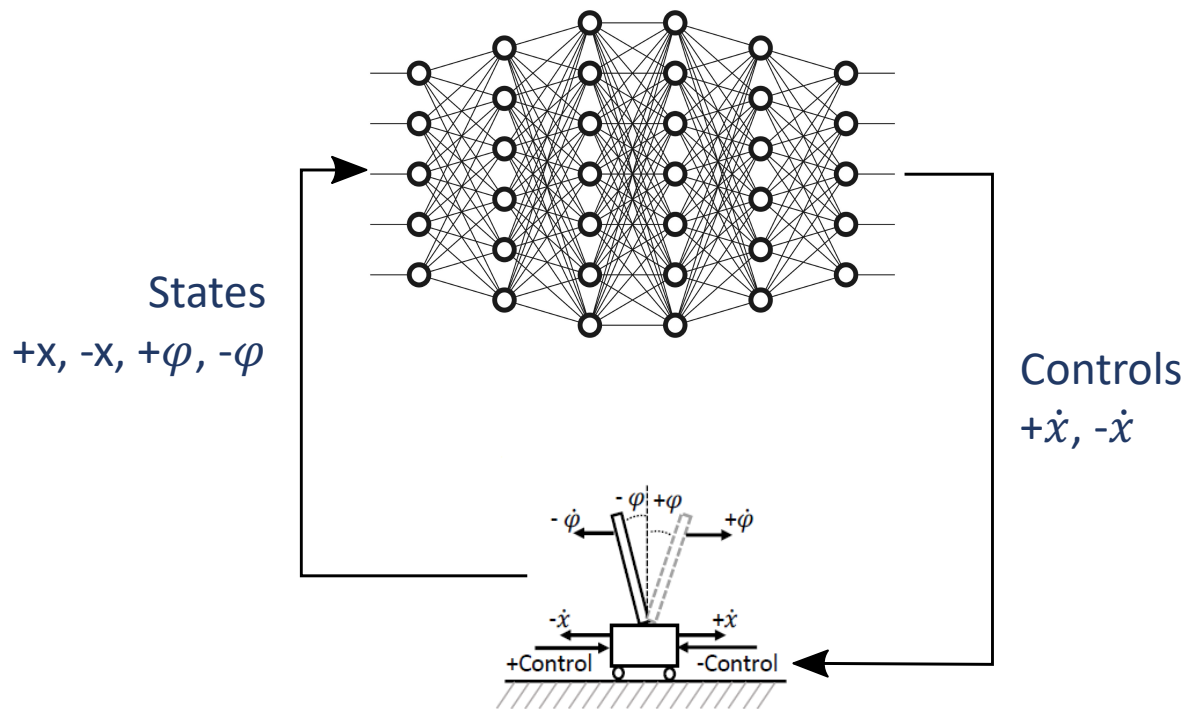


Risk of Crash



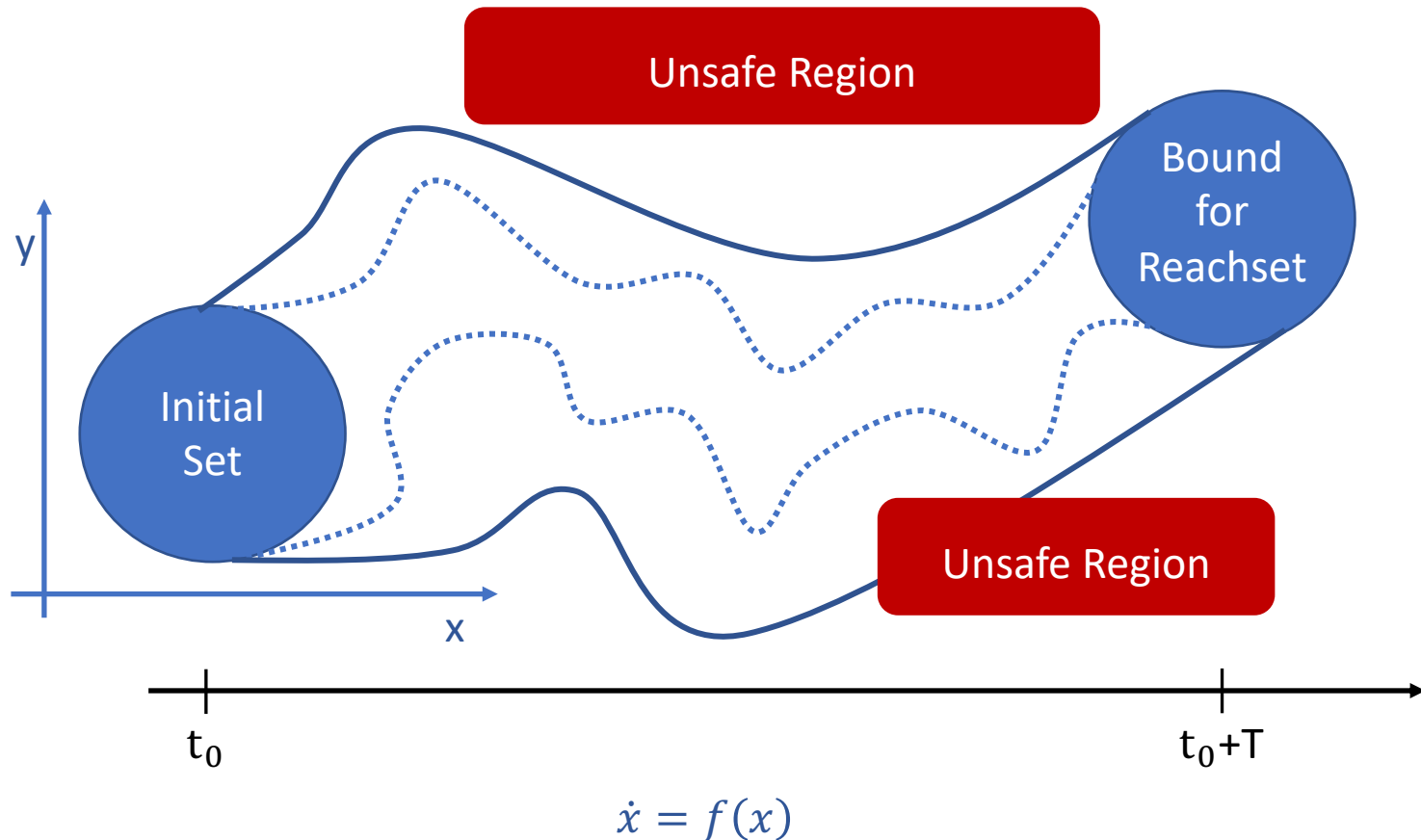
Safety of Cyber-Physical Systems

Neural Network Control Systems



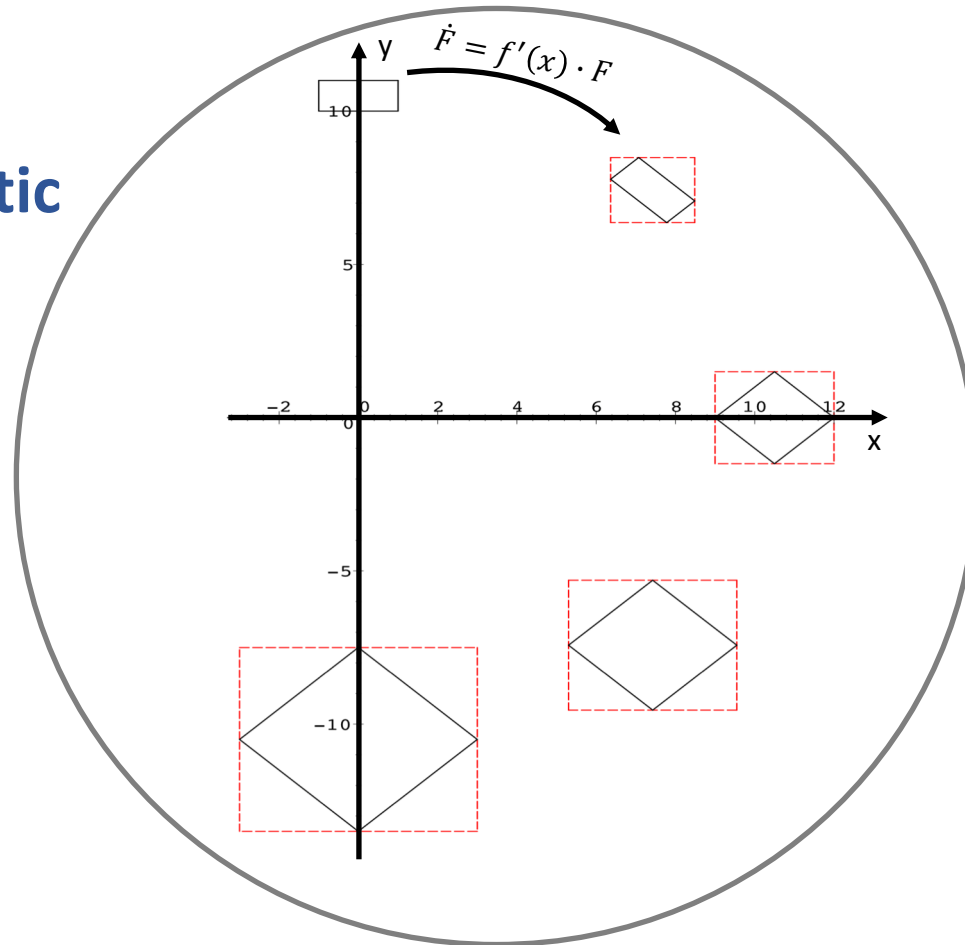
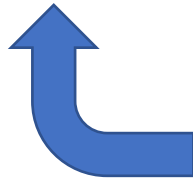
Lagrangian Reachtube (LRT)

Nonlinear dynamical System



Problem: Wrapping Effect

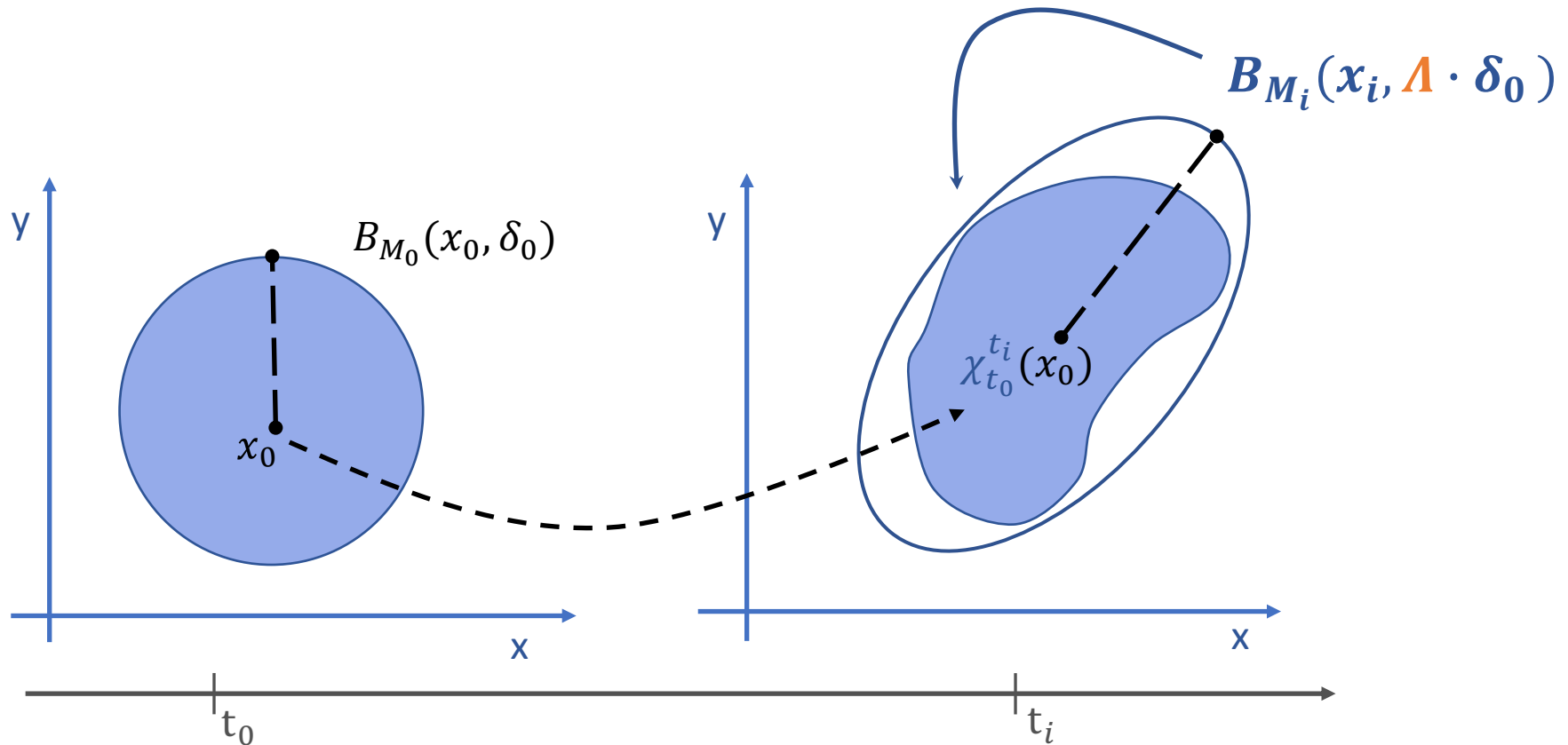
avoid:
Interval Arithmetic



N. Nedialkov, K. Jackson, and G. Corliss. Validated solutions of initial value problems for ordinary differential equations. Applied Mathematics and Computation, 105(1):21 – 68, 1999.

Wanted: tight conservative ellipse

Minimize volume of enclosing ellipse



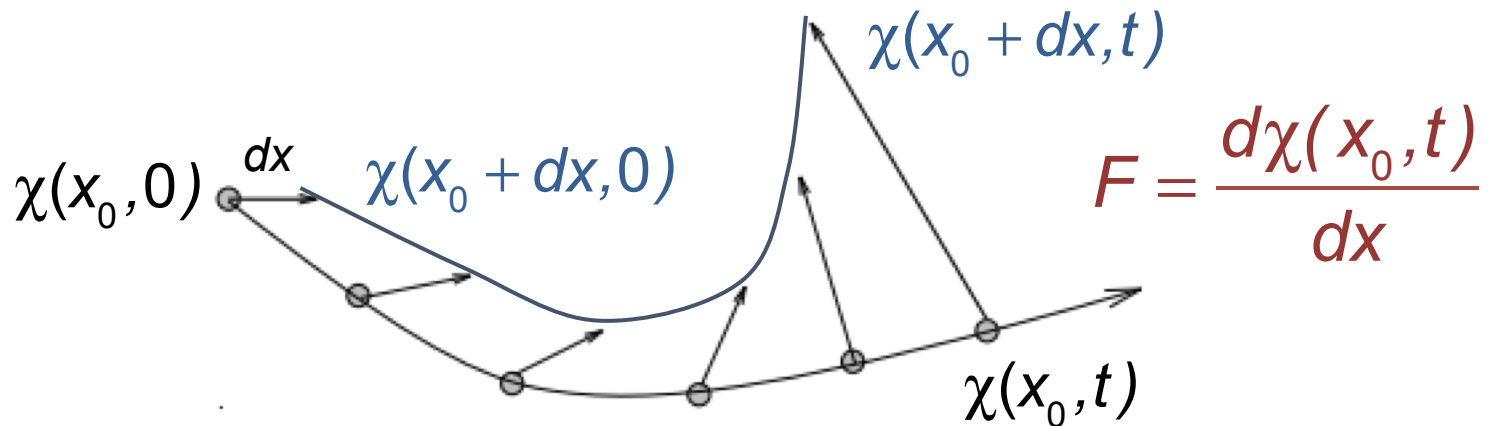
Main idea: Use sensitivity analysis

To every nonlinear, (time-dependent) ODEs

$$\dot{x} = f(x), \quad x_0 = x(0)$$

There is an associated variational equation:

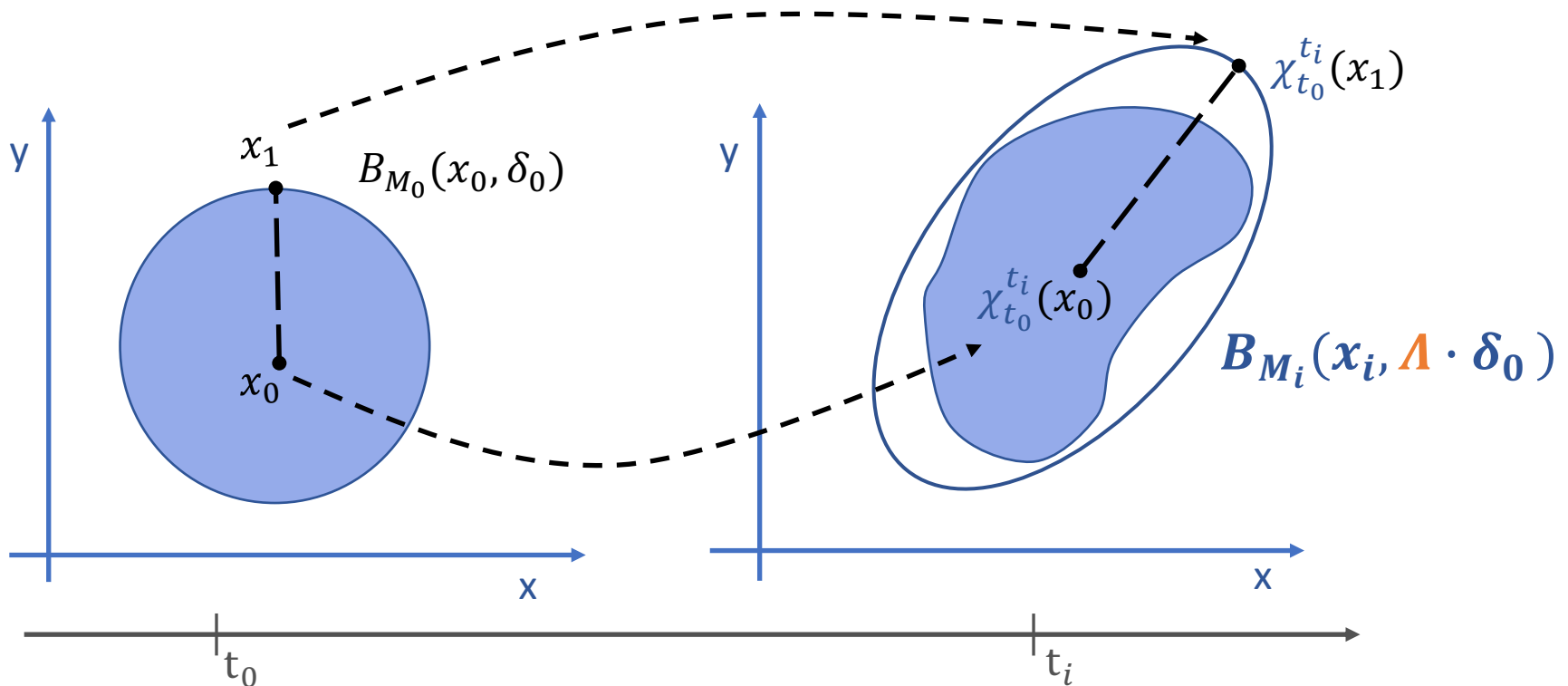
$$\dot{F} = J(x_0, t)F, \quad F(x_0, 0) = I$$



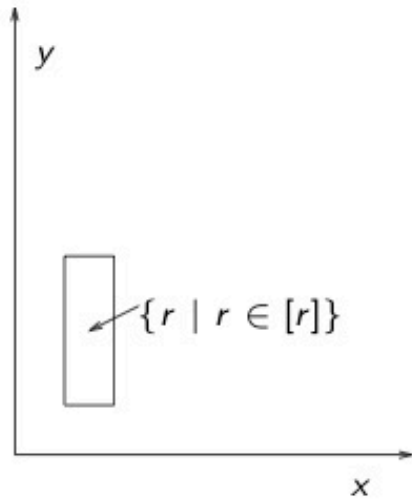
Stretching Factor

$$F = \frac{d\chi(x_0, t)}{dx}$$

$$\underbrace{\|(\chi_{t_0}^t(x_0), \chi_{t_0}^t(x_1))\|_{M_t}}_{\delta_t} \leq \underbrace{\|(F(t))\|_{M_{0,t}}}_{\Lambda} \cdot \underbrace{\|(x_0, x_1)\|_{M_0}}_{\delta_0}$$

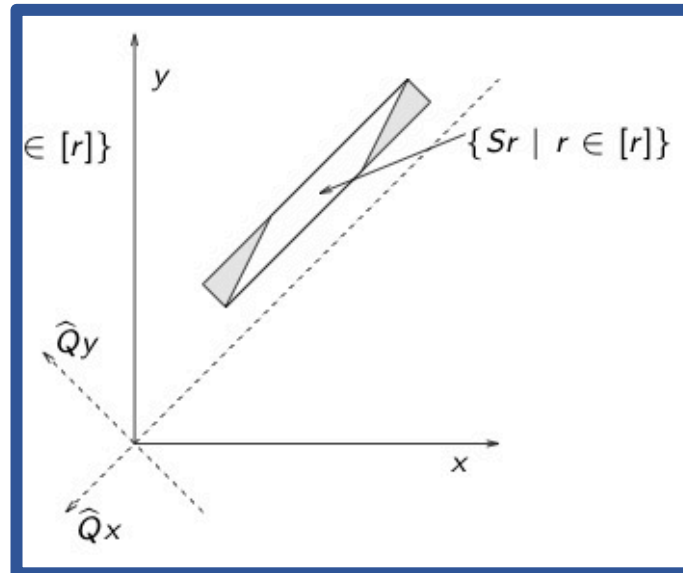
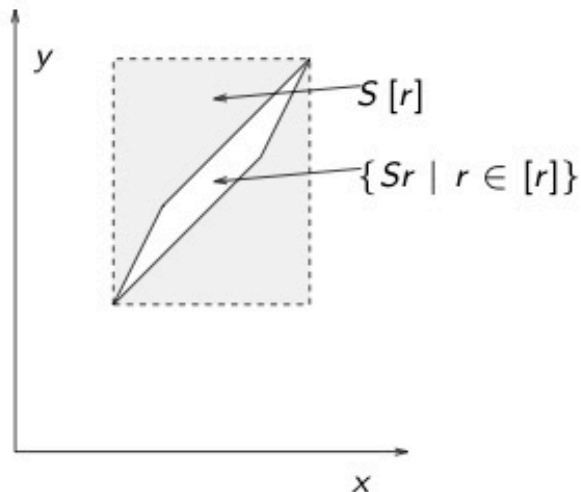


Lohner's QR method



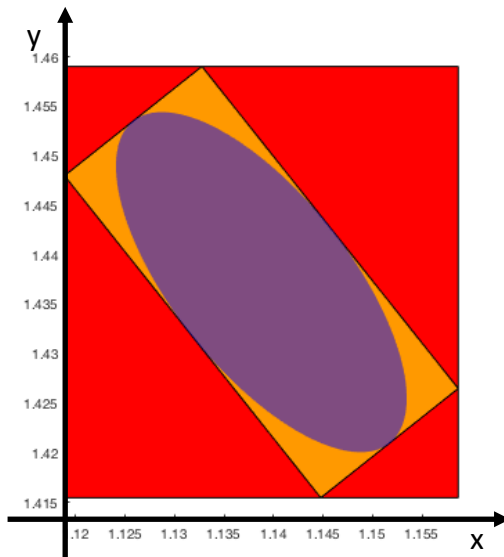
QR decomposition

Change to coordinate system Q

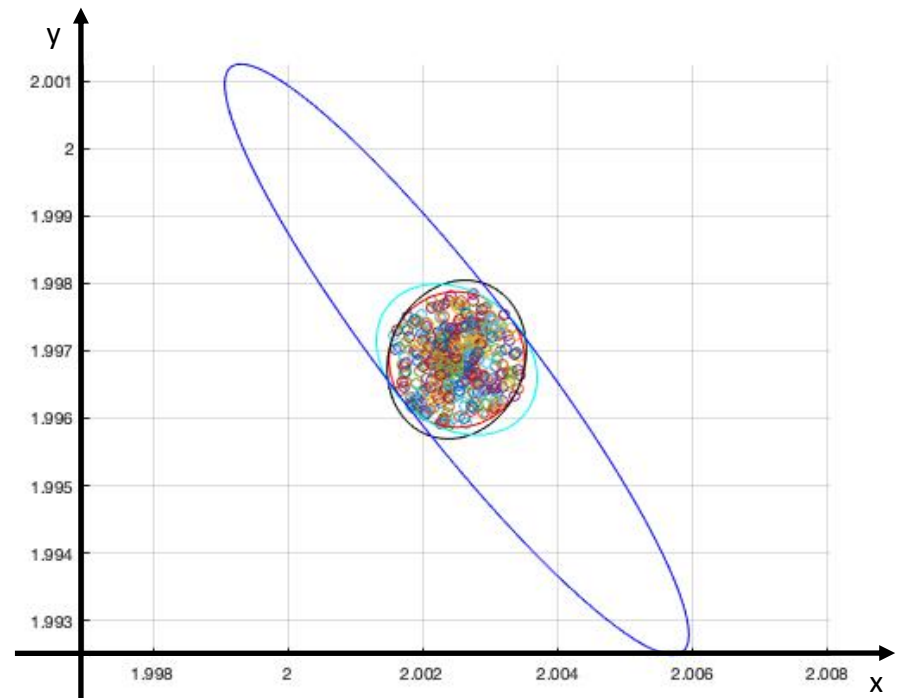


Work in progress

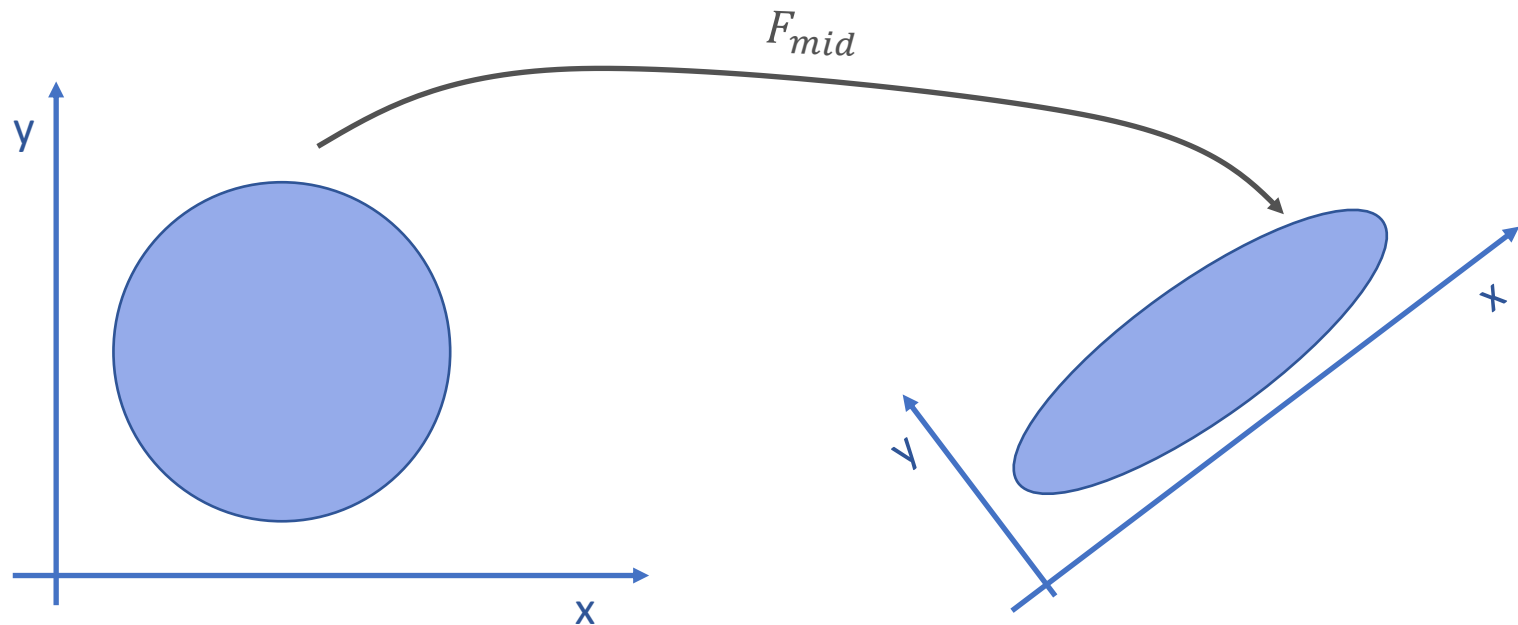
Wrapping of Reachset



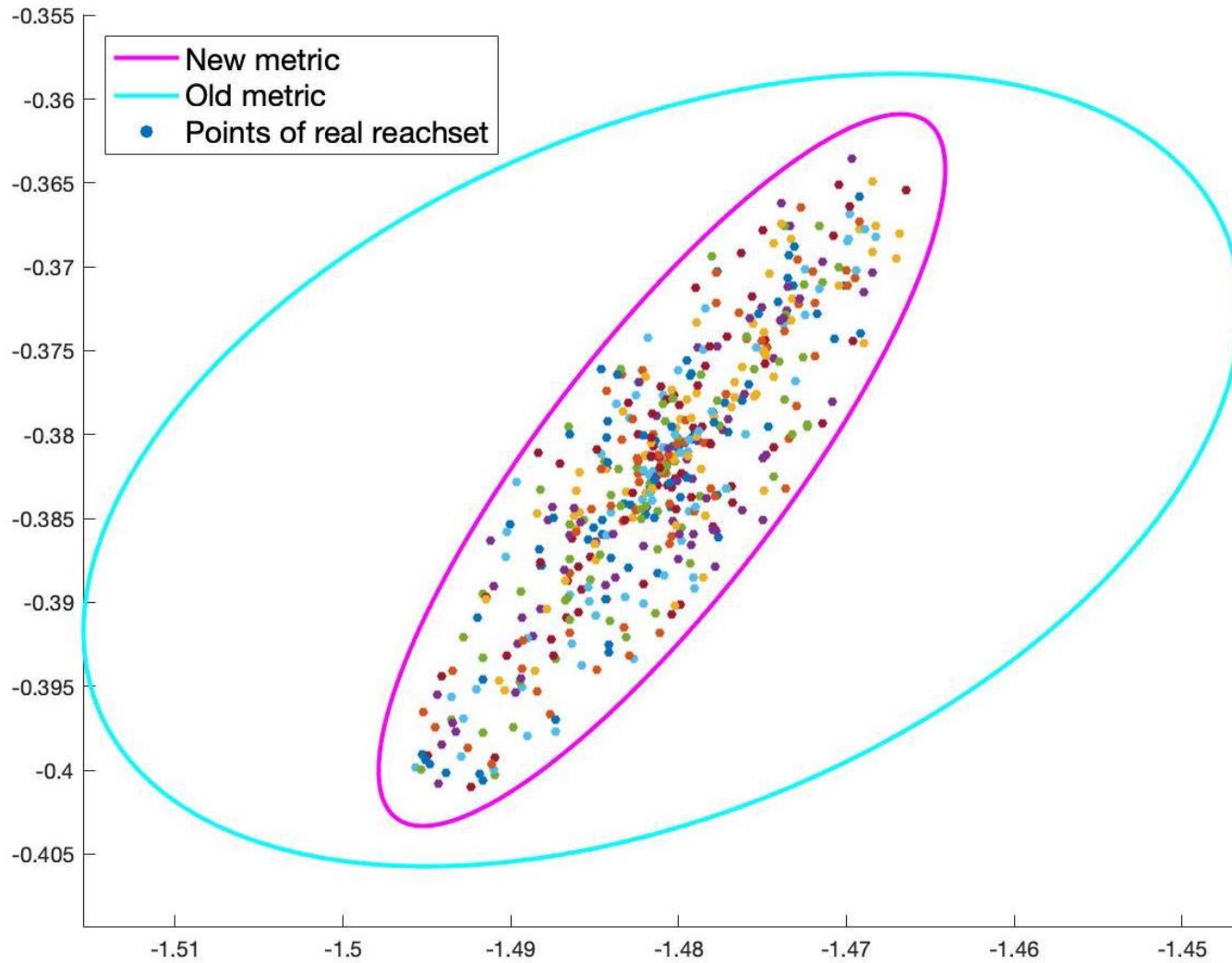
Choosing tightest ellipse



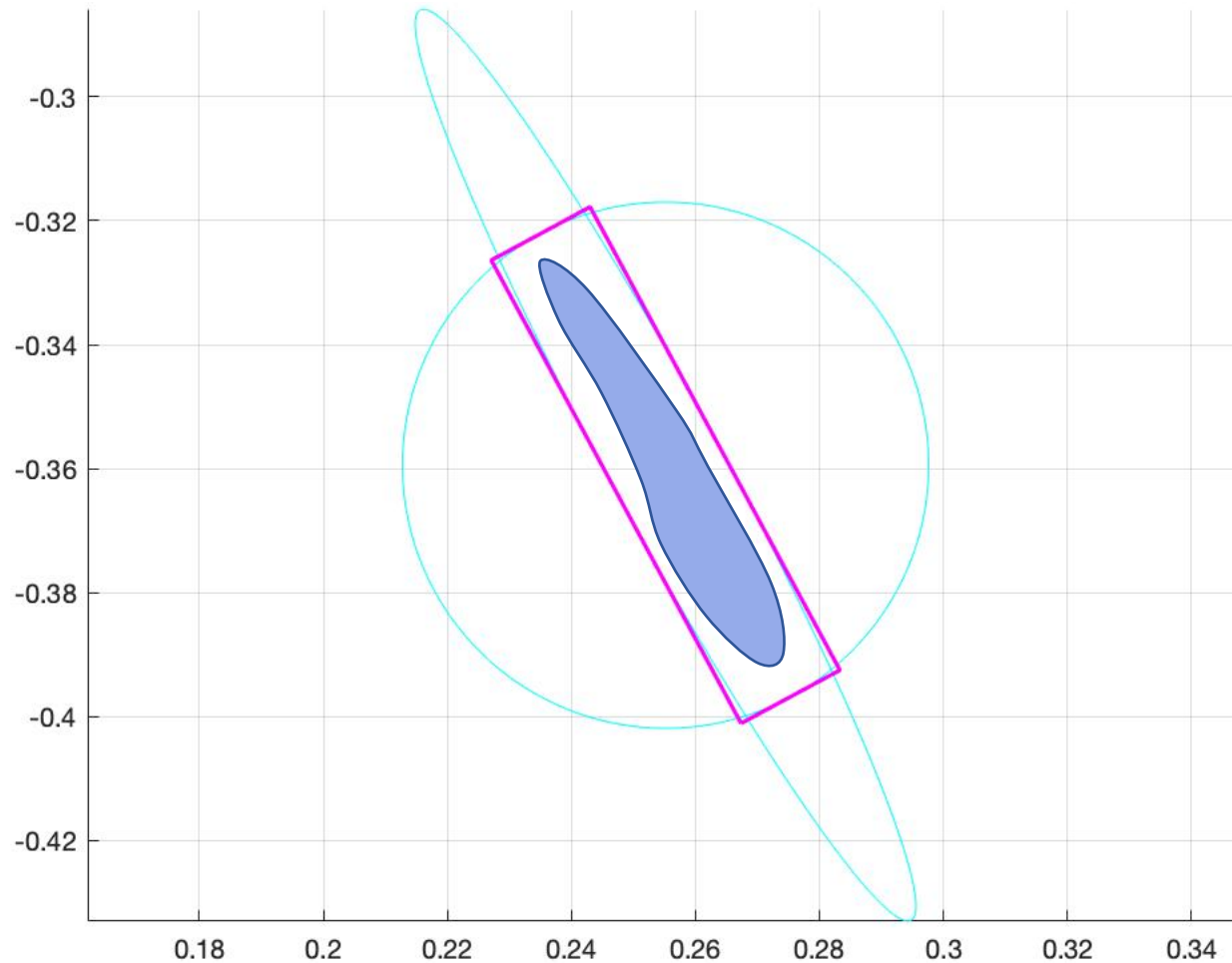
Choosing optimal metric



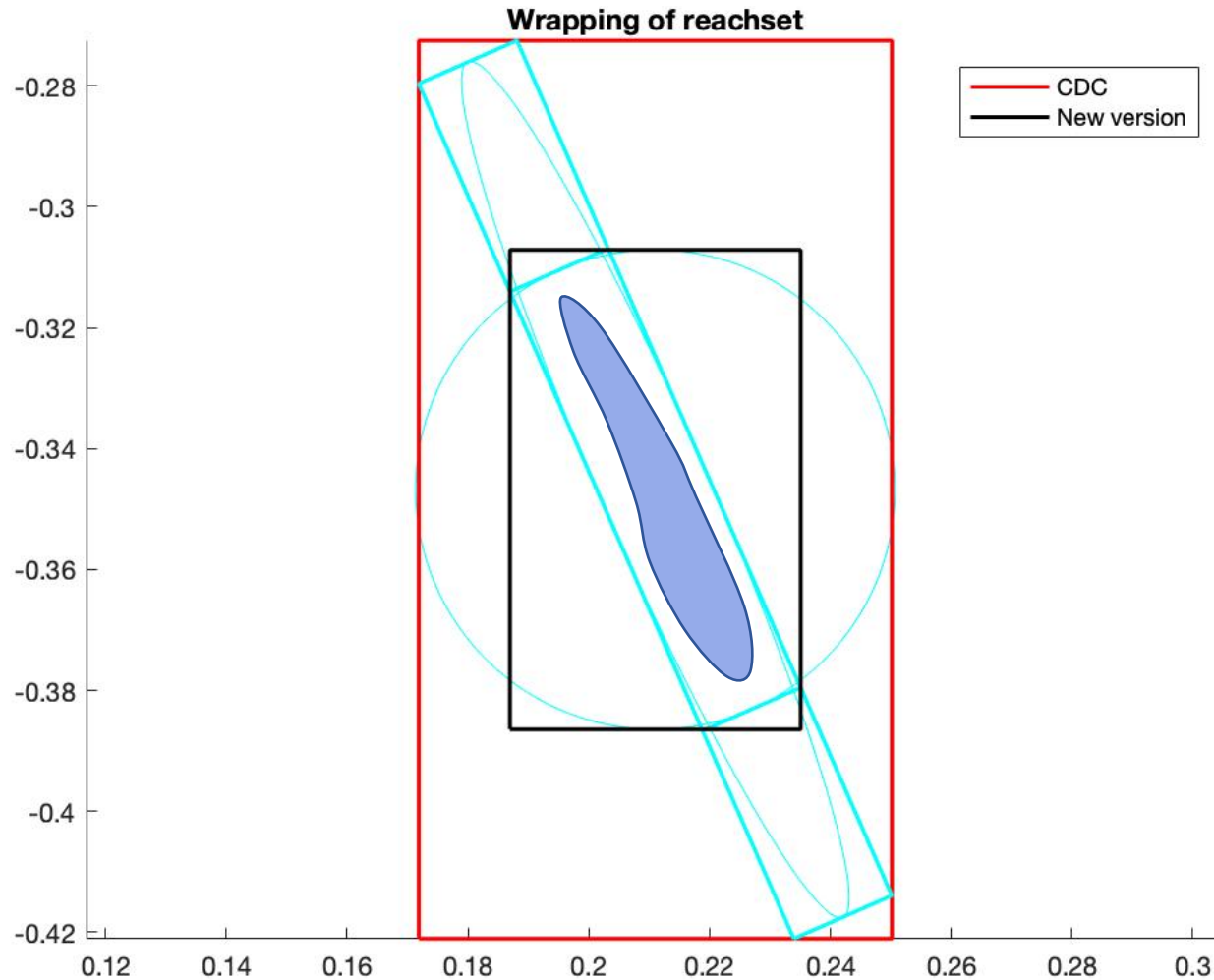
Choose tightest ellipse



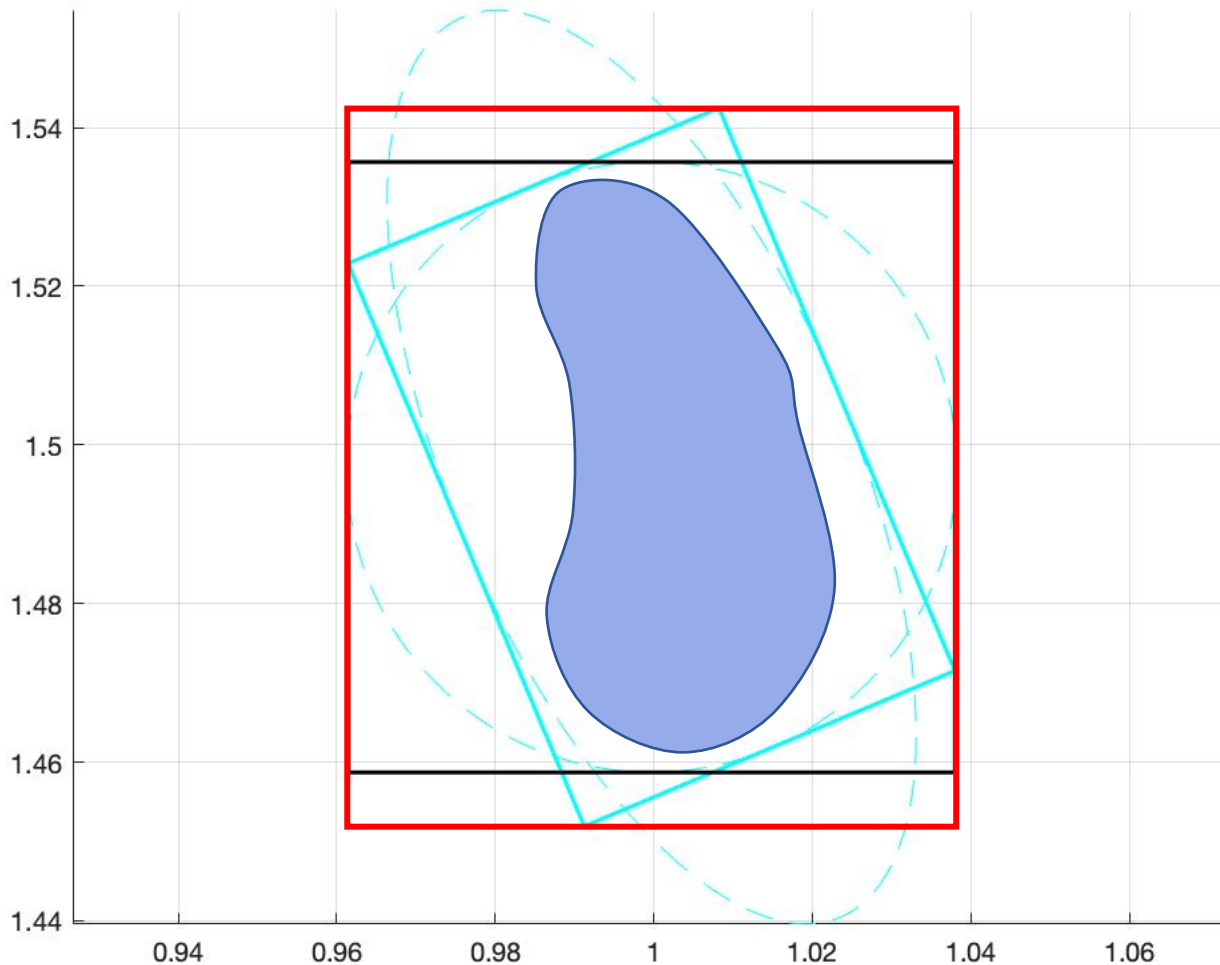
Wrapping of Reachset - Trick #1



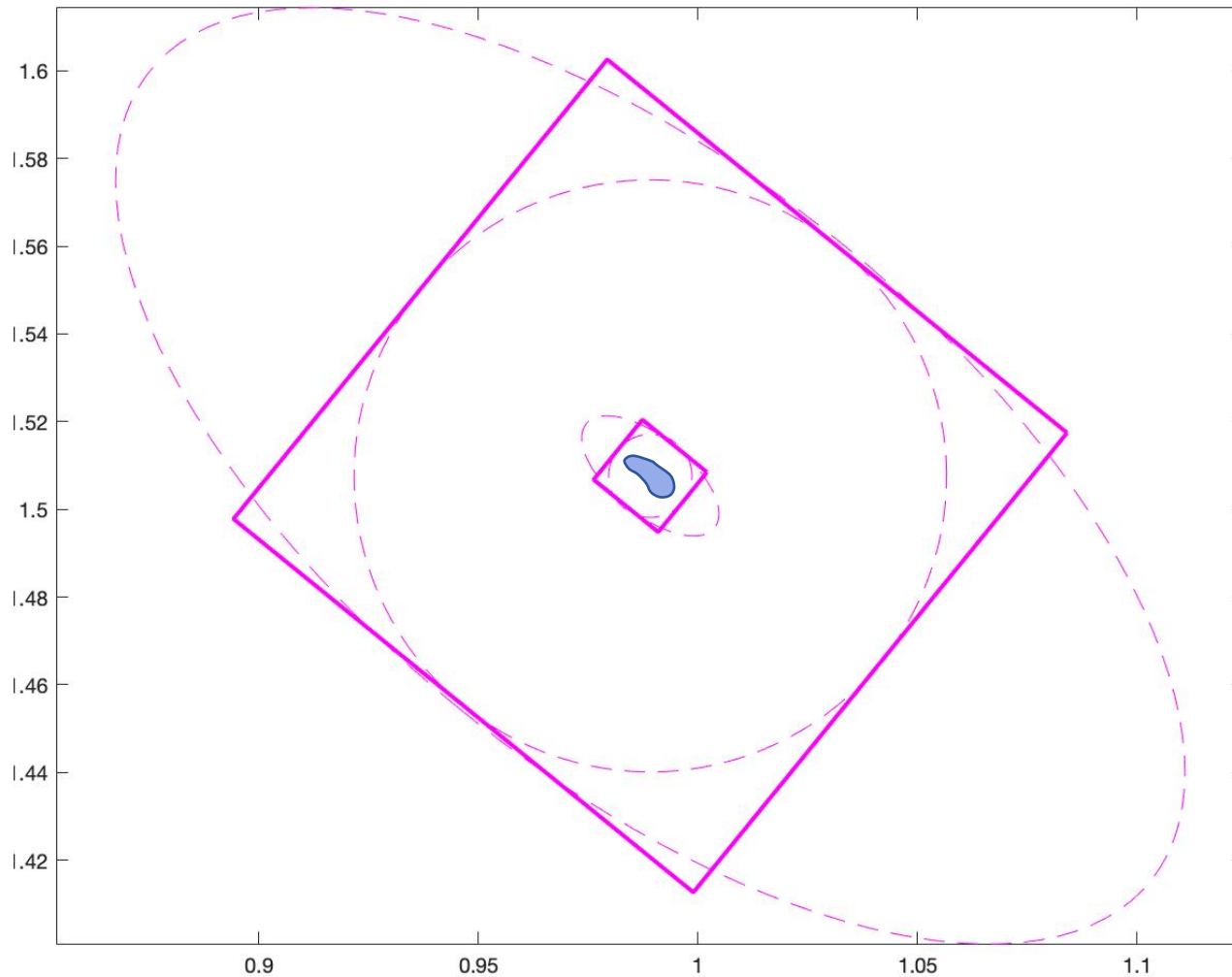
Tighter enclosing box (used in QR)



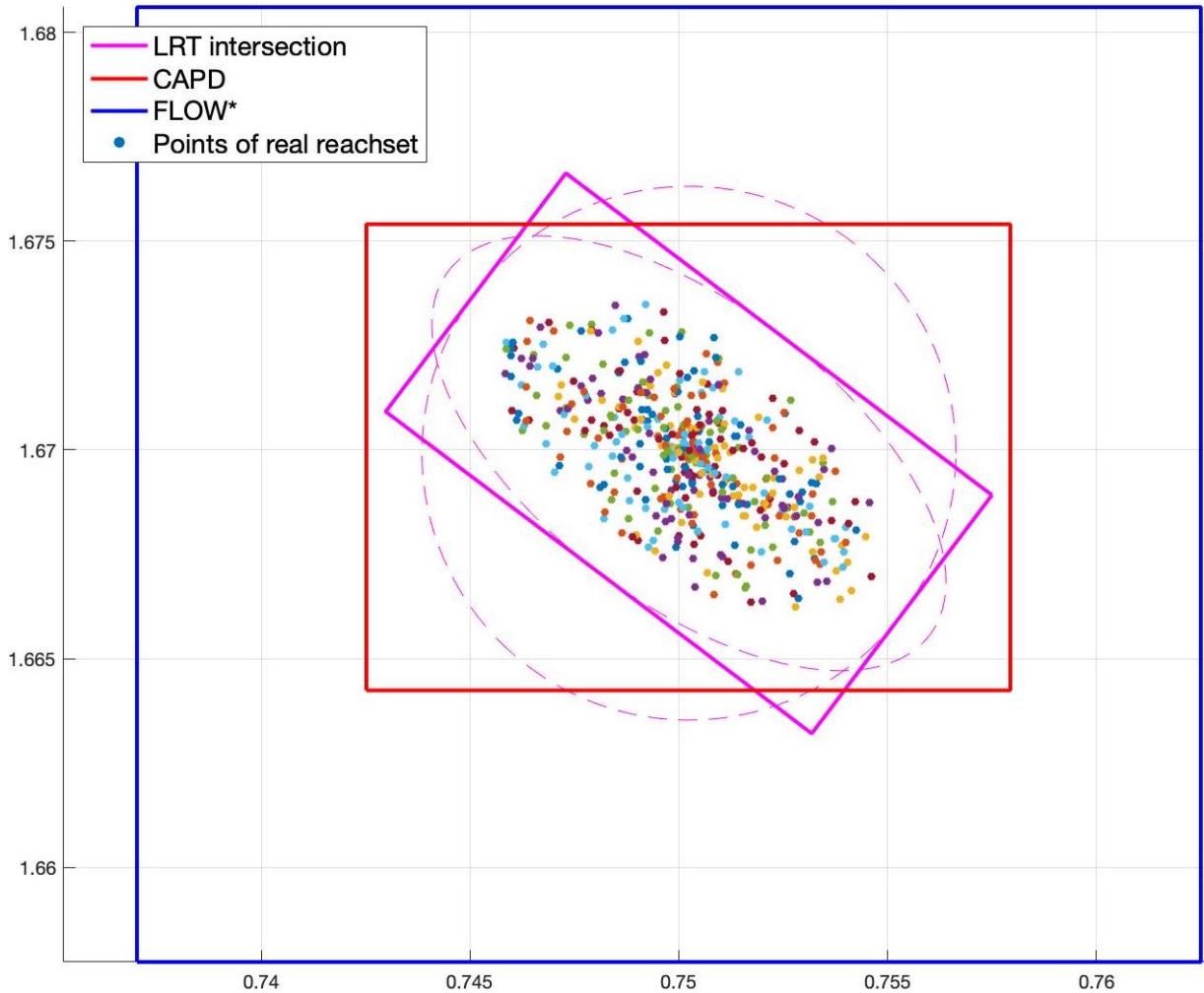
Wrapping of Reachset - Trick #2



Effects of intersection trick #2

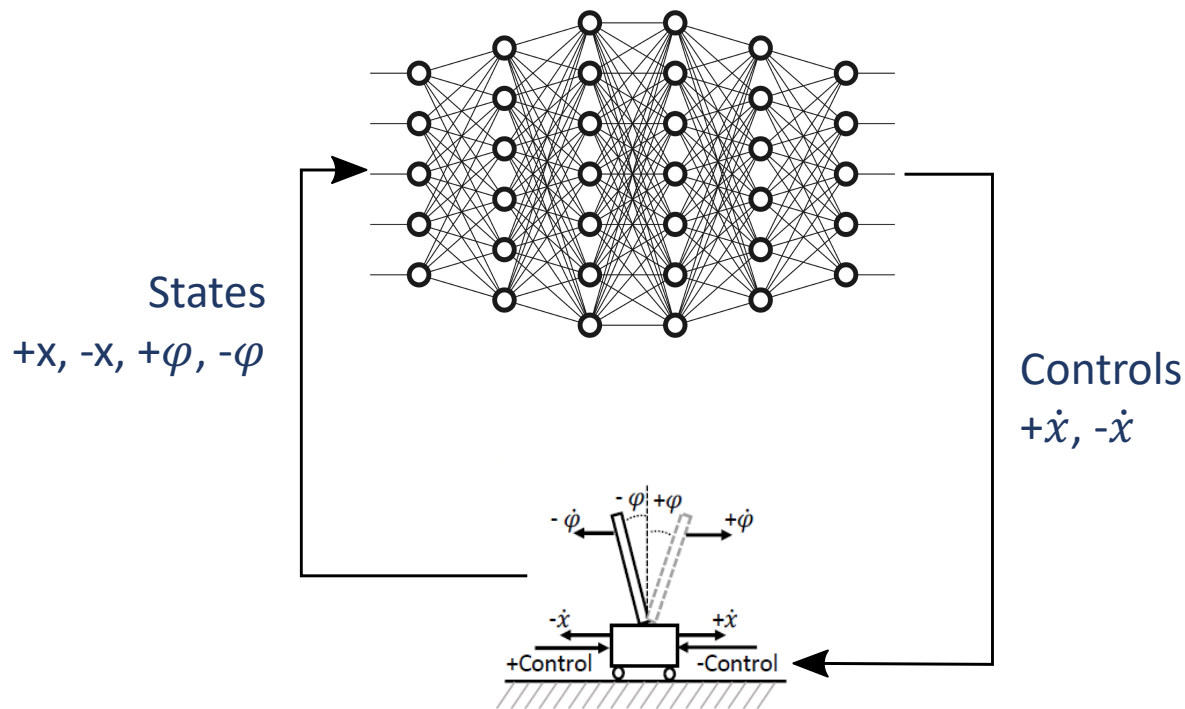


Comparison with other tools



Conclusion & Future Work

Scale up LRT  Verify **Neural Network Control Systems**



Thank you for your attention!

