Soundness in negotiations

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Joint work with
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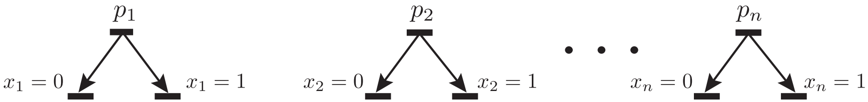
Verification of concurrent systems suffers from the state explosion problem.



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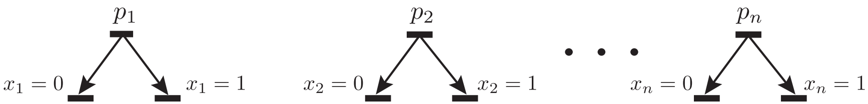
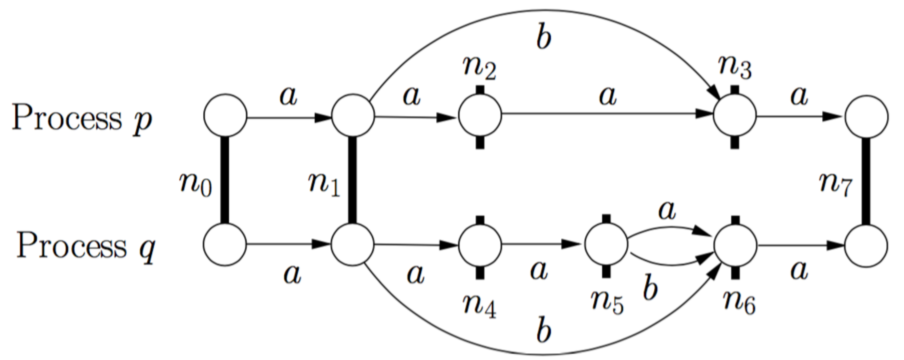
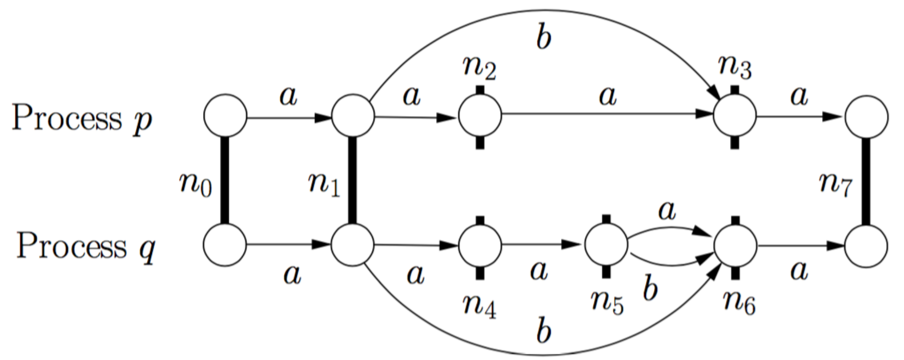


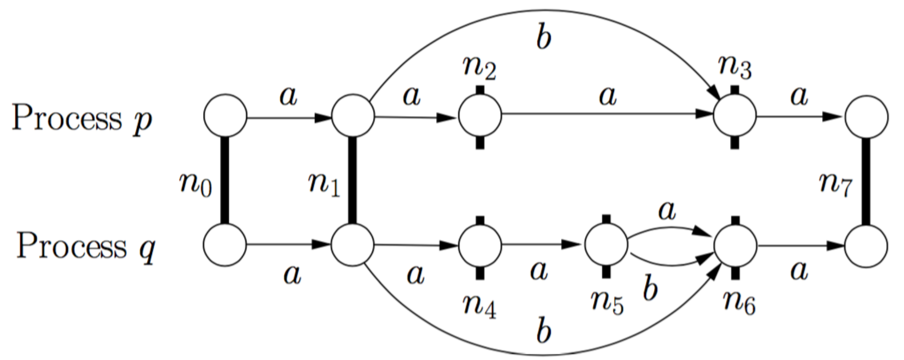
Negotiations is a restricted model for which some verification problems are much easier than usually.



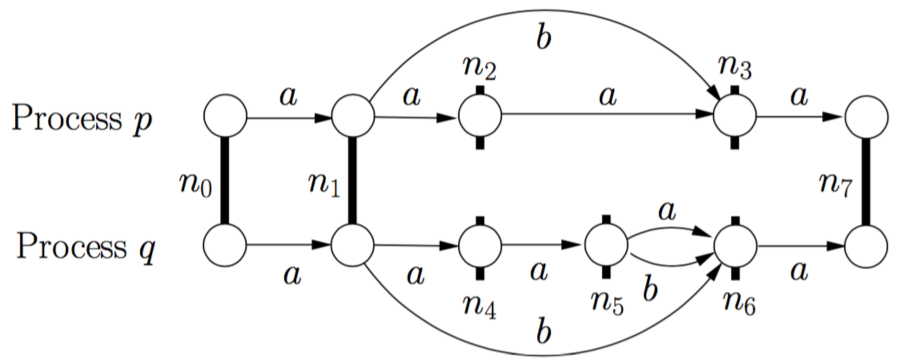
- $Proc$: processes.
- N : atomic negotiations (nodes); $dom : N \to \mathcal{P}(Proc)$.
- R : outcomes.
- $\delta : N \times R \times P \stackrel{\cdot}{\longrightarrow} \mathcal{P}(N)$: partial transition function
  $\delta(n, a, p)$ is a set of next atomic negotiations for process p;
  for every n, $a \in out(n)$, $p \in dom(n)$,



- A configuration $C : Proc \to \mathcal{P}(N)$
- n is enabled in C if $n \in C(p)$ for all $p \in dom(n)$.
- A run $C_1 \stackrel{(n_1,a_1)}{\longrightarrow} C_2 \stackrel{(n_2,a_2)}{\longrightarrow} C_3 \ldots$
- A successful run $C_{init} \stackrel{w}{\longrightarrow} C_{fin}$



- A negotiation is **sound** if every run $C_{init} \xrightarrow{w} C$ can be completed to a successful run.



- **Deterministic negotiation:** $\delta(n, a, p)$ is at most singleton.
- **Graph of a negotiation** (see above).
- **Local path** a path in the graph of a negotiation.
- **Acyclic negotiation** when its graph is acyclic.

Rem: For acyclic negotiations: sound $\equiv$ no-deadlock.

Soundness: every run can be completed to a successful run

Deterministic acyclic negotiations

- Soundness in NLOGSPACE
- $L(N) \cap L(A) \neq \emptyset$ NP-complete

A local path $n_0 \xrightarrow{p_0,a_0} n_1 \xrightarrow{p_1,a_1} \dots \xrightarrow{p_{k-1},a_{k-1}} n_k$ is **realizable** if it is a part of a run.

Lemma

Every local path is realizable.

Proof

Atomic negotiation n_0 is enabled in C_{init}.

Suppose n_i is enabled in C_i.
Let C'_i be the result of executing a_i. We have $C'_i(p) = n_{i+1}$
By soundness from C'_i we can reach C_{fin}.
So on the way we reach C_{i+1} where n_{i+1} is enabled.

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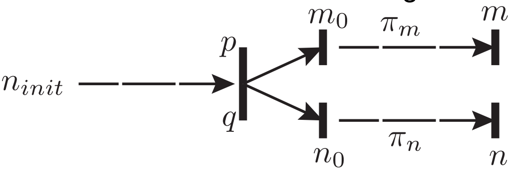
There is an execution containing m and n iff there is a pattern:



$\pi_m \cap \pi_n = \emptyset$

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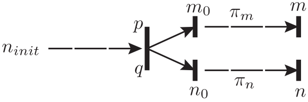

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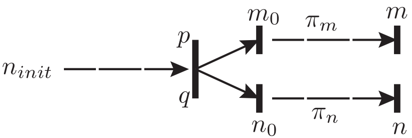
Lemma

Acyclic $\mathcal{N}$ is not sound iff its graph has a pattern:



$q \in dom(m)$

$m \preccurlyeq n$

$\pi_m \cap \pi_n = \emptyset$

Theorem

Soundness of acyclic deterministic negotiations is NLOGSPACE-complete.

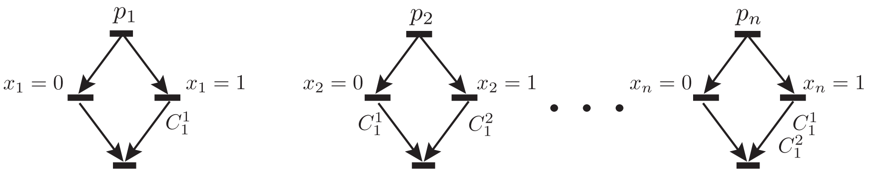
Not everything is easy to check for deterministic acyclic negotiations

Thm

$L(\mathcal{N}) \cap L(\mathcal{A}) \neq \emptyset$ is NP-complete, for $\mathcal{N}$ an acyclic deterministic negotiation and $\mathcal{A}$ a deterministic finite automaton.

1 in 3 SAT

$$(x_1 \vee \overline{x_2} \vee x_n) \wedge (x_2 \vee \overline{x_4} \vee x_n) \wedge \ldots$$



$$L(\mathcal{A}) = \{C_1^{i_1} C_2^{i_2} \ldots C_k^{i_k} : i_1, \ldots, i_k \in [n]\}$$

Soundness: every run can be completed to a successful run

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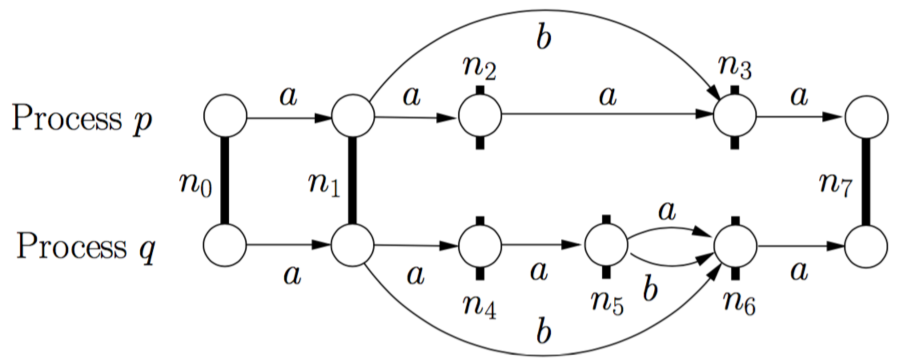
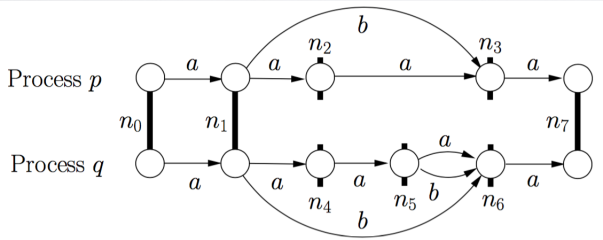
Verifying properties of sound acyclic deterministic negotiations

- some properties can be decided in PTIME
- races can be decided in PTIME



Atomic negotiations may have outcomes:
$alloc(x)$, $read(x)$, $write(x)$, and $dealloc(x)$.



(1) *Inconsistent data*: an atomic negotiation reads or writes a variable x while another atomic negotiation is writing, allocating, or deallocating it in parallel.

(2) *Never destroyed*: there is an execution in which a variable is allocated and then never deallocated before the execution ends.

(3) *Weakly redundant data*: there is an execution in which a variable is written and never read before it is deallocated or the execution ends.

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Thm

These properties can be checked in PTIME for acyclic, deterministic, sound negotiations.

Concurrency of two actions

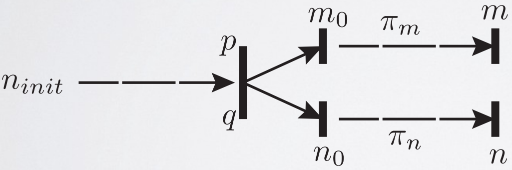
We write $m \parallel n$ if $\mathcal{N}$ has a reachable configuration C where both m and n are enabled.

Thm

We can decide in a linear time if in a given acyclic, deterministic, sound negotiation the two given atomic negotiations m, n satisfy $m \parallel n$.

Proposition

$m \parallel n$ iff there is a run containing m, n, and there is no local path from m to n or vice versa.



$$\pi_m \cap \pi_n = \emptyset$$

Thm [Kovalyov, Esparza]

For all deterministic negotiations there is a cubic algorithm for this problem.

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Soundness for bigger classes

- for weakly deterministic acyclic in PTIME
- without acyclicity coNP-hard

Thm [Espaza, Desel]

Soundness is PSPACE-complete for non-deterministic negotiations.
It is coNP-complete when they are acyclic.

Thm [Esparza, Desel]

Soundness is in PTIME for deterministic negotiations.

Thm

Soundness is in PTIME for acyclic weakly non-deterministic negotiations.

Thm

Soundness is coNP-complete for very weakly non-deterministic negotiations.

A process p is deterministic if $\delta(n, a, p)$ is at most a singleton, for all n, a.

A negotiation is weakly non-deterministic if for every $n \in N$ at least one of the processes in $dom(n)$ is deterministic.

Thm

Soundness can be decided in PTIME for acyclic, weakly non-deterministic negotiations.

A negotiation is weakly non-deterministic if for every $n \in N$ at least one of the processes in $dom(n)$ is deterministic.

Lemma

An acyclic weakly non-deterministic negotiation $\mathcal{N}$ is not sound if and only if:

- either its restriction $\mathcal{N}_D$ to deterministic processes is not sound,
- or, for some non-deterministic process p, its restriction to p and the deterministic processes is not sound.

Thm (Omitting)

It can be decided in PTIME if for a given deterministic, acyclic, and sound negotiation $\mathcal{N}$ and a set $B \subseteq N$ there is a successful run of $\mathcal{N}$ omitting B.

A negotiation is weakly non-deterministic if for every $n \in N$ at least one of the processes in $dom(n)$ is deterministic.

A negotiation is very weakly non-deterministic if for every $n \in N$ $a \in R$ and $p \in Proc$ there is a deterministic process q such that $q \in dom(n')$ for all $n' \in \delta(n, a, p)$. (q decides about the next negotiation)

det-acyclic: restriction to deterministic processes is acyclic.

Thm

Soundness of det-acyclic, very weakly non-deterministic negotiations is coNP-complete.

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