ASSUME ADMISSIBLE SYNTHESIS

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REACTIVE SYNTHESIS
Classical setting

$$\boxed{\text{Env}} \parallel \overset{\text{Sys}}{\boxed{?}} \models \phi$$

ϕ is the winning objective for Sys

Env is adversarial

$\Longrightarrow$ 2-player zero-sum game

Winning strategy = Correct Sys

$$\boxed{\;Env\;} \;\parallel\; \overset{Sys}{\boxed{\;?\;}} \;\models\; \phi$$

→ Env is completely adversarial

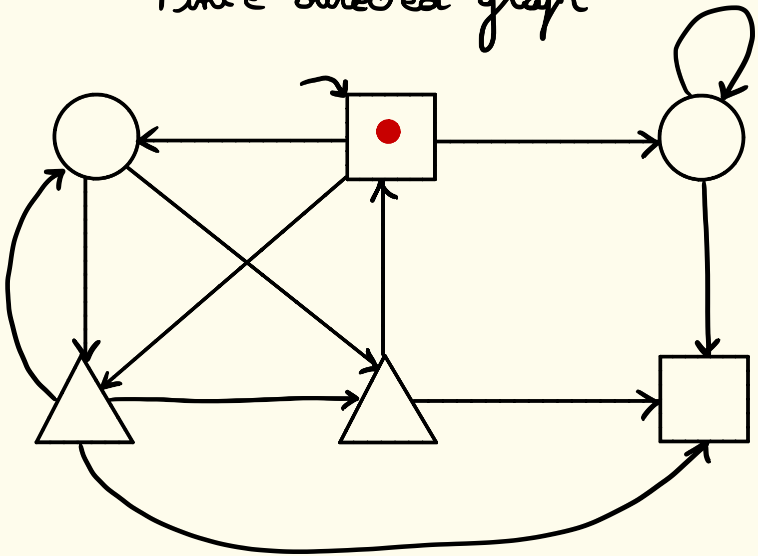
? what if Env = Rational user

→ Sys and/or Env can be made of several components, each with their own objective.

⇒ We need a richer setting

n-players turn-based graph games

Finite directed graph



Vertices are partitioned

$V = V_1 \uplus V_2 \uplus \dots \uplus V_n$

V_i = vertices of Player i

$E \subseteq V \times V$

Players and objectives

$$\mathcal{P} = \{1, 2, \dots, n\}$$

Players stand for:

- parts of the system to design
- parts of the environment

$$\boxed{1} \parallel \boxed{2} \parallel \dots \parallel \boxed{n}$$

$$\phi_1 \qquad \phi_2 \qquad \phi_n$$

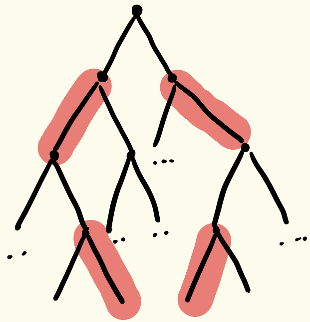
$$\boxed{\phi_i \subseteq V^\omega}$$

Strategies

$$\sigma_i : V^* . V_i \rightarrow V$$

s.t. $\forall \pi . v \in V^* . V_i$:

$$(v, \sigma_i(\pi . v)) \in E$$



Σ_i = set of strategies of Player i.

Strategy profiles and outcomes

$$\underbrace{(\sigma_1, \sigma_2, \dots, \sigma_N)}_{\text{profile}} \in \Sigma_1 \times \Sigma_2 \times \dots \times \Sigma_N$$

$$= (\sigma_i, \sigma_{-i})$$

$\sigma_{-i} \in \Sigma_{-i}$

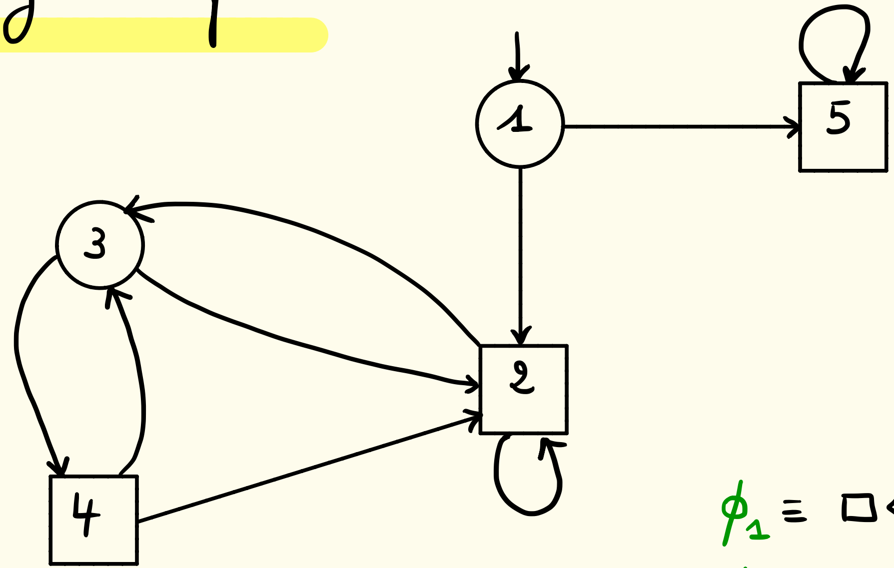
all strategies of the profile but σ_i

$$Out_v(\sigma_1, \sigma_2, \dots, \sigma_N) = v_0\, v_1\, v_2 \dots v_n \dots = \pi$$

s.t. :
- $v_0 = v$
- $\forall j \geq 0$: if $\pi(j) \in V_i$ then $\pi(j+1) = \sigma_i(\pi(0..j))$

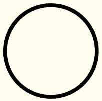
Running example

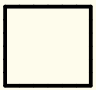
2 players



$\phi_1 \equiv \Box \Diamond 4$

$\phi_2 \equiv \Box \Diamond 3$

 = Player 1

 = Player 2

SYNTHESIS RULES

Synthesis rules

WIN

$$\overset{Sys}{\boxed{1}} \parallel \overbrace{\boxed{2} \parallel \cdots \parallel \boxed{N}}^{Env}$$

$$\phi_1 \qquad \{\phi_2 \quad \cdots \quad \phi_N\} \text{ ignored}$$

$$\exists \sigma_1 . \forall \sigma_2 \dots . \forall \sigma_N :$$

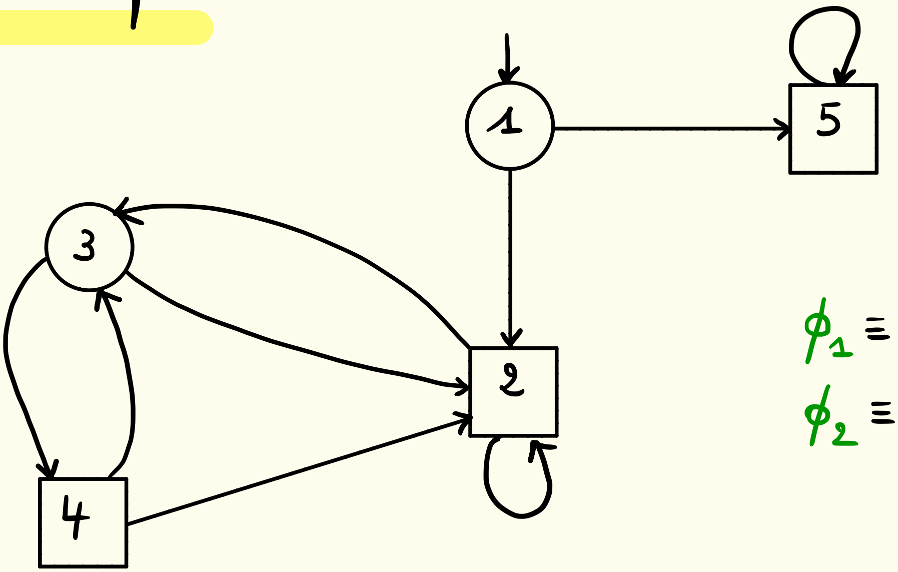
$$Out(\sigma_1, \sigma_2, \dots, \sigma_N) \models \phi_1$$

= 2-player zero-sum case

& can be instantiated for any players.

Running example

WIN



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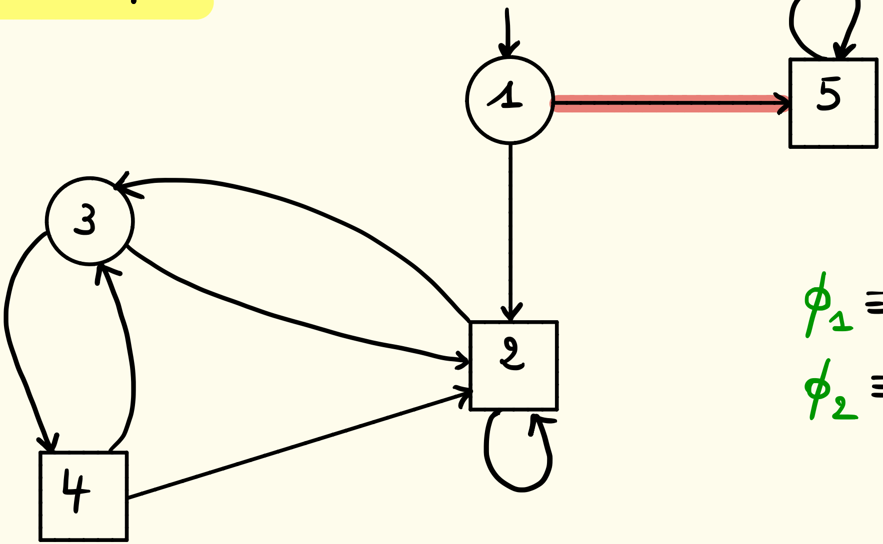
None of the players has a winning strategy!

Running example

WIN



$\phi_1 \equiv \Box \Diamond 4$

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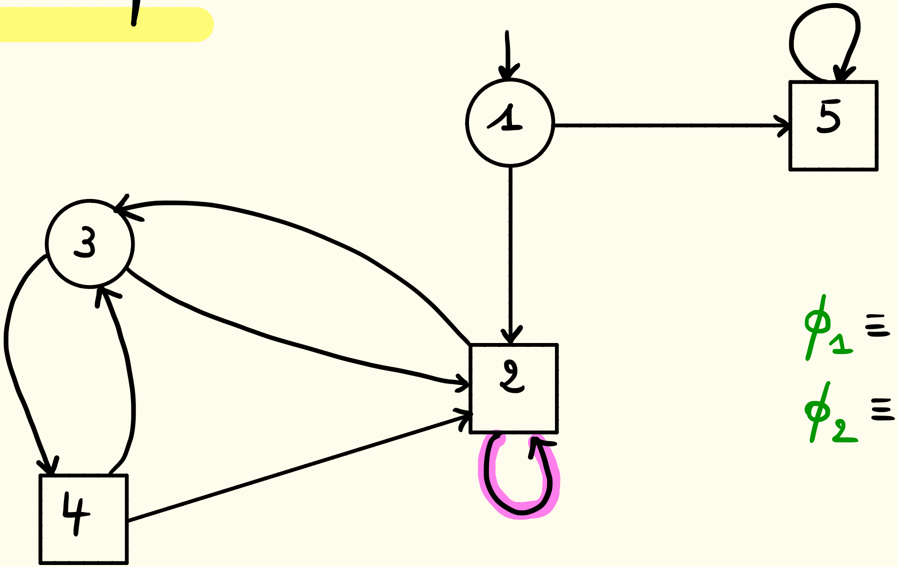
By playing $1 \longrightarrow 5$ Player 1 spoils $\phi_2 \equiv \Box \Diamond 3$

Running example

WIN



$\phi_1 \equiv \Box \Diamond 4$

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By playing $2 \longrightarrow 2$ Player 2 spoils $\phi_1 \equiv \Box \Diamond 4$

Synthesis rules

WIN-HYP

$$\overset{\text{Sys}}{\boxed{1}} \parallel \overbrace{\boxed{2} \parallel \cdots \parallel \boxed{N}}^{\text{Env}}$$

$$\phi_1 \quad \left\{ \phi_2 \quad \phi_N \right\} = \text{HYP}$$

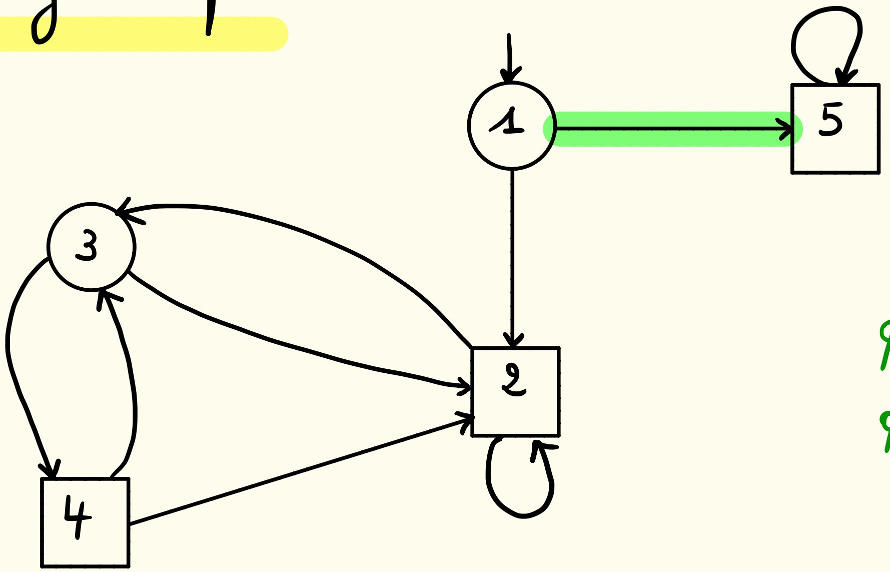
$$\exists \sigma_1 \cdot \forall \sigma_2 \ldots \cdot \forall \sigma_N :$$

$$\text{Out}(\sigma_1, \sigma_2, \ldots, \sigma_N) \models \underbrace{\phi_2 \wedge \ldots \wedge \phi_N}_{\text{HYP}} \rightarrow \phi_1$$

* Can be instantiated for any players.

Running example

WIN-HYP



$\phi_1 \equiv \Box \Diamond 4$

$\phi_2 \equiv \Box \Diamond 3$

Pl. 1 wins $\phi_2 \to \phi_1$ with $1 \to 5$

$\to$ useless solution !

ADMISSIBILITY

Dominated strategy

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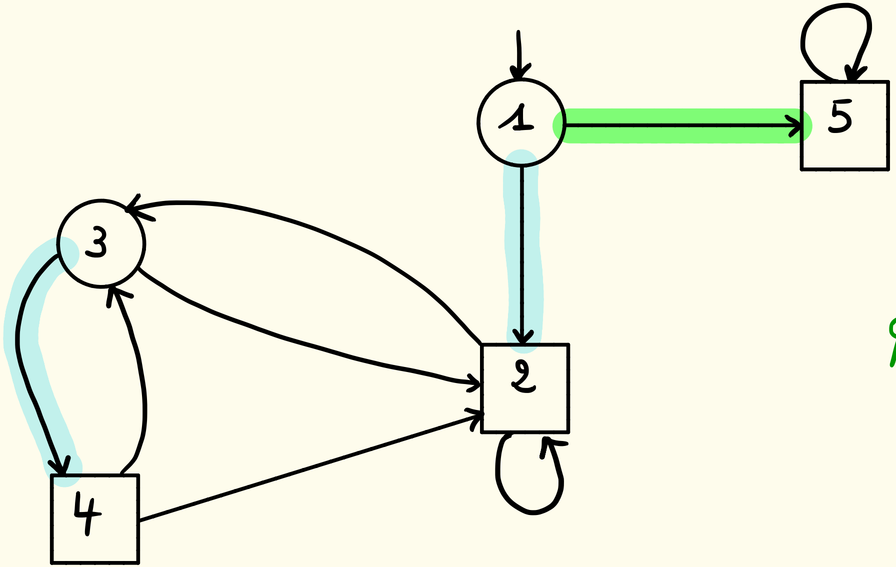
→ A rational player avoids dominated strategies



$\phi_1 = \Box \Diamond 4$

Any strategy that takes $1 \to 5$ is **dominated** by the strategy $\underbrace{1 \to 2 \,,\; 3 \to 4}_{\text{even if it is not a winning strategy}}$

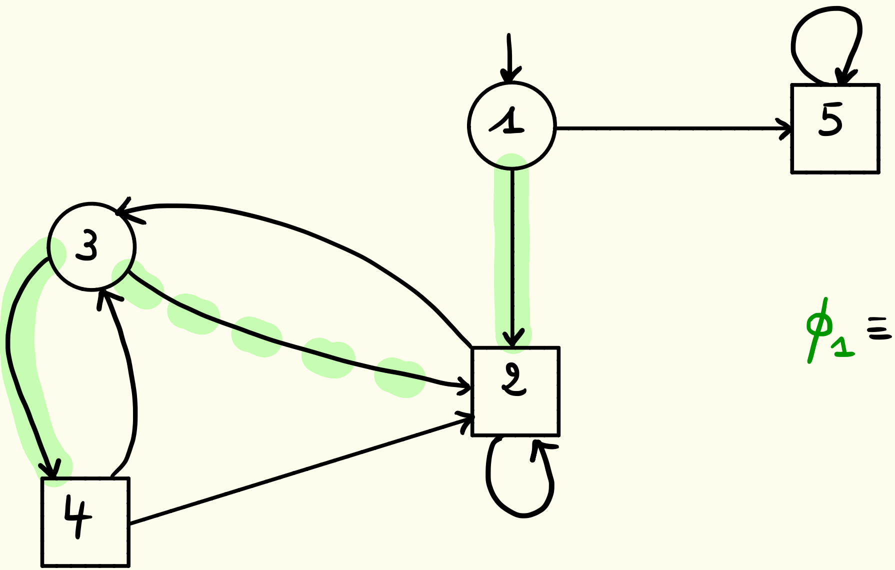
Admissible strategy

- $\boxed{\begin{array}{l}\sigma_i \in \Sigma_i \text{ is admissible for } \phi_i \\ \text{if } \sigma_i \text{ is not dominated by any } \sigma_i' \in \Sigma_i \text{ for } \phi_i\end{array}}$

- $Adm_i(\phi_i) \subseteq \Sigma_i$ is the set of admissible strategies of Player i for ϕ_i.

- $Adm_i(\phi_i) =$ the only $\left\{\begin{array}{l}\text{reasonable}\\ \text{rational}\end{array}\right\}$ strategies!

- $Adm_i(\phi_i) \neq \emptyset$, $\forall \phi_i$.



$\phi_1 \equiv \Box \Diamond 4$

σ_1 $\begin{cases} 1 \to 2, \quad 3 \to 4 \\ \text{is admissible} \end{cases}$

σ_1' $\begin{cases} 1 \to 2, \quad 3 \to 4 \quad \text{odd visits} \\ 3 \to 2 \quad \text{even "} \\ \text{also!} \end{cases}$

Synthesis rules

ASSUME ADMISSIBLE

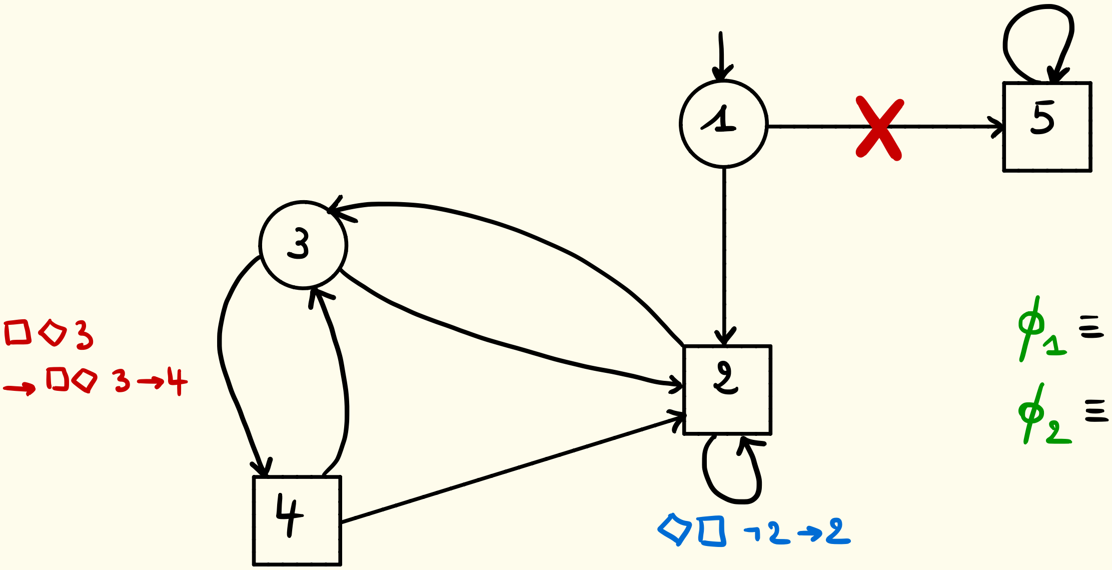
$$\begin{array}{ccccccc} \boxed{1} & \parallel & \boxed{2} & \parallel \cdots \parallel & \boxed{N} \\ \phi_1 & & \phi_2 & & \phi_N \end{array}$$

$\exists (\sigma_1, \sigma_2, \dots, \sigma_N)$:

① $\sigma_i \in Adm(\phi_i)$ for all i, $1 \leq i \leq N$.

② $\forall \sigma'_{-i} \in Adm_{-i}(\phi_{-i})$: $Out(\sigma_i, \sigma'_{-i}) \models \phi_i$

Each σ_i is winning against all admissible strategies of the others



$\phi_1 \equiv \square \Diamond 4$

$\phi_2 \equiv \square \Diamond 3$

Claim: $\forall \sigma_1 \in Adm_1(\phi_1) \cdot \forall \sigma_2 \in Adm_2(\phi_2) :$

$$Out(\sigma_1, \sigma_2) \models \phi_1 \wedge \phi_2$$

$\Rightarrow$ Assume Admissible rule applies!

Assume Admissible Synthesis

Theorem: for all AA-profiles $(\sigma_1, \sigma_2, \ldots, \sigma_n)$:

$$\mathrm{Out}(\sigma_1, \sigma_2, \ldots, \sigma_n) \models \phi_1 \wedge \phi_2 \wedge \ldots \wedge \phi_n$$

Theorem: the set of AA-profiles is rectangular.

$$= ST_1 \times ST_2 \times \ldots \times ST_n$$

$$ST_i = \{ \sigma_i \in \mathrm{Adm}(\phi_i) \mid \forall \sigma_{-i} \in \mathrm{Adm}_{-i}(\phi_{-i}) : \mathrm{Out}(\sigma_i, \sigma_{-i}) \models \phi_i \}$$

$\Rightarrow$ No need for synchronization!

$\Rightarrow$ Compositionality

Theorem: deciding if AA applies is Pspace-C for Safety, Reachability and Muller objectives.

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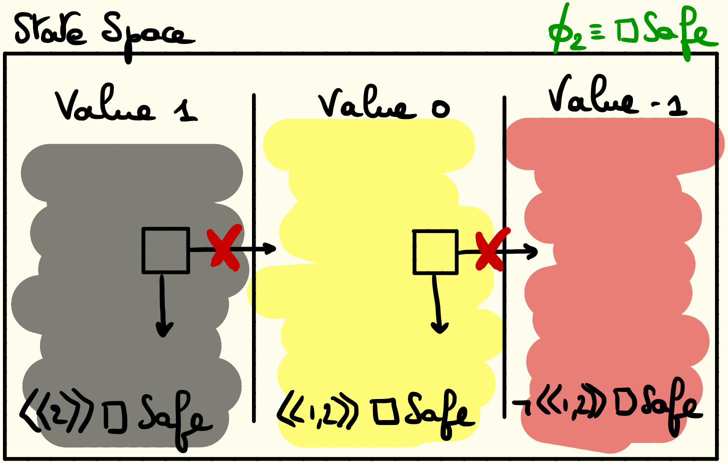
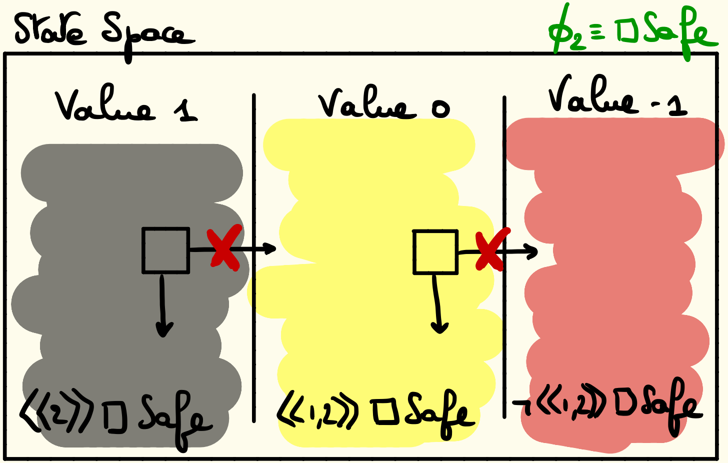


A player when playing admissible never decrease it's value.

Theorem: deciding if AA applies is Pspace-C for Safety, Reachability and Muller objectives.



For other properties

+ Value$_1$ $\Rightarrow$ Win!

Value$_0$ $\Rightarrow$ Win or Help!

A player when playing admissible never decrease its value.

$\Rightarrow$ The set of plays compatible with admissible strategies is ω-regular

Conclusion

- Assume admissible synthesis allows for compositional synthesis in N-player non-zero sum games.

- Rectangular sets of solutions.

- If WIN gives a solution then AA gives a solution.

- If WIN-HYP gives a solution and $\langle\langle 2 \rangle\rangle \phi_2$ then AA gives a solution.