On Higher-Order Cryptography

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Inria informatiques mathématiques

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September 20th, 2021

Cryptographic Reductions and Higher-Order Computation

- Security of Ψ in the computational model:

$$(\forall A \in \mathbf{PPT}.\neg(A\ \mathbf{Breaks}\ \Phi)) \Longrightarrow (\forall B \in \mathbf{PPT}.\neg(B\ \mathbf{Breaks}\ \Psi))$$

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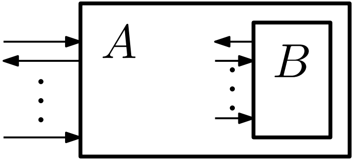

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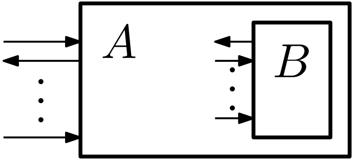

$$\texttt{Reduction} : \mathit{TypeOf}(B) \to \mathit{TypeOf}(A)$$

- `Reduction` is a *complexity preserving* higher-order function.

An Example from [KatzLindell2008]

DEFINITION 3.25 *Let $F : \{0,1\}^* \times \{0,1\}^* \to \{0,1\}^*$ be an efficient, length-preserving, keyed function. F is a* pseudorandom function *if for all probabilistic polynomial-time distinguishers D, there is a negligible function* negl *such that:*

$$\left| \Pr[D^{F_k(\cdot)}(1^n) = 1] - \Pr[D^{f(\cdot)}(1^n) = 1] \right| \leq \mathsf{negl}(n),$$

where the first probability is taken over uniform choice of $k \in \{0,1\}^n$ and the randomness of D, and the second probability is taken over uniform choice of $f \in \mathsf{Func}_n$ and the randomness of D.

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The first parameter is fixed to be k

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The Cryptographer's Notion of Efficiency [KatzLindell2008]

We equate "efficient adversaries" with randomized (i.e., probabilistic) algorithms running in time *polynomial in* n. This means there is some polynomial p such that the adversary runs for time at most $p(n)$ when the security parameter is n. We also require—for real-world efficiency—that honest parties run in polynomial time, although we stress that the adversary may be much more powerful (and run much longer than) the honest parties.

Equivalence-Preserving Cryptography?

- Programs, e.g. when hashed, are usually treated as strings:

$$\mathtt{P} \in \{0,1\}^* \longrightarrow \boxed{H_s} \longrightarrow \mathtt{R} \in \{0,1\}^*$$

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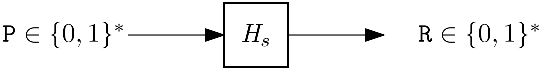
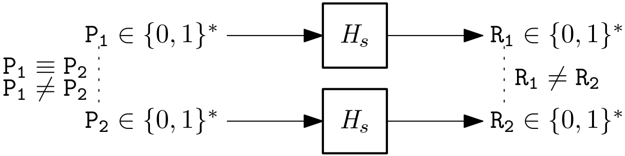


- If two programs P and Q are perfectly equivalent but distinct, they are thus seen as distinct strings, and mapped to distinct hashes:



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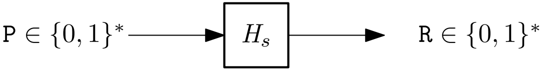

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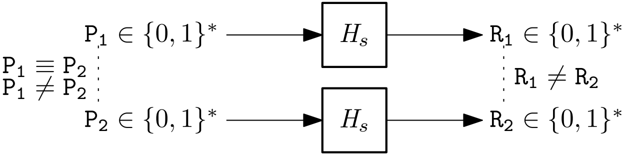

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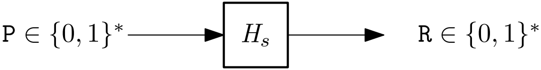

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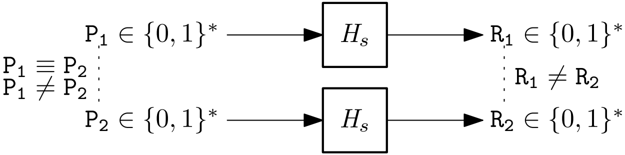

- The same argument holds when H_s is replaced by Enc_k (i.e. encryption) or Mac_k (i.e. authentication).
- Would it be possible to define any cryptographic primitive in such a way as to make it *equivalence preserving*?
  - That somehow amounts to turning H_s into a program of type $(\{0,1\}^* \to \{0,1\}^*) \to \{0,1\}^*$ (rather than $\{0,1\}^* \to \{0,1\}^*$).
  - E.g., hashing distinct but equivalent programs can be done *only once*.

This Talk

1. A **New Model** of Complexity-Bounded Higher-Order Computation Based on *Game Semantics*.
   - Second-order adversaries are everywhere in cryptography.
   - Defining the concept of an *efficient adversary* at order higher-than- 2 instead requires some care.
   - Game semantics [AJM00,HO00] offers a way to reduce higher-order computation to first-order computation.

This Talk

1. A **New Model** of Complexity-Bounded Higher-Order Computation Based on *Game Semantics*.
   - Second-order adversaries are everywhere in cryptography.
   - Defining the concept of an *efficient adversary* at order higher-than- 2 instead requires some care.
   - Game semantics [AJM00,HO00] offers a way to reduce higher-order computation to first-order computation.
2. Some *Negative* and *Positive* Results on the Feasibility of **Higher-Order Cryptography**.
   - Results about influential variables in decision trees imply that second-order pseudorandomness and collision-resistance are not attainable.
   - Some positive results can be obtained, but there is an high price to pay.

An Example: the Game Semantics of a Second-Order Function

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- The Player P must react to every move of the opponent O.
- It can do so based on *the whole* history, without any further constraint. This defines a strategy.
- The *length* of the interaction is in principle arbitrary.
- Multiple moves can be played *at the same site*, but they somehow need to be distinguished.

Cryptographic Game Semantics — I

Games Parametrized by a Security Parameter

- Games: $G = (O_G, P_G, (L_G^n)_{n\in\mathbb{N}})$
- Strategies: $f : \mathbb{N} \times (L_G^n \cap \mathrm{Odd}) \to P_G$

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Restricted Classes of Games and Strategies

Polynomially Bounded Games:
G such that there exists a polynomial P with positive coefficients, such that:
$\forall n \in \mathbb{N}, \forall s \in L_G^n, |s| \leq P(n)$.

Polytime Computable Strategies:
There exists a polynomial time Turing machine which on input $(1^n, s)$ returns $f(n, s)$.

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Constructing Games

From the games G, H, we can construct more complex games such as:

- $G \multimap H$, modeling functions from G to H;
- $G \otimes H$, modeling pairs of elements from G and H, respectively;
- $!_q G$ modeling $q(n)$ copies of G.

Cryptographic Game Semantics — II

Proposition (Compositing Strategies)

If f, g polytime strategies on $G \multimap H$ and $H \multimap K$ (respectively), one can form $g \circ f$ as a ***polytime*** *strategy on $G \multimap K$. Composition is associative.*

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YES!

The whole sequence of probabilistic choices is available, and strategies compose.

An Example: a Simple Randomized Strategy

$$!_1\mathbf{B} \quad\multimap\quad !_2(\mathbf{S}[n] \quad\multimap\quad \mathbf{B}) \quad\multimap\quad \mathbf{B}$$

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Then:

- If $X_n = \{0,1\}^{r(n)}$, then the distinguisher is just a polytime TM.
- If $X_n = \{0,1\}^n \to \{0,1\}^n$, then the distinguisher is a polytime (in n) OTM.
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Higher-Order Pseudorandomness?

- Intuitively, it is **impossible** to build deterministic polytime objects of type

$$\{0,1\}^n \to (\{0,1\}^n \to \{0,1\}^1) \to \{0,1\}^n$$

  which "look random": a truly random function "query" its argument exponentially many times, namely on all the strings in $\{0,1\}^n$.
- How to turn this into a formal argument in the just introduced model?

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- The *type* of a (candidate) **pseudorandom function** could be

  $$SOF_{p,q,r} = \mathbf{S}[n] \multimap !_p(\mathbf{S}[q] \multimap \mathbf{B}) \multimap \mathbf{S}[r],$$

  while the type of an **adversary** for it, being randomized, should be

  $$ADV_{s,t,p,q,r} = !_s\mathbf{B} \multimap !_t(!_p(\mathbf{S}[q] \multimap \mathbf{B}) \multimap \mathbf{S}[r]) \multimap \mathbf{B}$$

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We say that a polytime strategy f for the game $SOF_{p,q,r}$ is **pseoudorandom** iff for any polytime strategy $\mathcal{A}$ for the game $ADV_{s,t,p,q,r}$ it holds that

$$|Pr[\mathcal{A} \circ (f \circ rand) \Downarrow 1] - Pr[\mathcal{A} \circ (rand) \Downarrow 1]| \leq \varepsilon(n)$$

where ε is a negligible function.

The Two Results

Theorem

For every δ there is a strategy $coll_\delta$ on the game

$$!_t(!_p(\mathbf{S}[n] \multimap \mathbf{B}) \multimap \mathbf{S}[r]) \multimap (\mathbf{S}[n] \multimap \mathbf{B}) \otimes (\mathbf{S}[n] \multimap \mathbf{B})$$

such that for every deterministic strategy f, the composition $(!_s f) \circ coll_\delta$, with probability at least $1 - \delta$, computes two functions g, h such that:

1. $H(g, h) \geq 0.1$;
2. $f \circ g$ *and* $f \circ h$ *behave the same;*
3. *For every function e on which $coll_\delta$ queries its argument, it holds that $H(e, g) \geq 0.1$ and $H(e, h) \geq 0.1$.*

The Two Results

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Theorem

If there is a one-way function, then there is a pseudorandom strategy for $\mathbf{S}[n] \multimap !_n(\mathbf{S}[\log_2(n)] \multimap \mathbf{B}) \multimap \mathbf{S}[r]$.

Conclusion

Main Contributions

- A novel *game-theoretic framework* for higher-order, randomized, complexity bounded computation.
- *Impossibility* of building second-order functions having the expected type, (i.e. taking in input characteristic functions on $\{0,1\}^n$) and having good cryptographic properties.
- *Existence*, under standard cryptographic assumptions, of secord-order pseudrandom functions taking in input characteristic functions on $\{0,1\}^{\log_2(n)}$.

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Future Work

- How about **encryption**?
- Is it that our game-semantic framework can be seen as a methodology for proving higher-order cryptographic **reduction arguments** to be not only *complexity preserving*, but even *correct*?

Thank You!

Questions?

Example

$F : \{0,1\}^n \to \{0,1\}^{P(n)}$ a one-way function.

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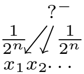
$g : \mathbf{1} \multimap \mathbf{S}[X]$



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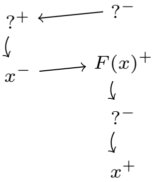

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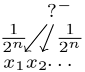

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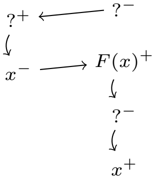

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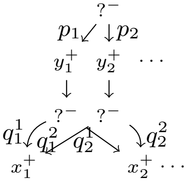

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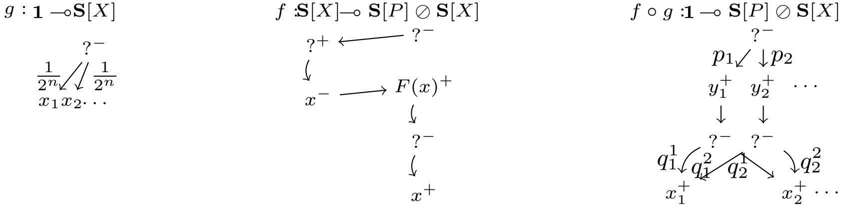

Compute $f \circ g(?y_i)$ boils down to finding an element in $F^{-1}(y_i)$.

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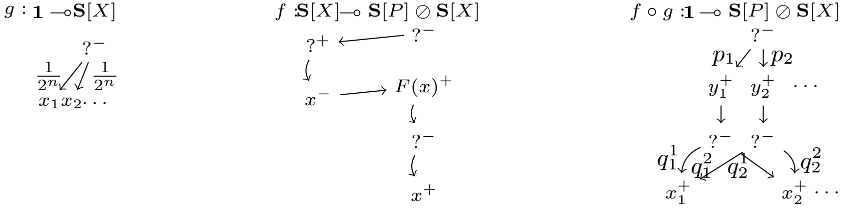

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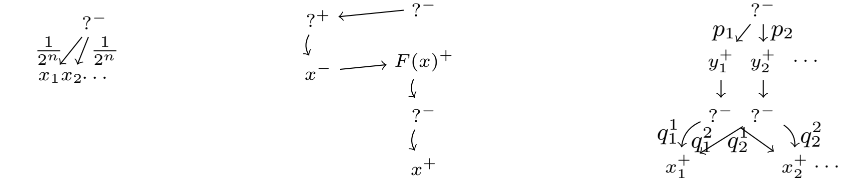

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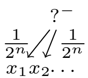

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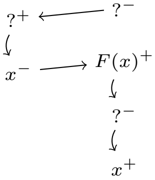

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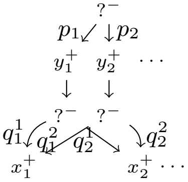

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- f, g are polytime computable functions on bounded games.
- $f \circ g : \mathbf{1} \multimap \mathbf{S}[P] \oslash \mathbf{S}[X]$ is not polytime computable.