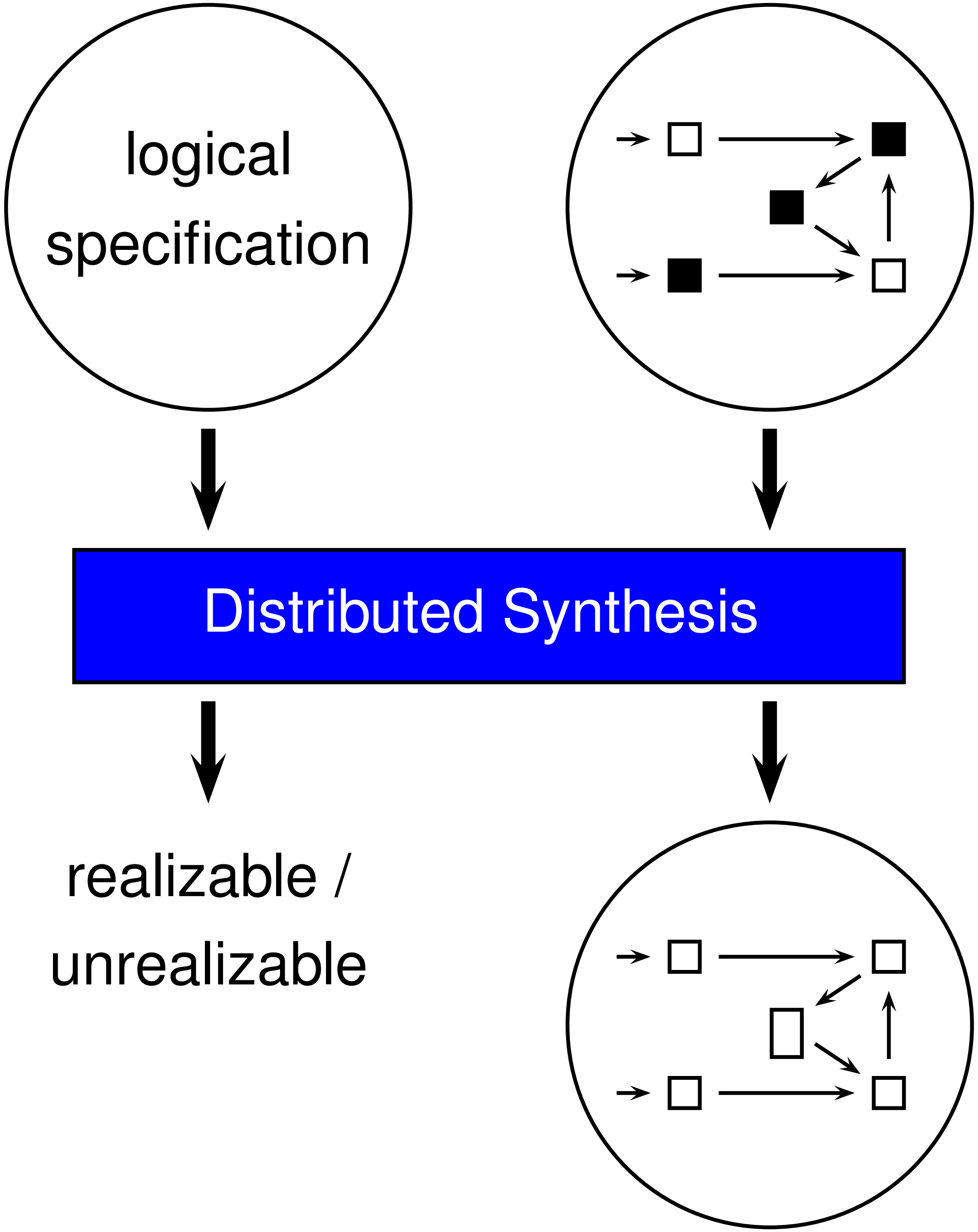
Petri Games:

Synthesis of Distributed Systems with Causal Memory

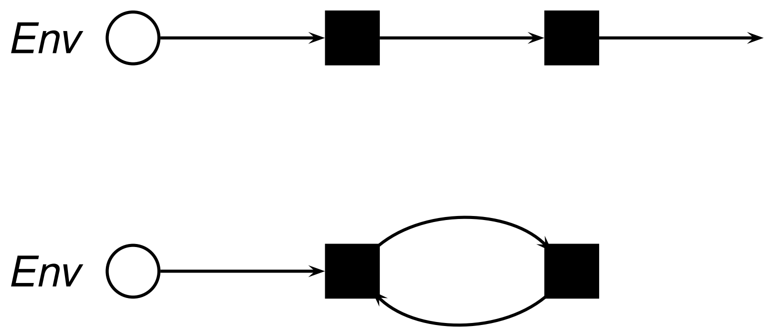
Bernd Finkbeiner & Ernst-Rüdiger Olderog
Saarbrücken & Oldenburg


logical
specification
Distributed Synthesis
realizable /
unrealizable

Models of Distributed Systems

Pnueli-Rosner model: synchronous concurrency with partial observation of shared variables

- distributed synthesis: in general undecidable
- pipelines and rings: nonelementary



A. Pnueli & R. Rosner (1990): Distributed Reactive Systems are Hard to Synthesize.

B. Finkbeiner & S. Schewe (2005): Uniform Distributed Synthesis.

Models of Distributed Systems

Zielonka automata: asynchronous concurrency with shared actions and causal memory

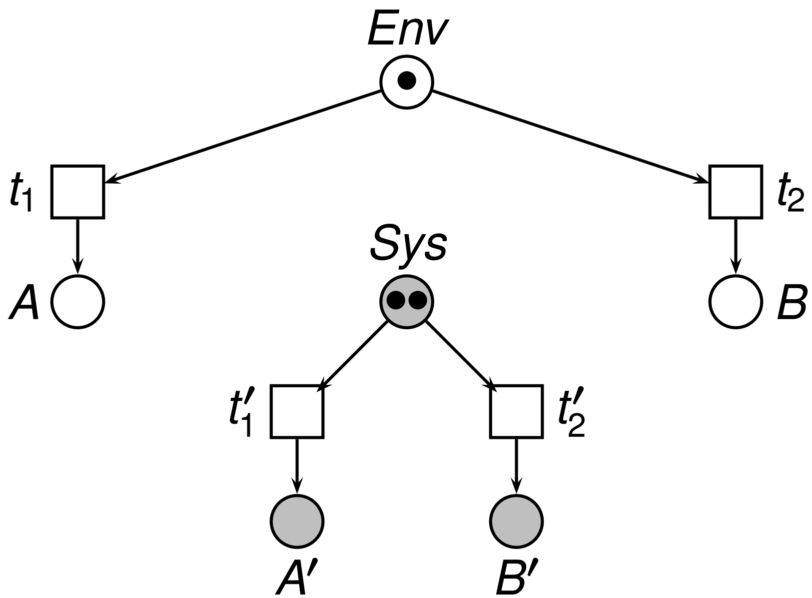
- distributed synthesis: in general decidability open
- tree architectures: nonelementary

Gastin, Lerman & Zeitoun (2004):
Distributed Games with Causal Memory are Decidable for Series-Parallel Systems.

Genest, Gimbert, Muscholl & Walukiewicz (2013):
Asynchronous Games over Tree Architectures.

Petri Game: Example

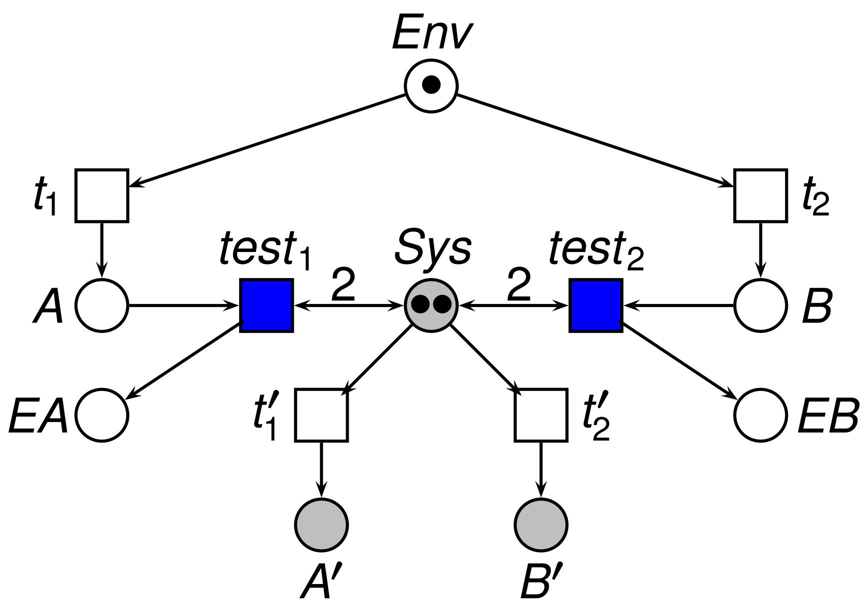
Modelling principle: causality



Goal of *Sys*: achieve the same decisions as *Env*.

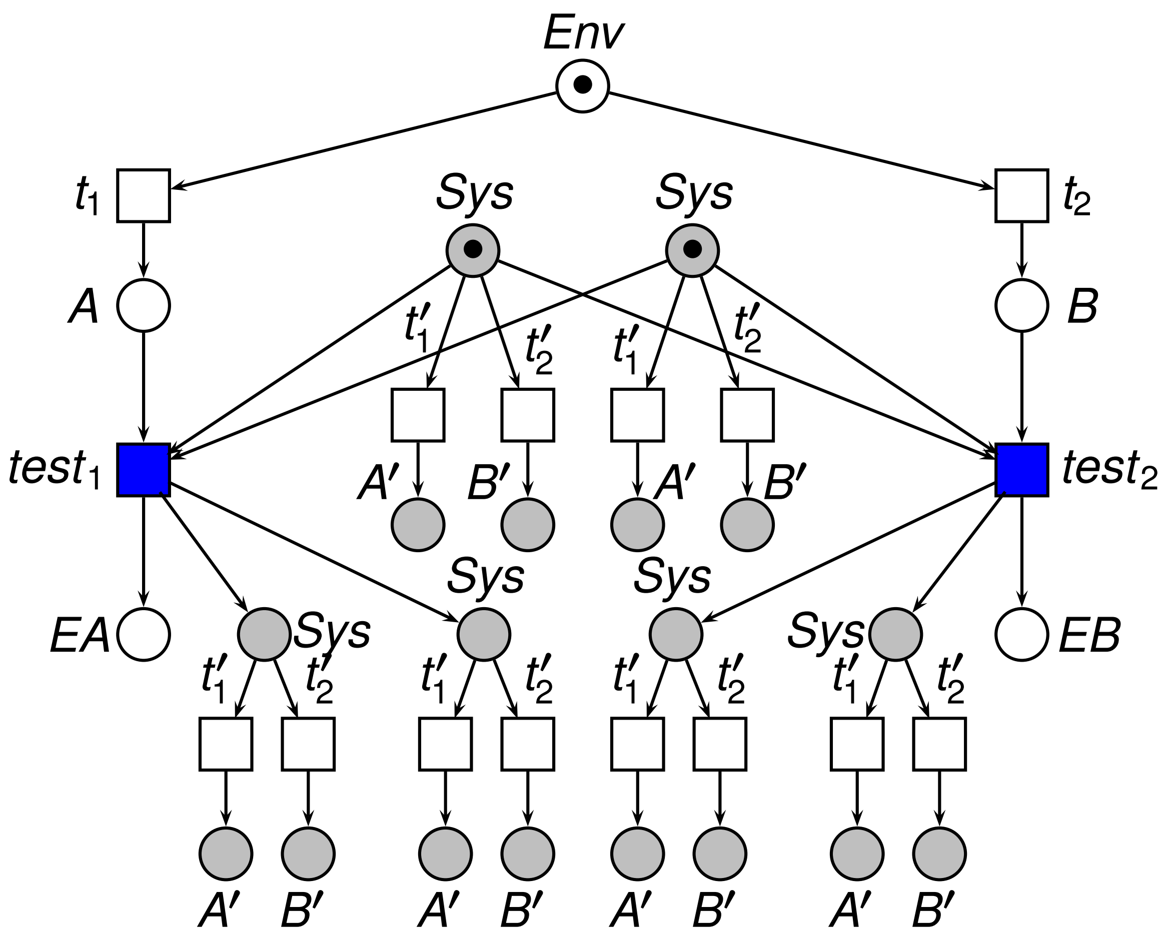
Problem: *Sys* cannot observe decisions of *Env*.

Petri Game with Tests



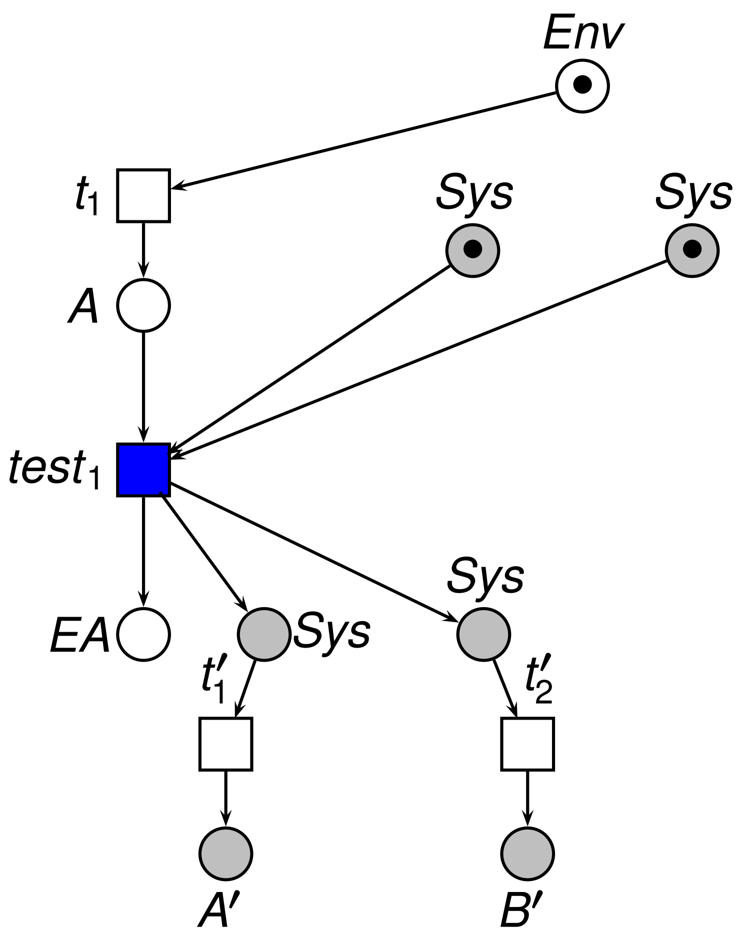
By communicating via transitions $test_1$ or $test_2$, *Sys* learns about decisions of *Env*.

Causality: Unfolding



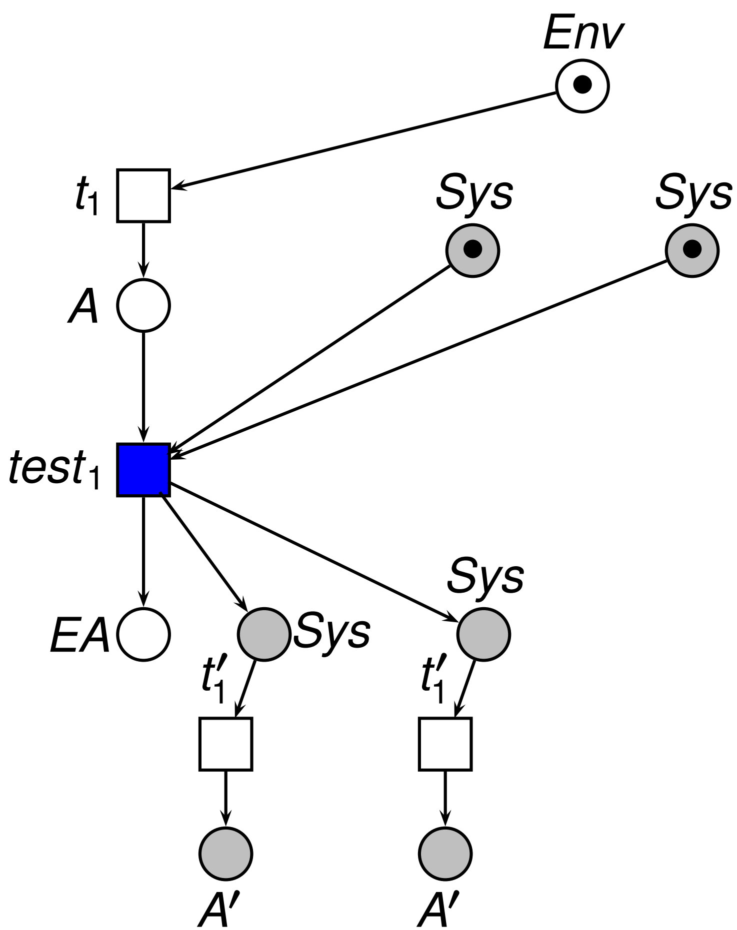
J. Engelfriet (1991): Branching processes of Petri nets.

Play 1



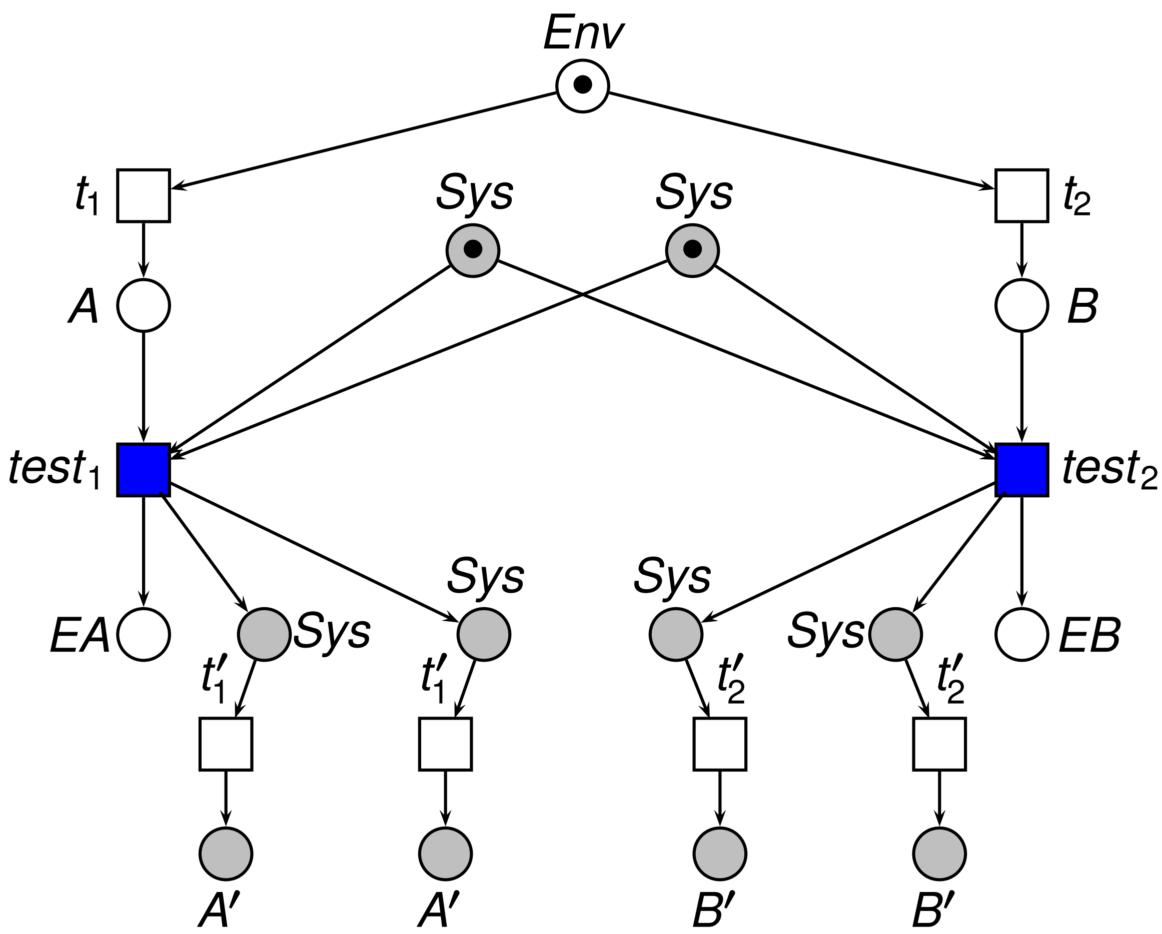
Env wins this play.

Play 2



Sys win this play.

Global Strategy



This is a winning strategy for *Sys*.

Definition: Global Strategy

A global strategy for the system players is a subprocess σ of the unfolding subject to the following conditions:

(S1) at every system place, σ is deterministic,

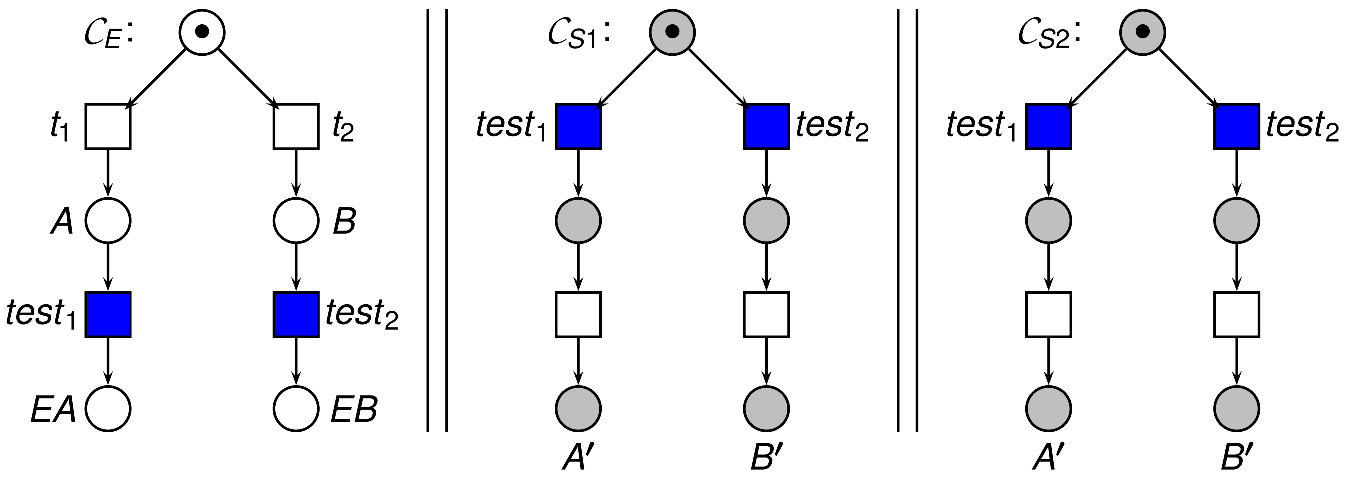
(S2) at every environment place, σ does not restrict any local transitions.

A strategy is deterministic at a place p if in every reachable marking at most one transition t with $p \in pre(t)$ is enabled.

A strategy is deadlock-avoiding if in every reachable marking if there is a transition is enabled in the unfolding then there is some transition enabled in the the strategy.

Strategy Distribution

A global strategy σ is distributable if σ can be represented as the parallel composition of local controllers (finite automata) for *Env* and *Sys*.



Distribution Lemma. Every global strategy for a concurrency-preserving Petri game is distributable.

Existence of Winning Strategy

Question: Is it decidable ?

Although the reachability problem is decidable
even for unbounded P/T nets, we have:

Theorem. For general Petri games, the question whether the system players have a winning strategy is undecidable.

Idea: Games on Vector Addition Systems with States are undecidable and can be reduced to
Petri games with unbounded markings.

Solving Petri Games

EXPTIME-completeness

For bounded Petri games with

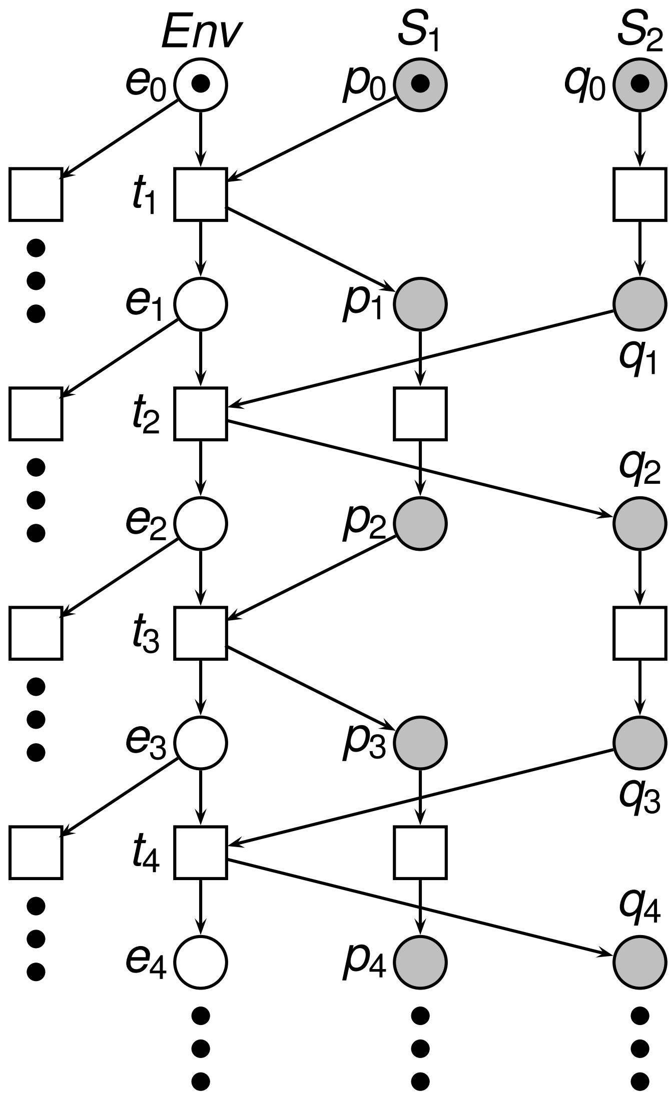
- one environment player,
- a bounded number of system players,
- a safety objective,

the question whether the system players have a winning strategy is EXPTIME-complete.

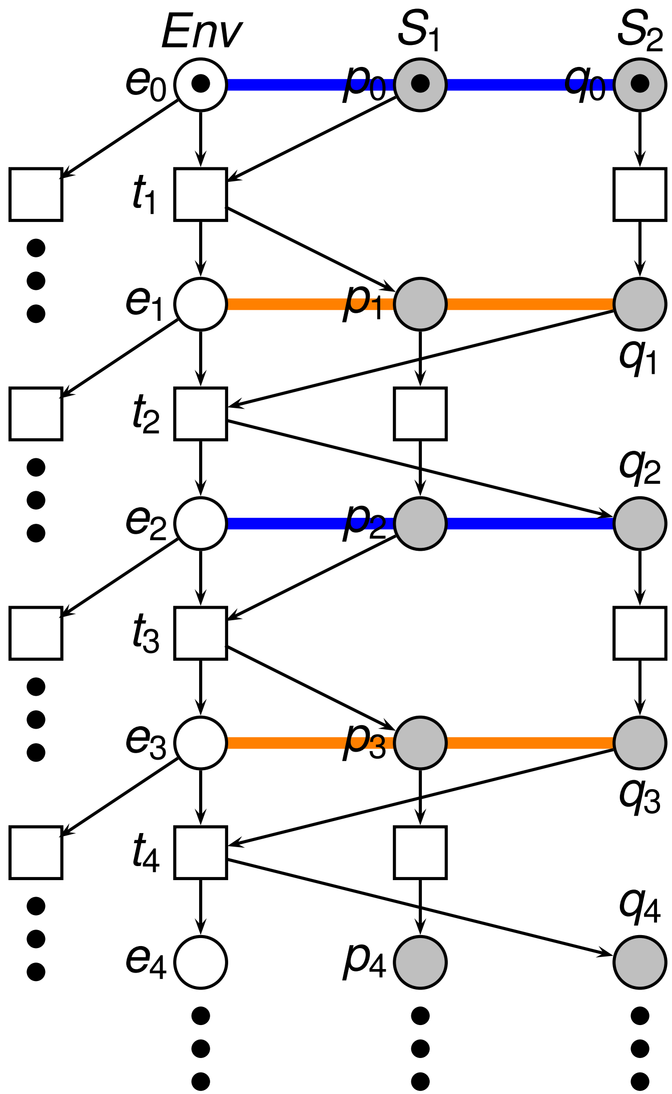
If a winning strategy for the system players exists, it can be constructed in exponential time.

- Lower bound: Reduction from combinatorial games
- Upper bound: Reduction to 2-player game on finite graphs

Infinite Strategy

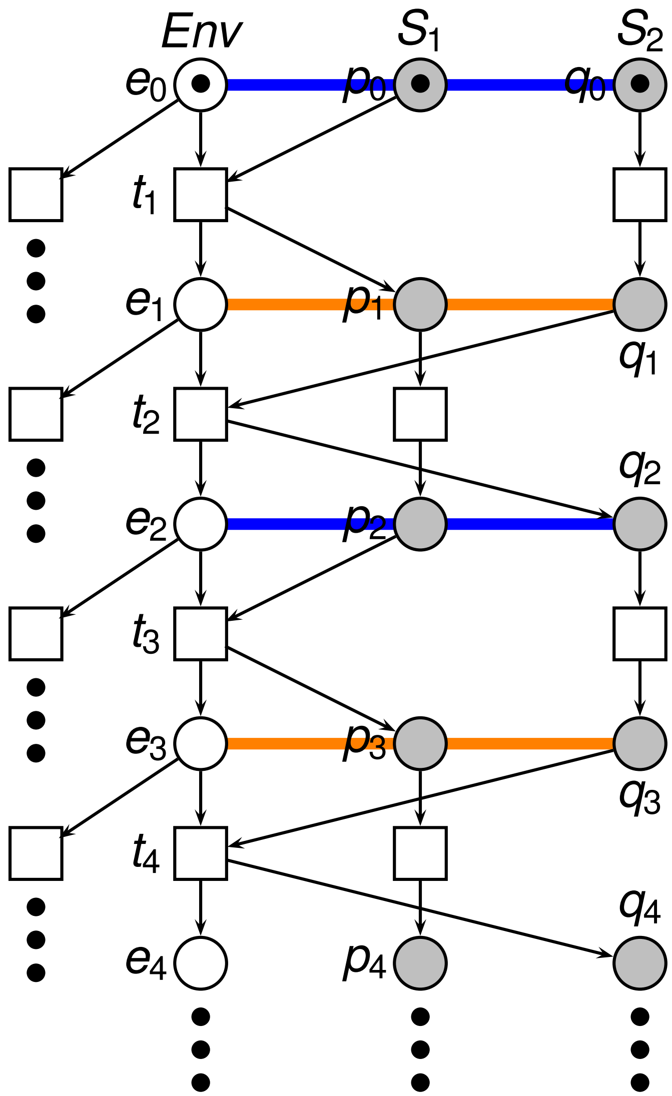


Cuts



J. Esparza (1994): Model checking using net unfoldings.

Cuts: changing informedness



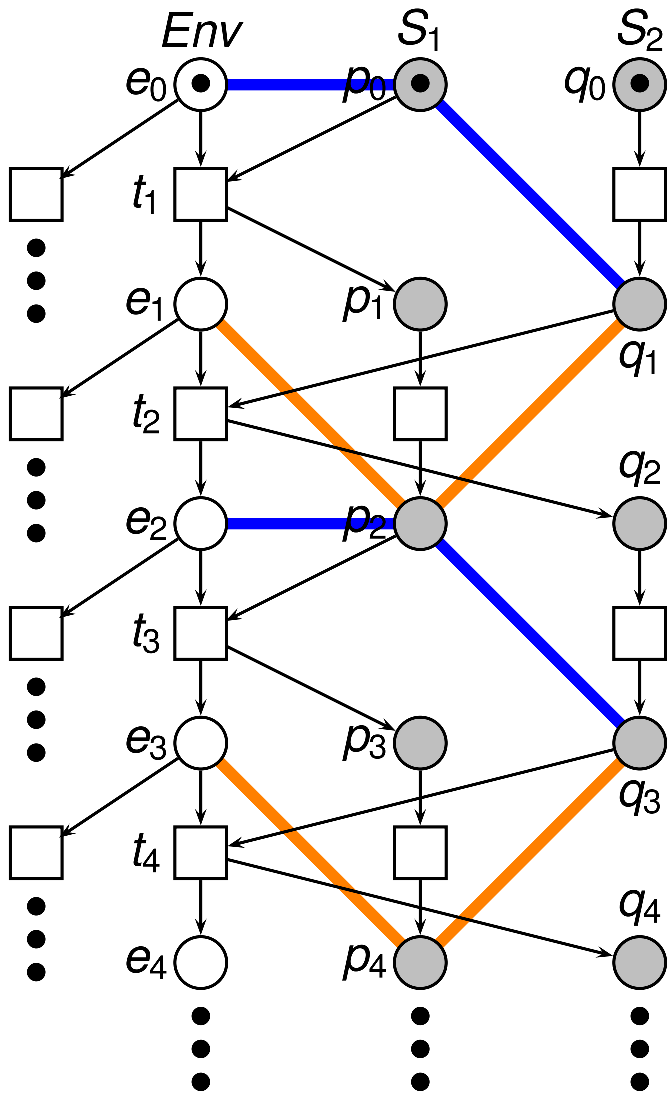
S_1 and S_2 know the same.

S_1 knows more than S_2.

S_2 knows more than S_1.

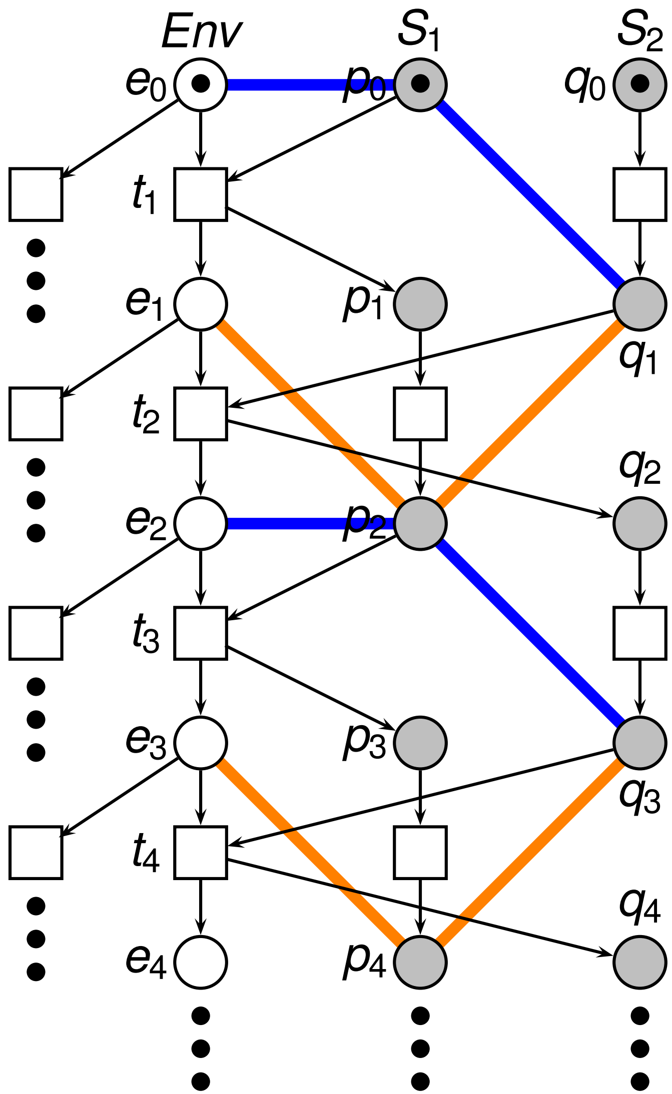
S_1 knows more than S_2.

Mcuts



System players have maximally progressed.

Mcuts: equal information



System players have maximally progressed.

Reduction to Finite-Graph Game

Reduction Theorem

Every bounded Petri game can be transformed into
a finite-graph game of exponential size such that

- the system players have a deadlock-avoiding winning strategy

iff

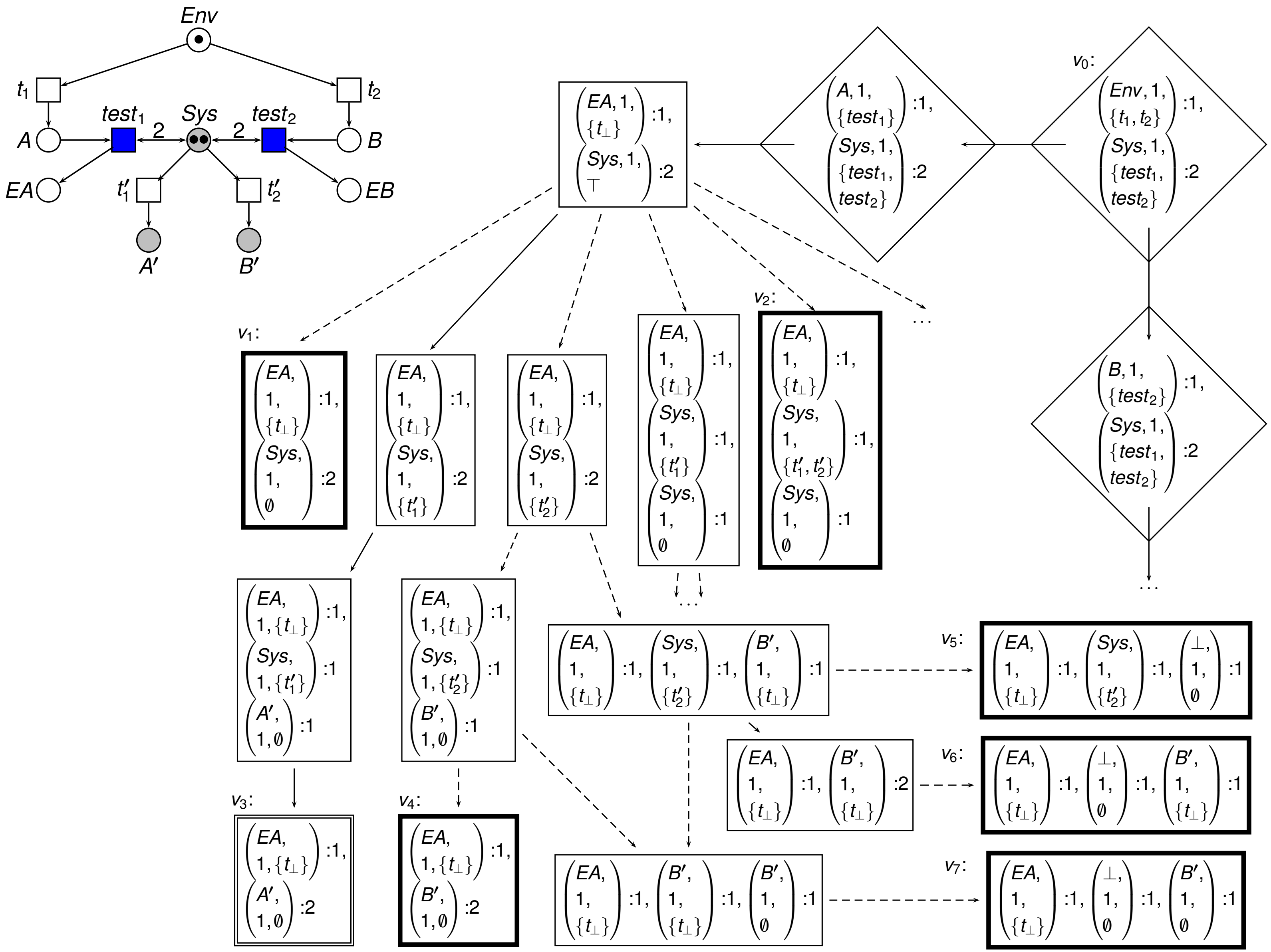
- player 0 has a winning strategy in the finite-graph game.

Idea:

States of the finite-graph game simulate cuts in the net unfolding, more precisely: cuts annotated with sets of outgoing transitions.

Environment decisions are delayed until an mcut is reached.

Example of a Reduction



Tool ADAM ...

implements symbolic game solving algorithm. It

- inputs a Petri game,
- decides whether a winning strategy exists, and
- if so outputs a winning Petri net strategy for the system players.

ADAM:

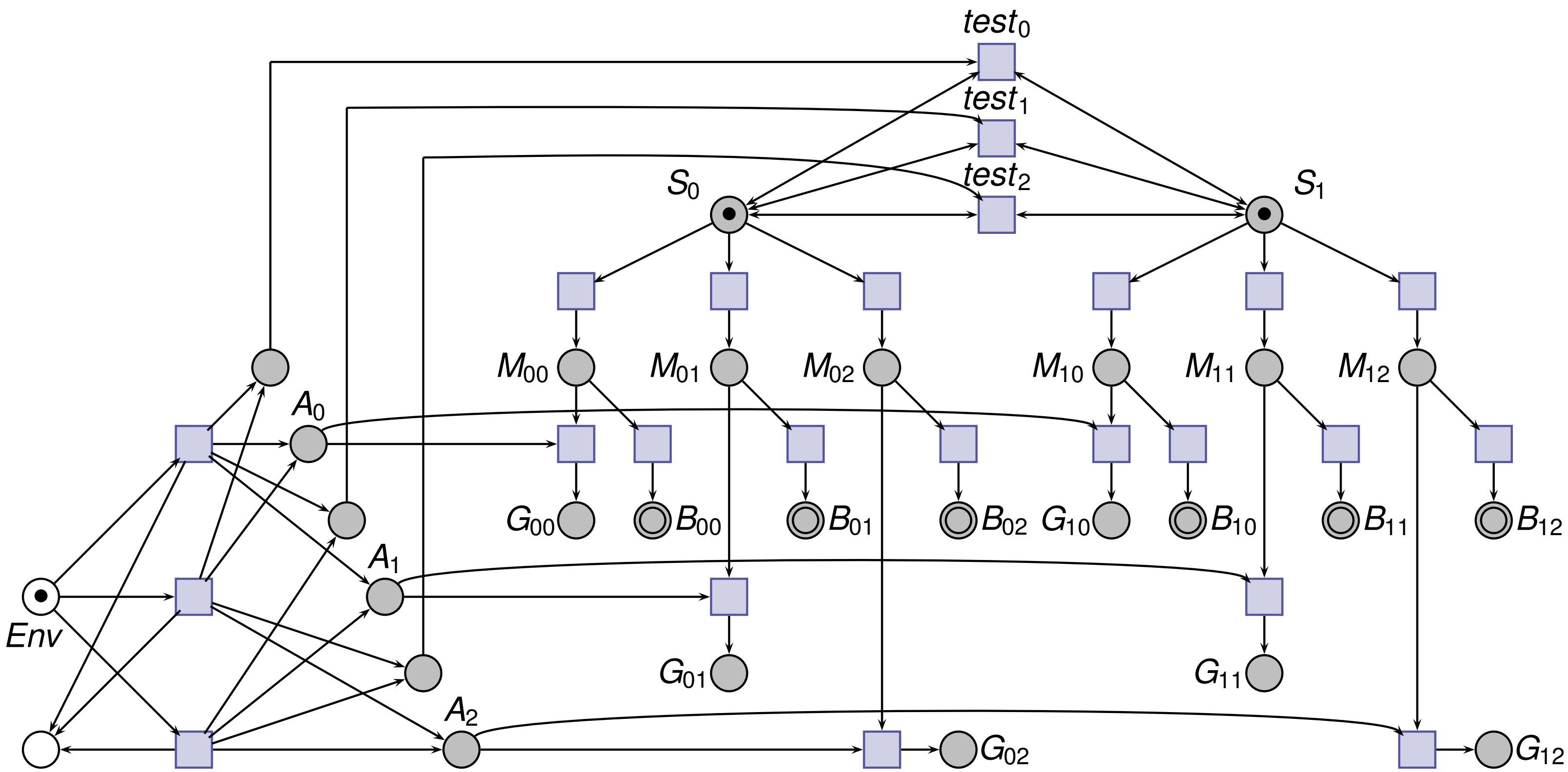
- running time only single-exponential in # processes
- automatic synthesis of distributed systems with $>$ 30 processes

B. Finkbeiner, Manuel Gieseking & E.-R. Olderog:
ADAM: Causality-Based Synthesis of Distributed Systems, CAV 2015.

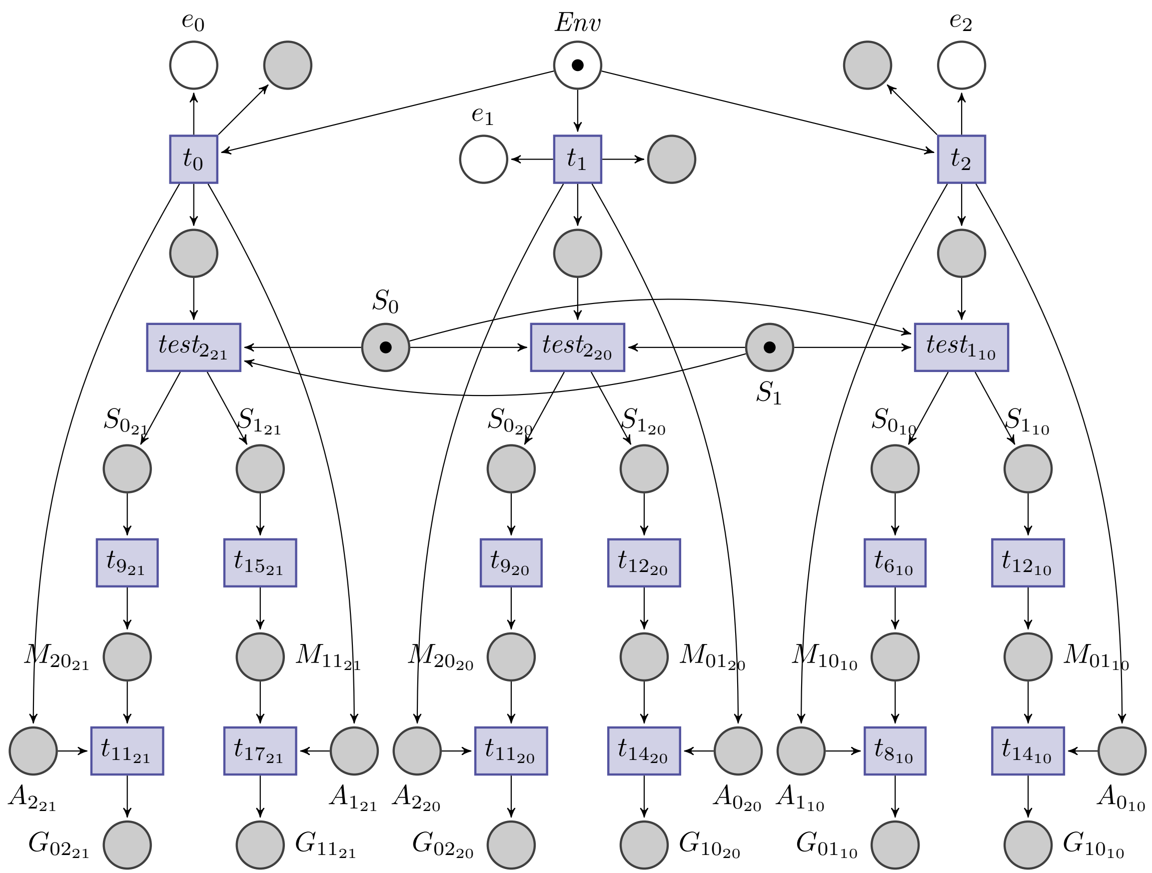
Concurrent Machines

Robots execute *k* orders on *n* machines
in an environment, which may disable an arbitrary machine.

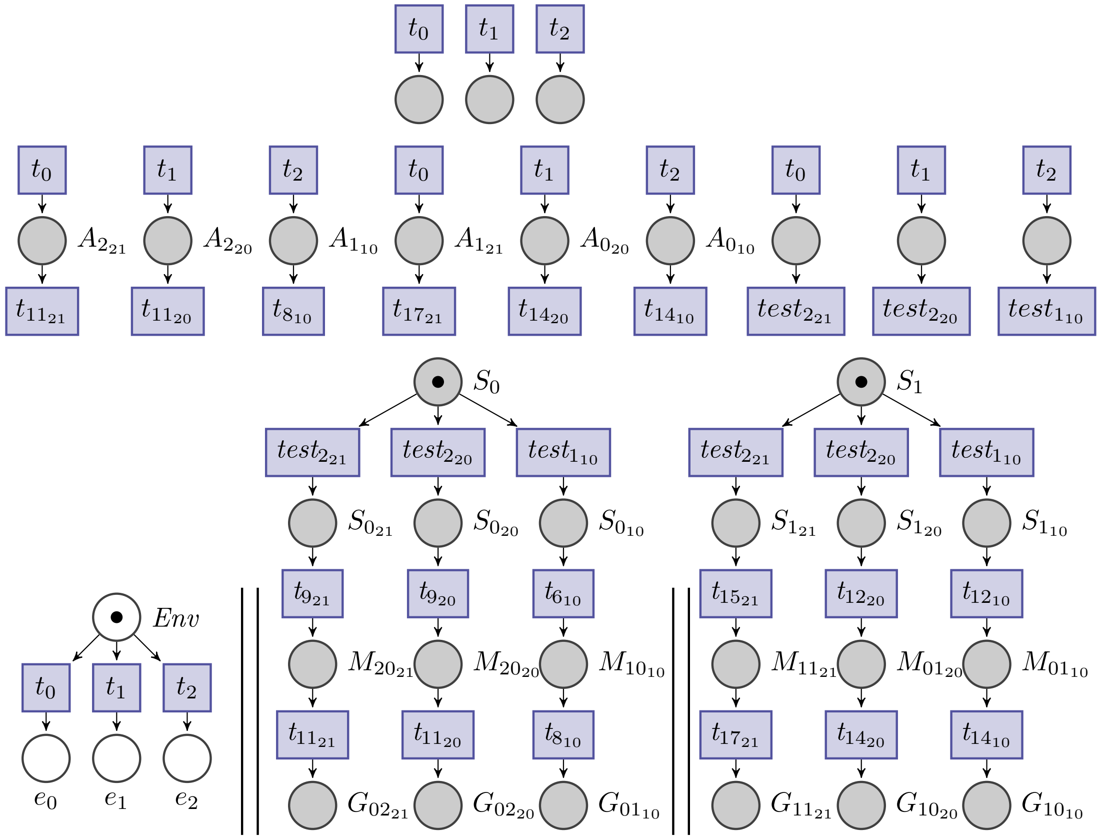
Petri game for $k = 2$ and $n = 3$:



Petri Game Strategy



Distributed Controllers



Further Benchmarks

- ## SR: Self-reconfiguring Robots

  Self-reconfiguration of n robots on which the environment destroys up to k tools.

- ## JP: Job Processing

  Processing of a job by a subset of n processors chosen by the environment.

- ## DWs: Document Workflow simple

  Workflow of a document among n clerks starting at a clerk selected by the environment.

Conclusion

Petri games with tool support:

Distributed strategies in single-exponential time

Context:

- one environment token = one source of information
- causality $\neq$ partial information
- synthesis without predefined interfaces

References

A. Pnueli & R. Rosner:
Distributed Reactive Systems are Hard to Synthesize, FOCS 1990.

B. Finkbeiner & S. Schewe:
Uniform Distributed Synthesis, LICS 2005.

B. Finkbeiner & E.-R. Olderog:
Petri Games: Synthesis of Distributed Systems with Causal Memory, GandALF 2014.

B. Finkbeiner, M. Gieseking & E.-R. Olderog:
ADAM: Causality-Based Synthesis of Distributed Systems, CAV 2015.

P. Gastin, B. Lerman & M. Zeitoun:
Distributed Games with Causal Memory are Decidable for Series-Parallel Systems, FSTTCS 2004.

B. Genest, H. Gimbert, A. Muscholl & I. Walukiewicz: Asynchronous Games over Tree Architectures, ICALP 2013.