Dynamic Causality (in Event Structures)

Youssef Arbach, David S. Karcher[1],
Uwe Nestmann[2], Kirstin Peters

September 22, 2015

[1] slides
[2] presenter

Motivation

Process modelling for health care scenarios, formally !

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- Why?
  - *Analysis* of processes (Proofs of properties)
  - *Improvement* of processes

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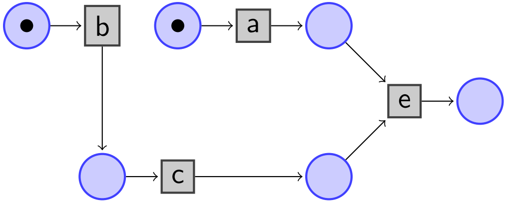
Process modelling for health care scenarios, formally !

- Why?
  - *Analysis* of processes (Proofs of properties)
  - *Improvement* of processes
- How?
  - Application domain is „rule“-based
  - Formal model with focus on causal dependencies

What is a Suitable Model?



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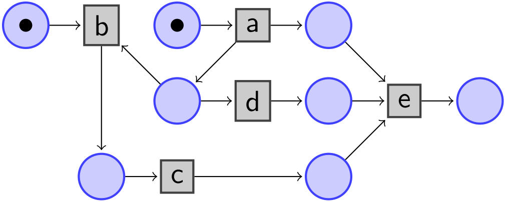

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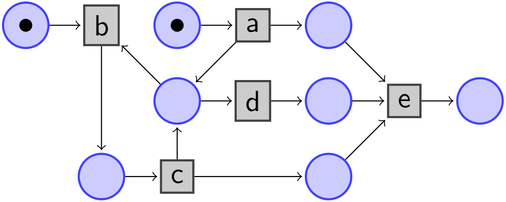

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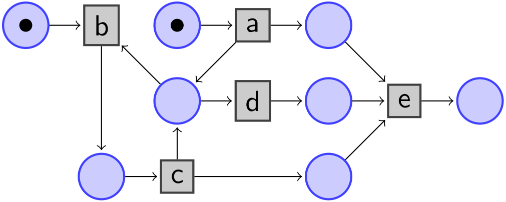

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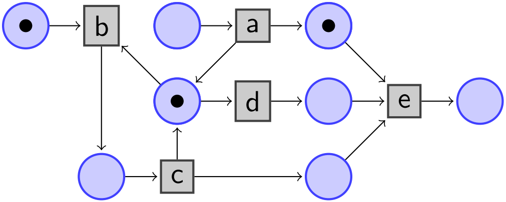

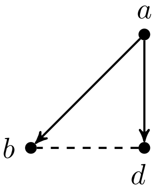

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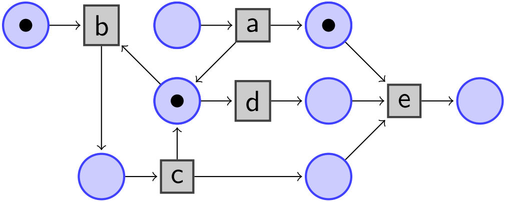

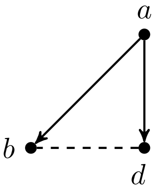

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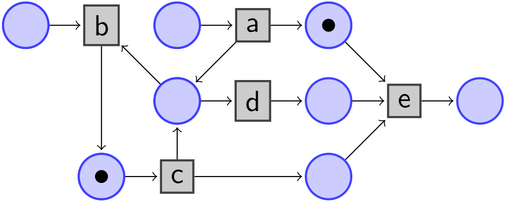

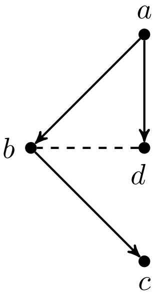

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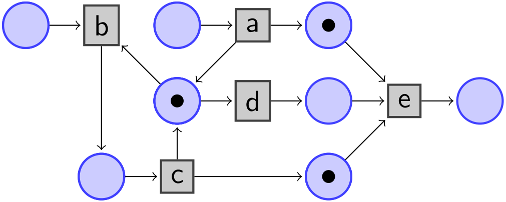

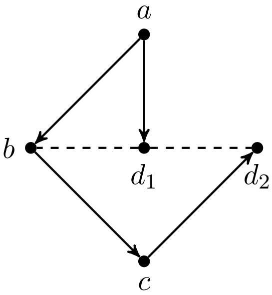

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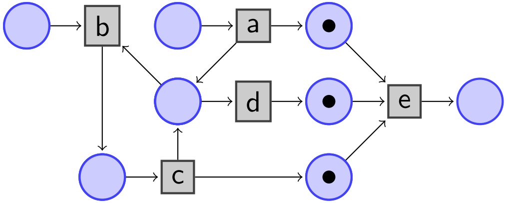

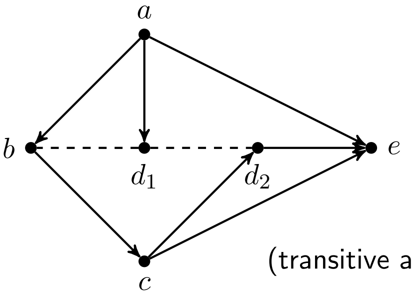

(transitive arrows omitted)

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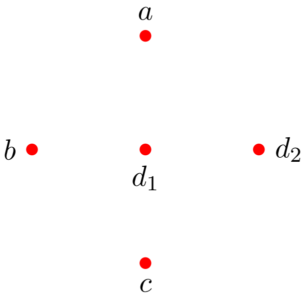

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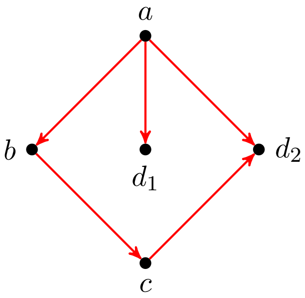

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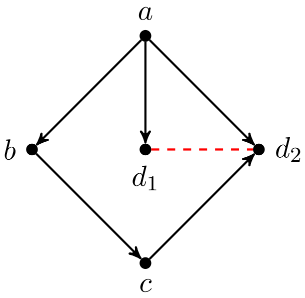

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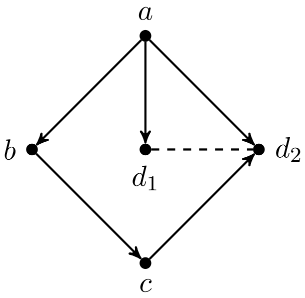

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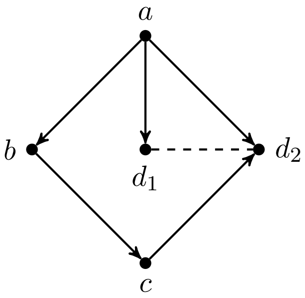

traces:

transitions:

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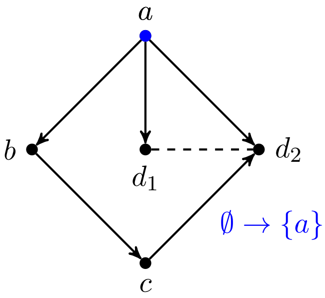

traces:

a

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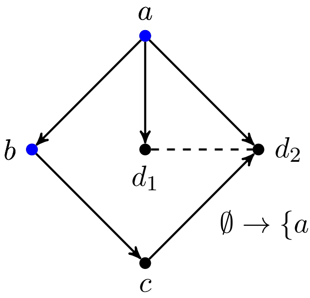

traces:

ab

transitions:

$\emptyset \to \{a\} \to \{a, b\}$

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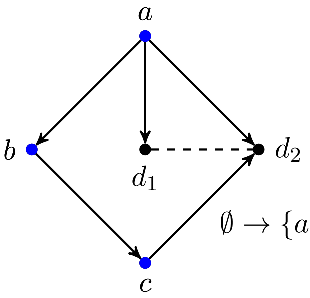

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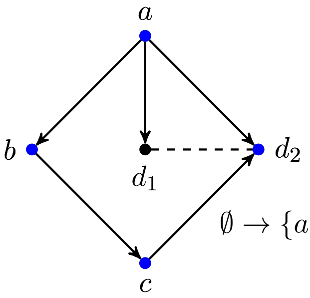

traces:

$abcd_2$

transitions:

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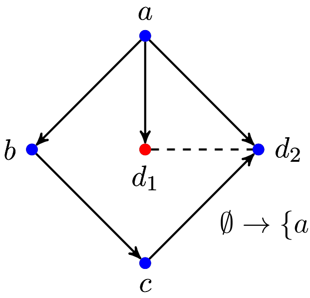

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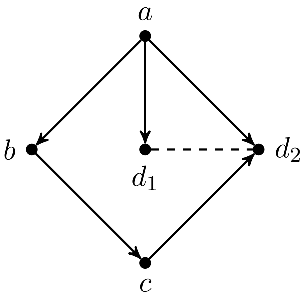

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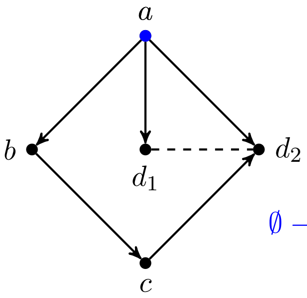

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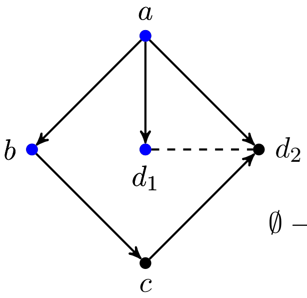

traces:
$abcd_2$
abd_1

transitions:
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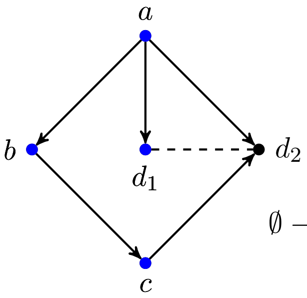

traces:

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abd_1c

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Prime Event Structures (PES)

First ES by Winskel 1980s, $(E, \leq, \#)$:

- *causality/enabling* relation $\leq$
- *conflict* relation $\#$

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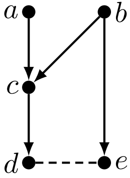
  $\forall\, e, e', e'' \in E \,.\, e \# e' \wedge e' \leq e'' \Rightarrow e \# e''$
- Finite causes:
  $\forall\, e \in E \,.\, \{e' \mid e' \leq e\}$ is finite

Semantics

- A trace is a sequence of events that can occur in an event structure, with respect to causality and conflicts.



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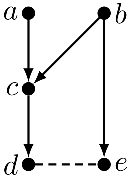

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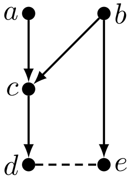

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No traces: $bc, c, abcde, \ldots$

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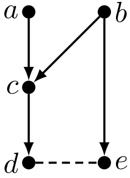
- A configuration of an event structure is a set of events, that can occur in one system run.



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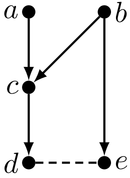

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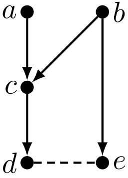

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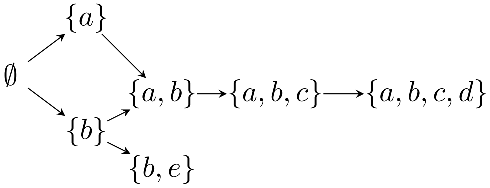
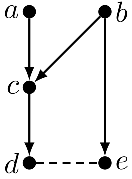
No configurations: $\{a, c\}, \{d, e\}$

Families of Configurations

A *family of configurations*
is a set of configurations ordered by inclusion:





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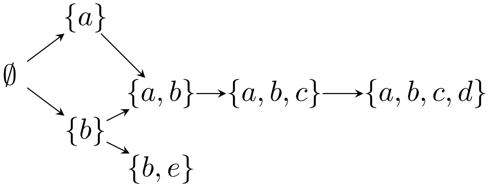
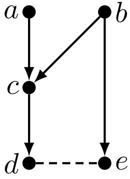
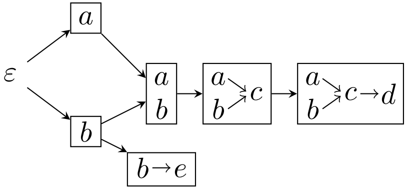


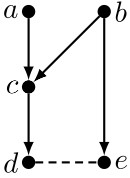


- Reflexive and transitive arrows are omitted.

Families of Posets

A *family of posets (partially ordered sets)* consists of several posets and a prefix relation on posets.





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Dual ES (DES):

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Extended BES:

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Bundle event structures have a new enabling relation $\mapsto \subseteq \mathcal{P}(E) \times E$, for each $X \mapsto e$, an $x \in X$ must precede e.
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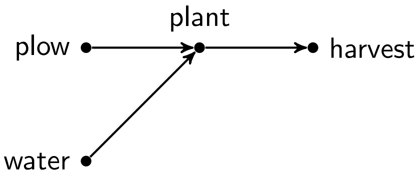
Other Event Structures ...

- Prioritized ES [Arbach, Peters, Nestmann 13]
- Several other extensions ...

Motivating Example

- Simple work-flow/process with regular behaviour



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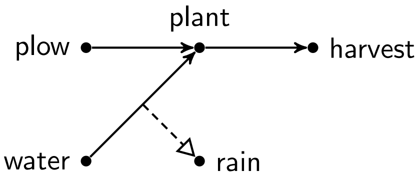

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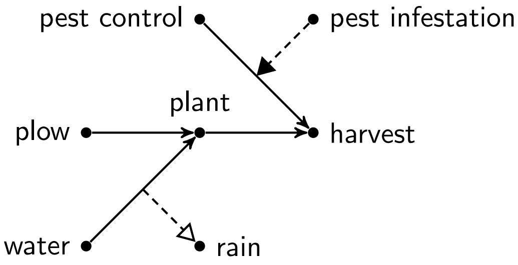
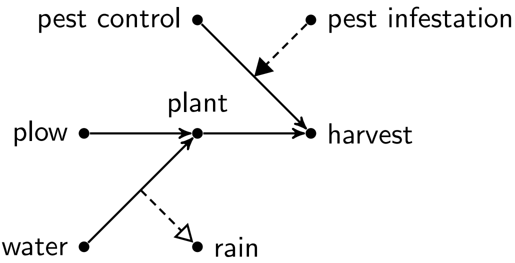


Shrinking Causality

- Events may drop causal dependencies



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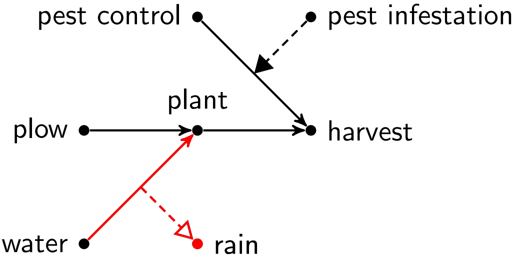

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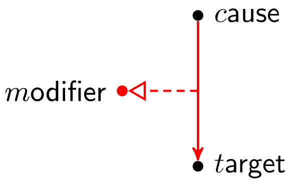

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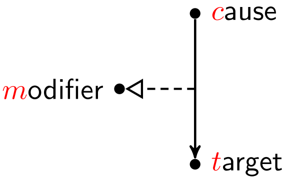

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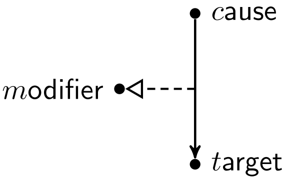

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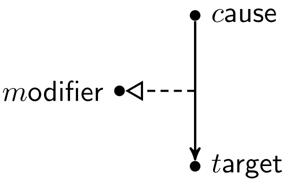

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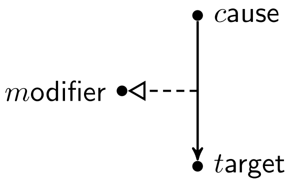

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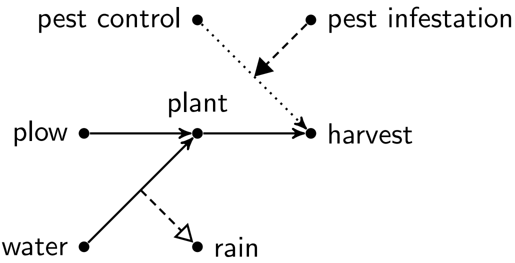

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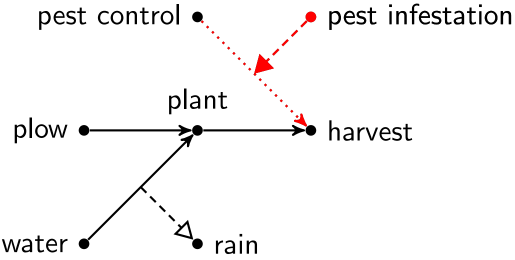

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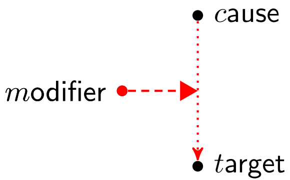

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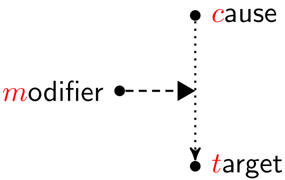

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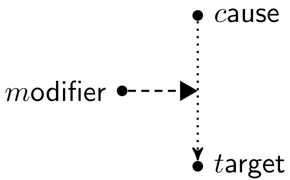

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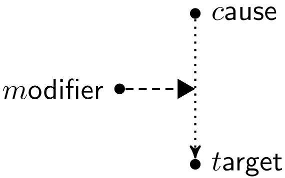

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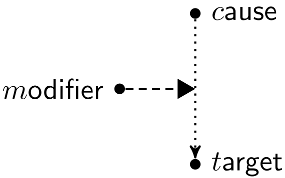

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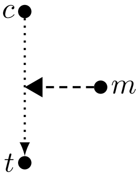

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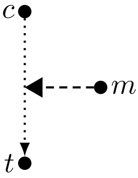

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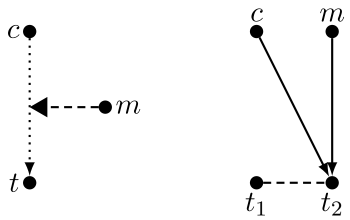

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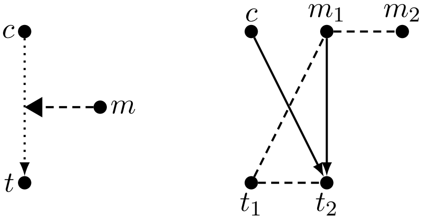

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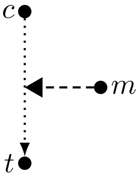

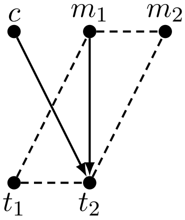

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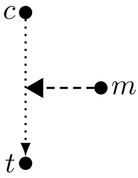

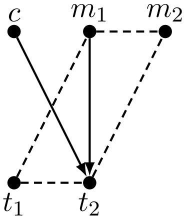

Simulation is ugly.

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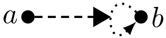

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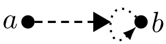

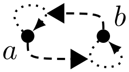

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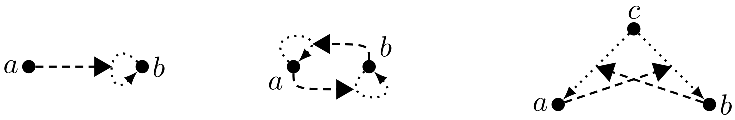
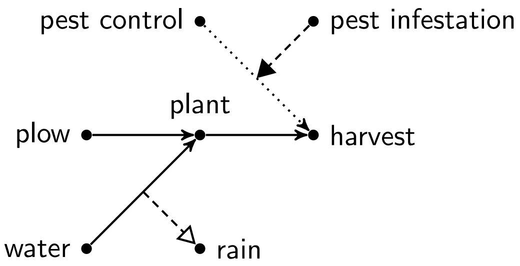


- Disabling
- binary conflict
- resolvable conflict

Dynamic Causality

- Combining shrinking and growing causality.



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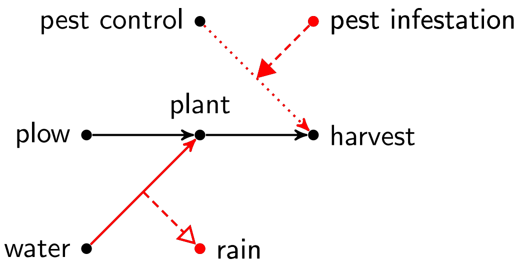

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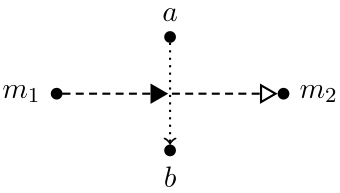

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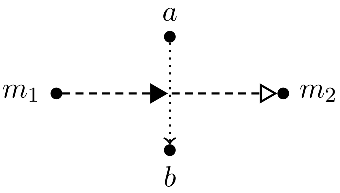

- First trace:

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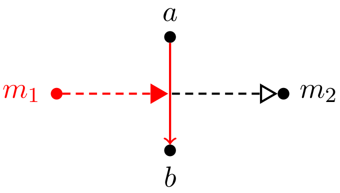

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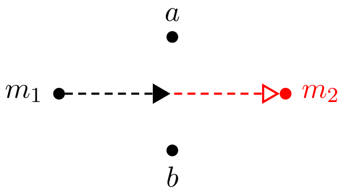

- First trace: $m_1 m_2$

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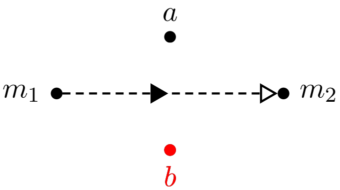

- First trace: $m_1 m_2 b$

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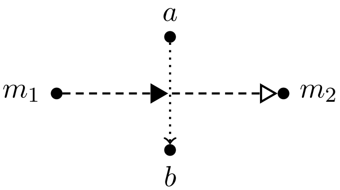

- First trace: $m_1 m_2 b$
- Second trace:

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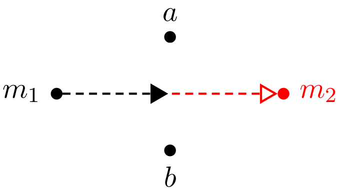

- First trace: $m_1 m_2 b$
- Second trace: m_2

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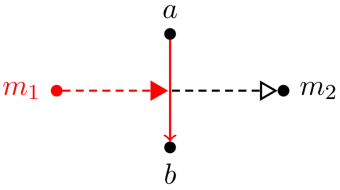

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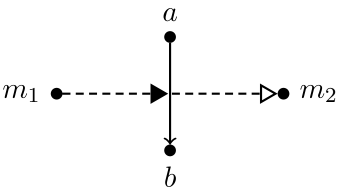

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- Now b is not allowed!

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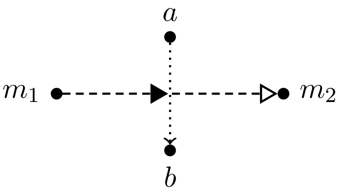
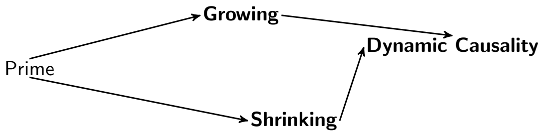
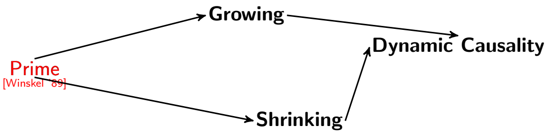
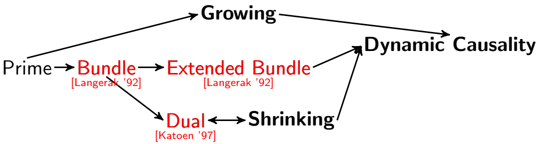


- First trace: $m_1 m_2 b$
- Second trace: $m_2 m_1$
- Now b is not allowed!
- order sensitive

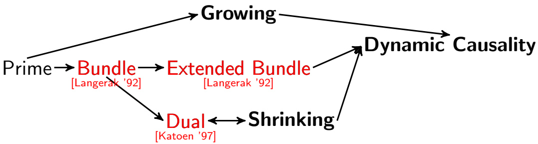

Growing
Dynamic Causality
Prime
Shrinking



- The ES from the introduction (with some additional properties)



- Bundle enabling: $X \mapsto e$, one $x \in X$ must precede e



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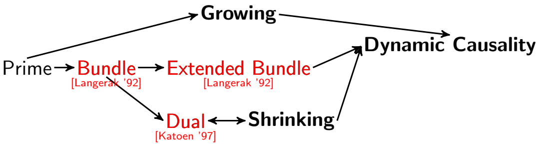
- Extended Bundle ES: Disabling instead of conflict



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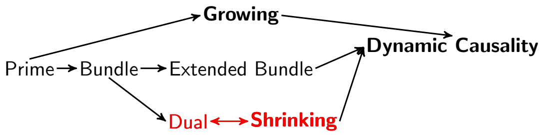

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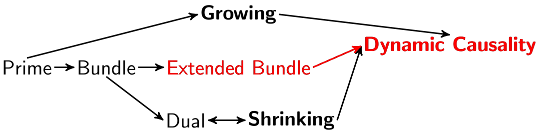
- Dual ESs and Shrinking Causality ESs equivalent



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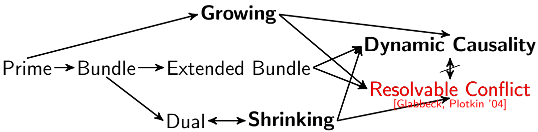
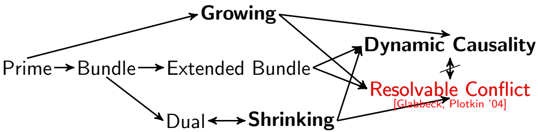
- Extended Bundle ESs strictly less expressive than Dynamic Causality ESs



- Set of events and witness relation $\vdash \subseteq 2^E \times 2^E$, transitions between configurations



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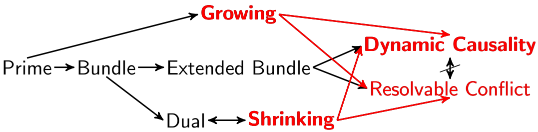

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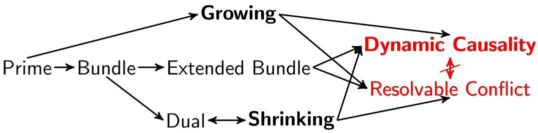

- Set of events and witness relation $\vdash \subseteq 2^E \times 2^E$, transitions between configurations
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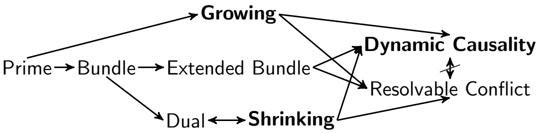
Growing
Prime
Bundle
Extended Bundle
Dynamic Causality
Resolvable Conflict
Dual
Shrinking

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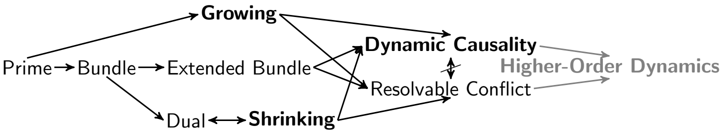
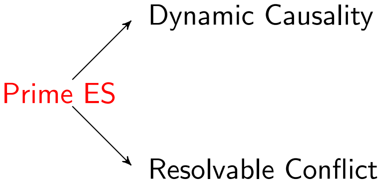


- Solution:
  set-based and higher-order dynamic causality ES

Classification



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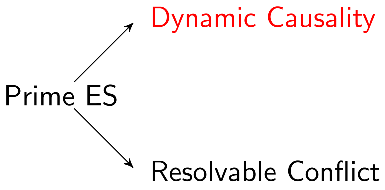

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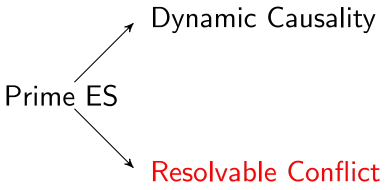

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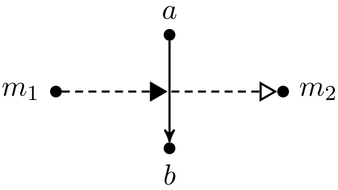

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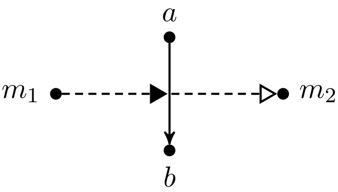

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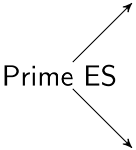

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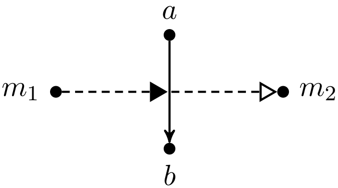

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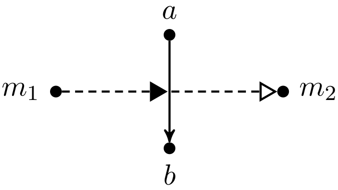

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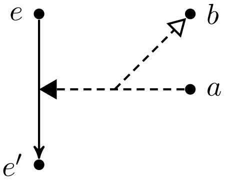

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- The rules for higher-order dynamics over E are generated by the following grammar:

$$S ::= A \rightarrow A \mid M \blacktriangleright [S] \mid M \triangleleft [S] \qquad M \subseteq E$$

$$A ::= e \qquad e \in E$$

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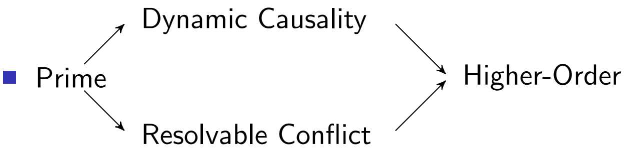
Conclusion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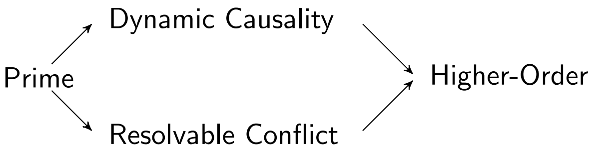
- 

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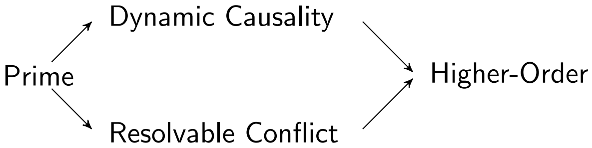

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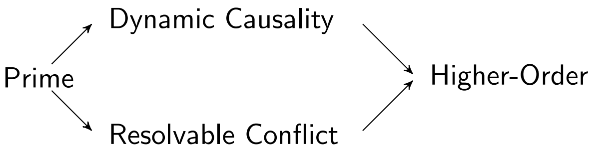
- 

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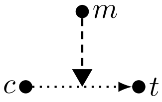
- If the events a and b may happen concurrently, then they may also happen in either order.
- In *true concurrency* semantics, as opposed to *interleaving* semantics, events a and b may happen at the same time.

Asymmetric Concurrency



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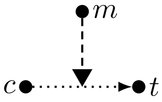

- Up to now:
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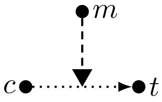

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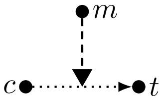

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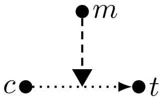

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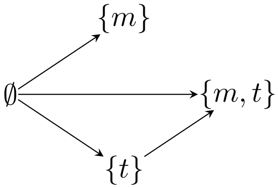

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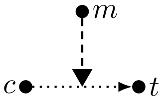

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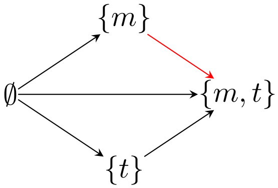

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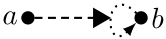

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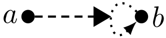

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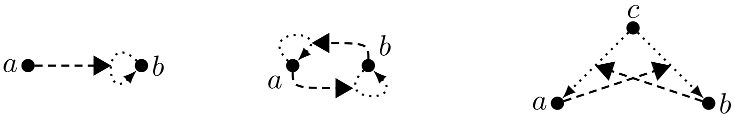

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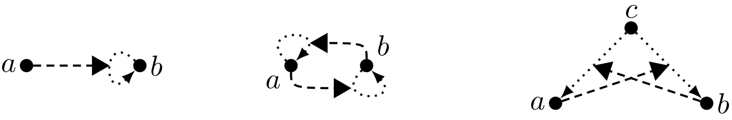

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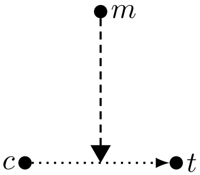
- Resolvable conflict
  becomes ‘concurrent or resolvable conflict’.

Suitable semantics ?!

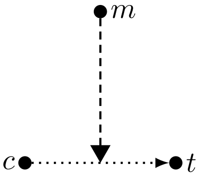


- No traces

tm is a trace, but mt is not.

Because m and t can occur concurrently, traces arent suitable.

Suitable semantics ?!

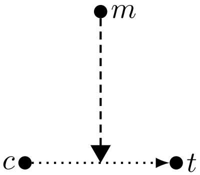


- No traces
- No families of configurations

Because $\{m, t\}$ and $\{m\}$ are configurations, there would be a transition from $\{m\}$ to $\{m, t\}$.

Therefore families of configurations are not suitable.

Suitable semantics ?!

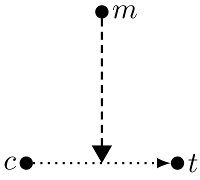


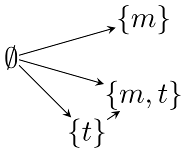
- No traces
- No families of configurations
- No families of posets

No poset can express the causal relation between m and t.

Therefore families of posets are not suitable.

Suitable semantics





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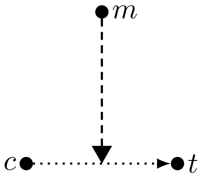
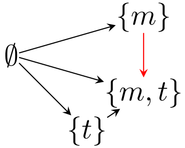
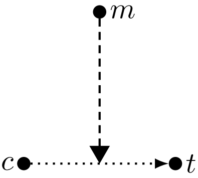


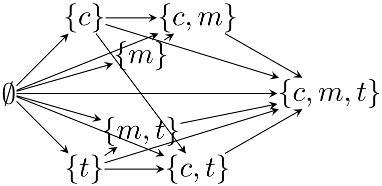


Dynamic Causality

YA, DSK, UN, KP

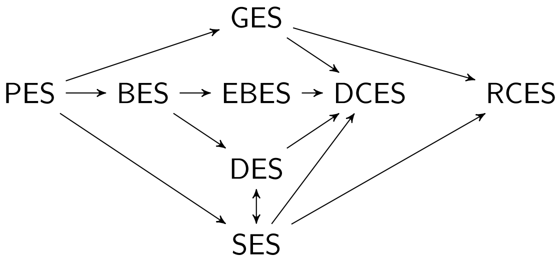
Suitable semantics





- Transitions on configurations!

Landscape revisited



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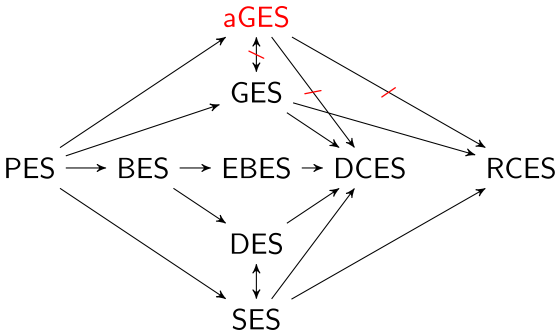

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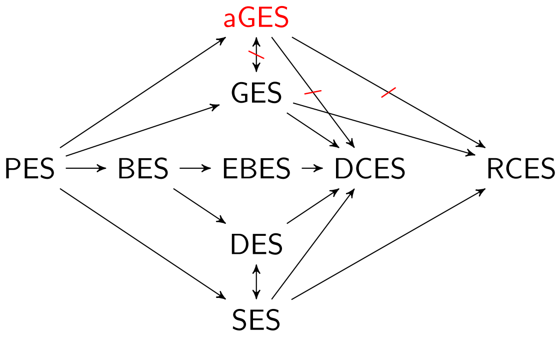

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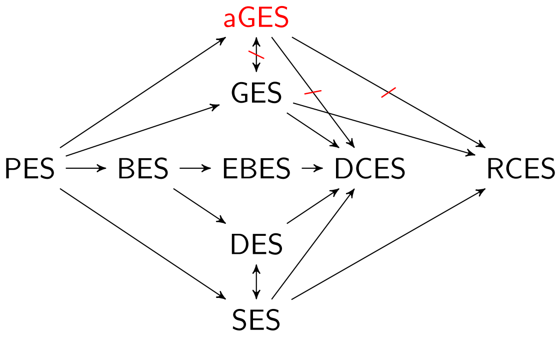
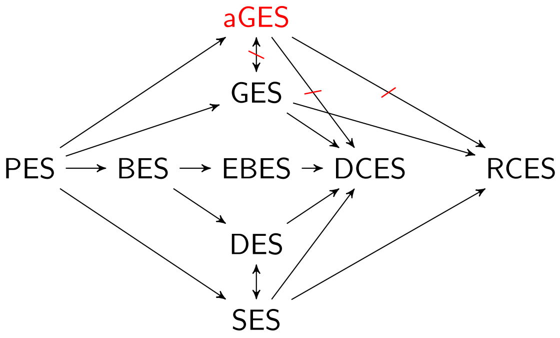


- Any more properties to be proved?
- What is the ‘supremum’?

Landscape revisited



- Any more properties to be proved?
- What is the ‘supremum’?
- Similarities in other concurrency models?