Proving Safety of Traffic Manoeuvres on Country Roads

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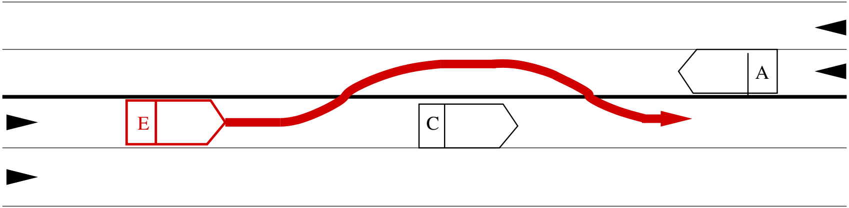
The Challenge

Prove safety (collision freedom) of
traffic on country roads including overtaking:



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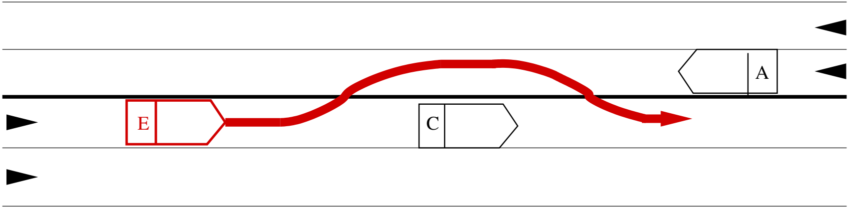

Hybrid system verification problem:

car dynamics + car controller(s) + assumptions $\models$ safety

Our Approach

Abstract model of multi-lane road traffic
based on spatial properties hiding car dynamics.

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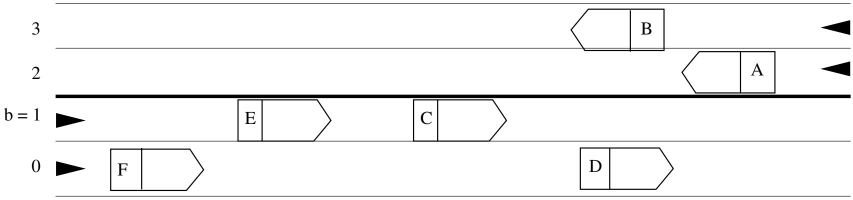
Properties expressed in a Multi-Lane Spatial Logic inspired by:

- Moszkowski's interval temporal logic [Mos85]
- Zhou, Hoare and Ravn's Duration Calculus [ZHR91]
- Schäfer's Shape Calculus [Sch07]

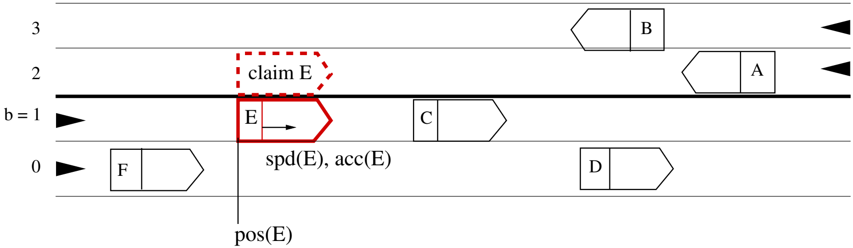
Model



Preliminaries:

- Car identifiers globally unique: $A, B, \ldots$
  Set of all car identifiers: $\mathbb{I}$
- Infinite road ($\mathbb{R}$)
- Lanes: $\mathbb{L} = \{0, \ldots, N\}$
  Border: $b \in \mathbb{L}$

Model



A traffic snapshot is a structure $\mathcal{TS} = (res, clm, pos, spd, acc)$, where

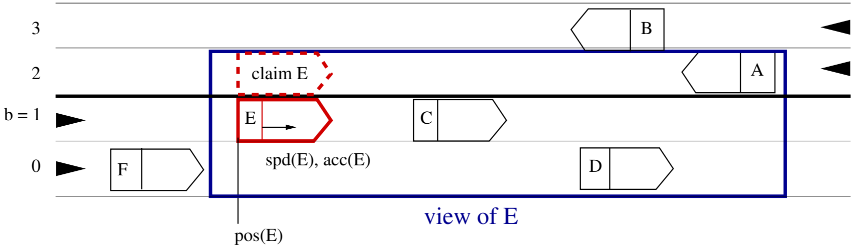
- $res : \mathbb{I} \to \mathcal{P}(\mathbb{L})$ reserved lanes,
- $clm : \mathbb{I} \to \mathcal{P}(\mathbb{L})$ claimed lanes,
- $pos : \mathbb{I} \to \mathbb{R}$ car positions,
- $spd : \mathbb{I} \to \mathbb{R}$ current speeds,
- $acc : \mathbb{I} \to \mathbb{R}$ current accelerations.

Transitions

$\mathcal{TS} \xrightarrow{\alpha} \mathcal{TS}'$ for an action α of the following type:

$\mathcal{TS} \xrightarrow{t} \mathcal{TS}'$	time passes
$\mathcal{TS} \xrightarrow{\mathsf{acc}(C,a)} \mathcal{TS}'$	accelerate
$\mathcal{TS} \xrightarrow{\mathsf{c}(C,n)} \mathcal{TS}'$	claim
$\mathcal{TS} \xrightarrow{\mathsf{wd\ c}(C)} \mathcal{TS}'$	withdraw claim
$\mathcal{TS} \xrightarrow{\mathsf{r}(C)} \mathcal{TS}'$	reserve
$\mathcal{TS} \xrightarrow{\mathsf{wd\ r}(C,n)} \mathcal{TS}'$	withdraw reservation

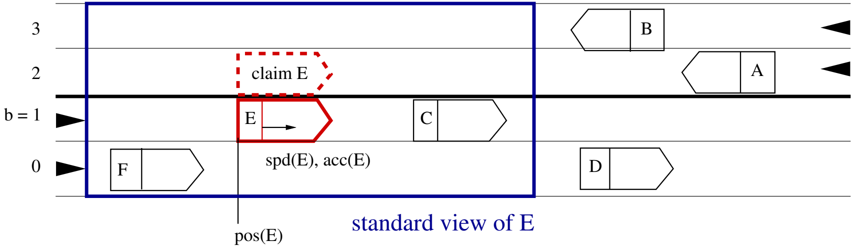
Local View



View $V = (L, X, E)$, where

- $L = [m, n]$ subinterval of $\mathbb{L}$,
- $X = [r, t]$ subinterval of $\mathbb{R}$,
- $E \in \mathbb{I}$ identifier of car under consideration.

Local View

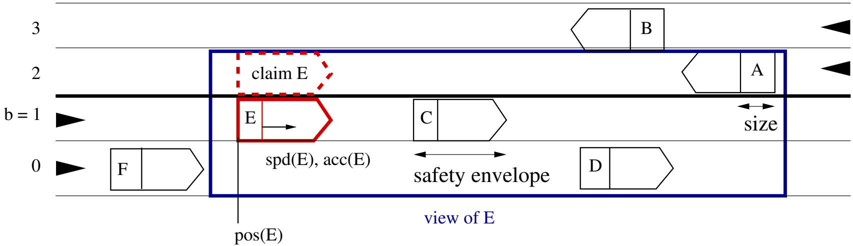


Standard view $V_s = (L, X, E)$, where

- $L = \mathbb{L}$,
- $X = [pos(E) - h, pos(E) + h]$,
- $E \in \mathbb{I}$,

and h is the horizon.

Sensor Function



Sensor function covering directions:

$$\Omega_E : \mathbb{I} \times \mathbb{TS} \to \mathbb{R}$$

e.g., perfect knowledge

$$\Omega_E(I, \mathcal{TS}) \equiv se(I, \mathcal{TS}).$$

Syntax: MLSL + ℓ

Multi-Lane Spatial Logic with length measurements:

- Car variables: c, d, special variable ego
- Real variables: x, y

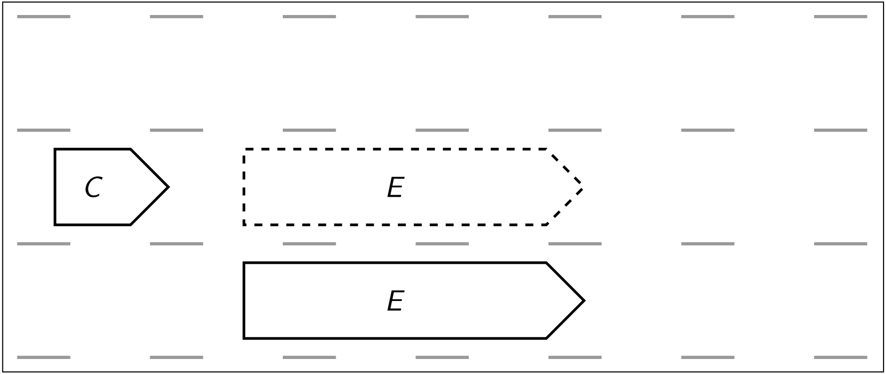
Real-valued terms θ

$$\theta ::= r \mid x \mid f(c_1, \ldots, c_n) \mid g(\theta_1, \ldots, \theta_n),$$

Formulae ϕ

$$\begin{aligned} \phi ::= \; & \mathit{true} \mid c = d \mid \ell = \theta \mid \mathit{free} \mid \mathit{re}(c) \mid \mathit{cl}(c) && (\mathit{Atoms}) \\ & \mid \phi_1 \land \phi_2 \mid \neg \phi_1 \mid \exists c \colon \phi_1 && (\mathit{FOL}) \\ & \mid \phi_1 \frown \phi_2 \mid \begin{matrix} \phi_2 \\ \phi_1 \end{matrix} && (\mathit{Spatial}) \end{aligned}$$

Semantics



$$\phi \equiv \begin{pmatrix} \mathit{true} \\ \mathit{free} \frown \mathit{re}(\mathrm{ego}) \frown \mathit{free} \end{pmatrix}$$

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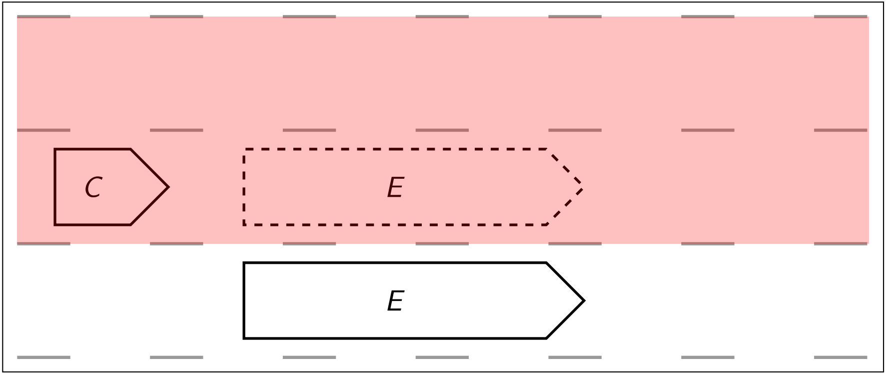

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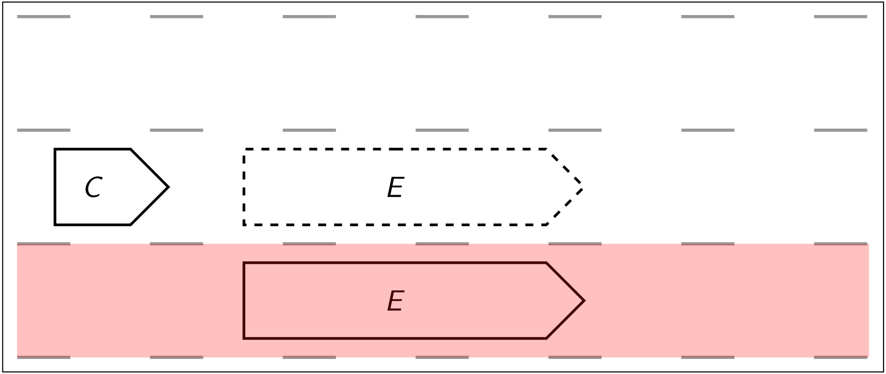

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Example: Collision Check

Somewhere: $\langle \phi \rangle \equiv true \frown \begin{pmatrix} true \\ \phi \\ true \end{pmatrix} \frown true$

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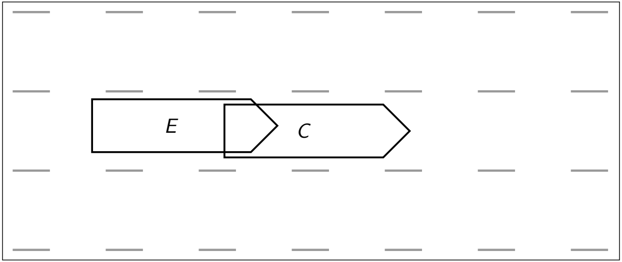

$$\langle re(\mathrm{ego}) \land re(c) \rangle$$

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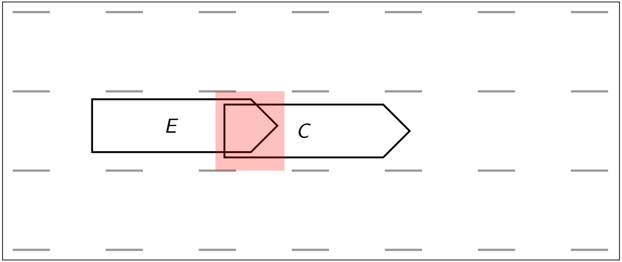

$$\langle re(\mathrm{ego}) \wedge re(c) \rangle$$

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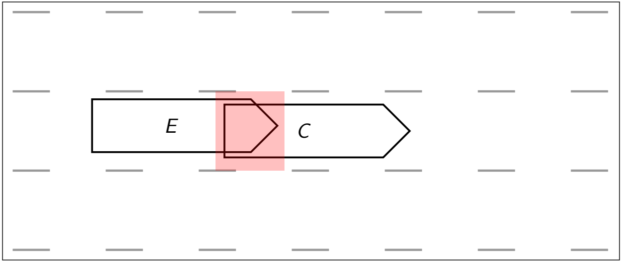

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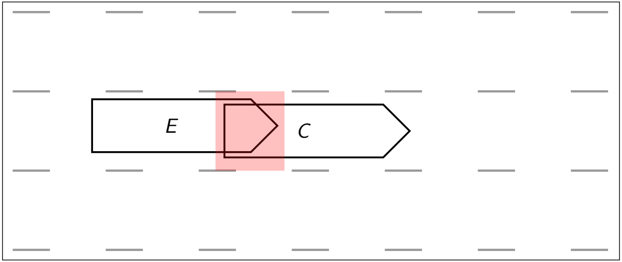

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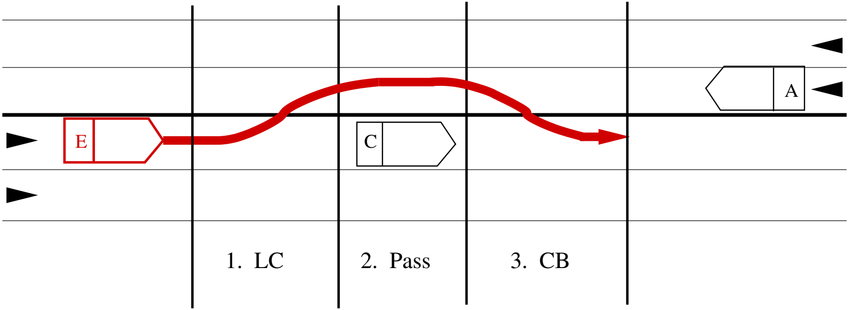
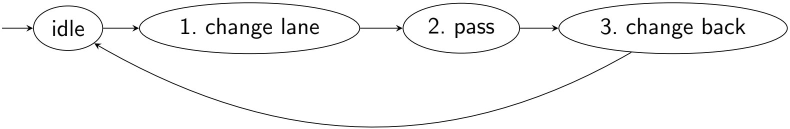
Safety from ego's perspective: $\neg cc$

Controller: General Idea

- Perfect knowledge, i.e.,
  sensors return full safety envelopes of all cars
- Instantaneous broadcast communication
- Timed automaton with data variables:
  - variable n: original lane,
  - variable ℓ: target lane,
  - clock x,
  - guards and invariants:
    MLSL formulae and clock/data constraints,
  - actions:
    transitions of cars, clock/data updates.

Protocol for Overtaking





Aim: Safety of Overtaking

A traffic snapshot safe if it satisfies

$$Safe \equiv \forall c, d : c \neq d \Rightarrow \neg \langle re(c) \wedge re(d) \rangle .$$

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Assumptions:

A1. There is an initial safe traffic snapshot.

A2. Every car C is equipped with a distance controller.

A3. Every car is equipped with a contoller implementing the protocol for overtaking.

A4. The horizon in the standard view is sufficiently large.

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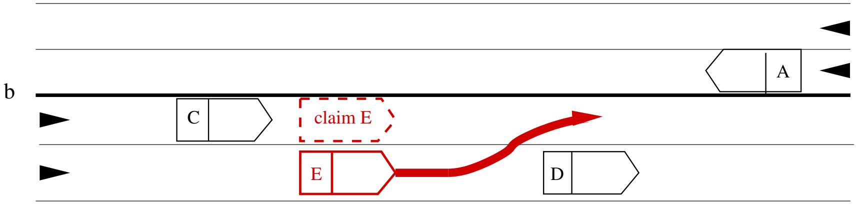
Safety of Overtaking

Under the assumptions A1 to A4,
the protocol specifying the overtaking procedure is safe.

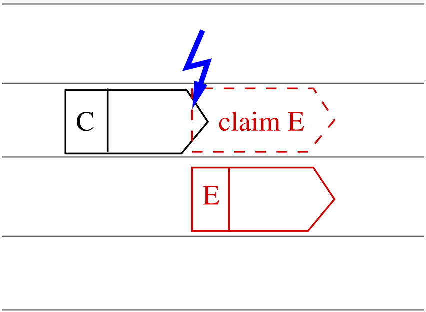
Lane Change on Non-Borders



Relevant traffic in one direction as on motorways: [HLOR11]

Lane Change on Non-Borders

- Potential collision: $pc \equiv \exists c : c \neq \mathrm{ego} \land \langle re(c) \land cl(\mathrm{ego}) \rangle$



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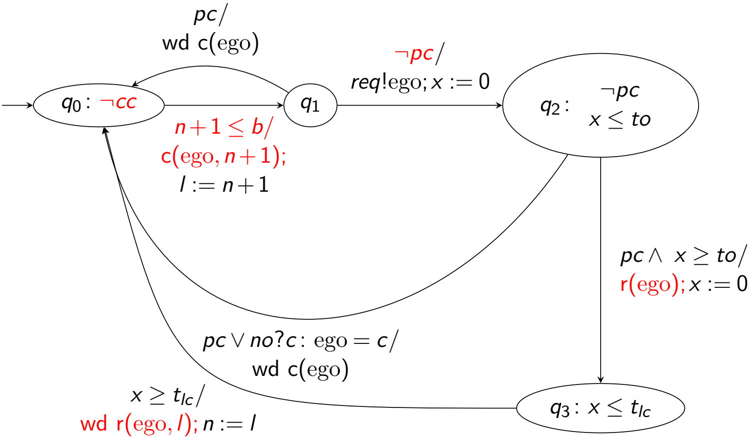
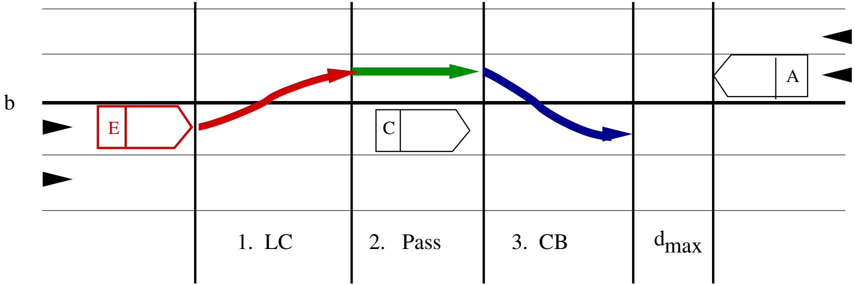


Lane Change into Opposing Traffic



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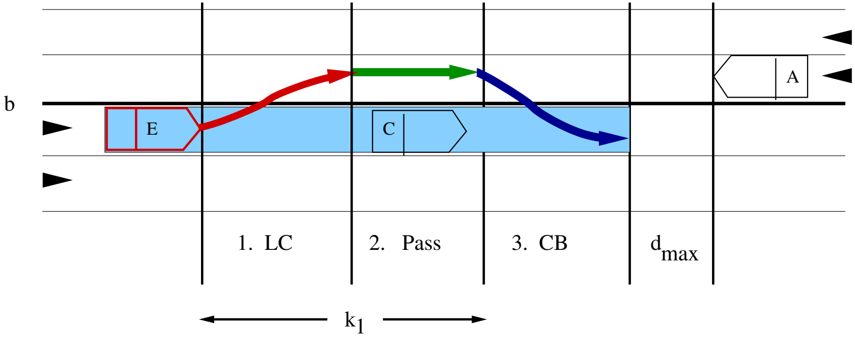
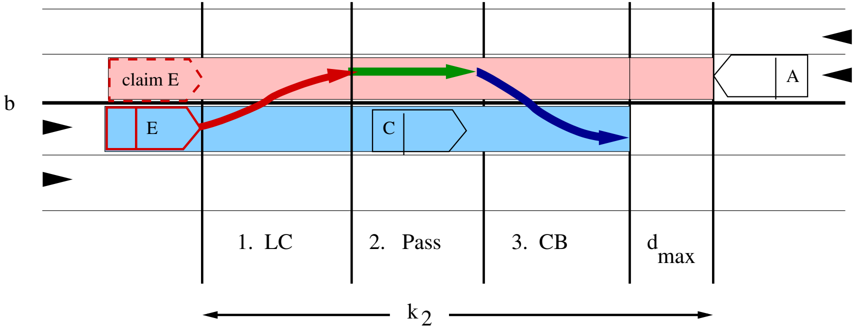


- Enough space on original lane:

$$esol(c) \;\equiv\; \langle re(\mathrm{ego}) \frown (free \frown re(c) \frown free)^{k_1} \frown free^{d_{lcb}} \rangle$$

where $\phi^{\theta} \;\equiv\; \phi \wedge \ell = \theta$.

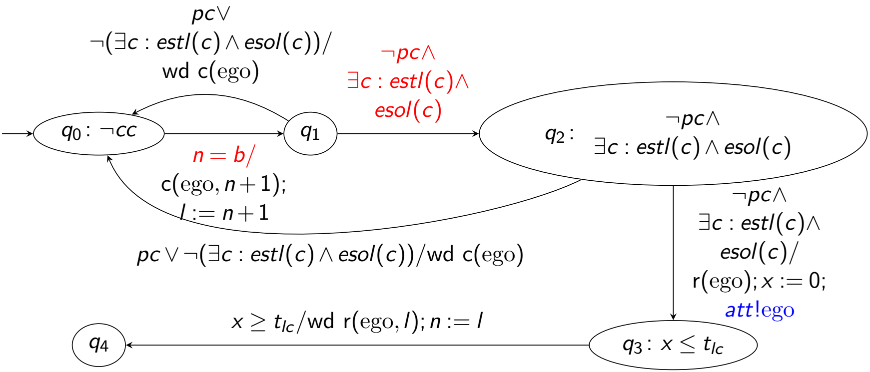
Lane Change into Opposing Traffic



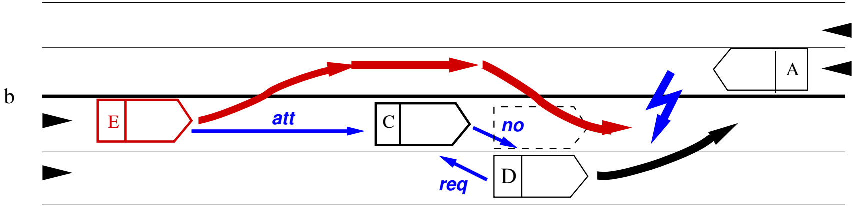
- Enough space on target lane:

$$estl(c) \equiv \langle cl(\mathrm{ego}) \frown free^{k_2} \rangle$$

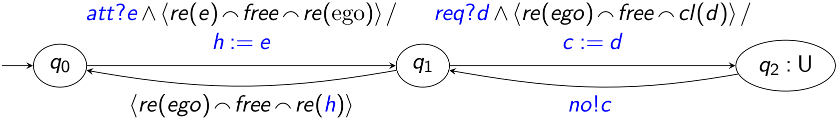
Lane Change into Opposing Traffic



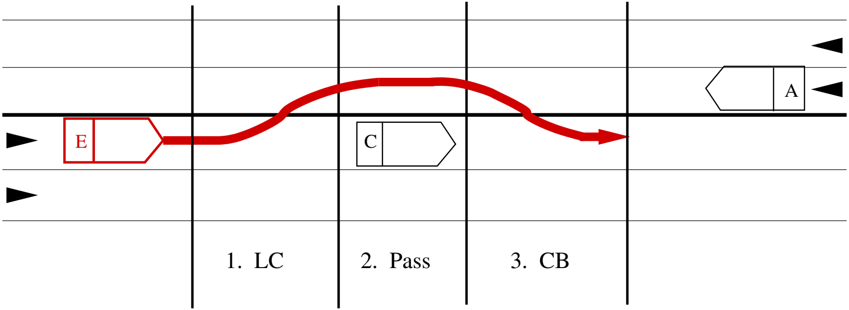
Additional Helper Controller



Inside car C in the role of ego:



Pass and Change Back



Simple controllers for phases 2 and 3

Safety of Overtaking

A traffic snapshot $\mathcal{TS}$ safe if it satisfies

$$Safe \equiv \forall c, d : c \neq d \Rightarrow \neg \langle re(c) \wedge re(d) \rangle .$$

Assumptions:

A1. There is an initial safe traffic snapshot $\mathcal{TS}_0$.

A2. Every car C is equipped with a distance controller that keeps *Safe* invariant under time and accelaration transitions.

A3. Every car is equipped with a contoller implementing the protocol for overtaking.

A4. The horizon in the standard view is $h = se_{max} + 2 \cdot d_{max}$.

Theorem (Safety of Overtaking)

Under the assumptions A1 to A4,
the protocol specifying the overtaking procedure is safe.

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Under the assumptions A1 to A4, the protocol specifying the overtaking procedure is safe.

Proof idea.

We show that every traffic snapshot $\mathcal{TS}$ that is reachable from $\mathcal{TS}_0$ by time and acceleration transitions and transitions allowed by the controller implementing the overtaking protocol is safe.

Future and Further Work

- Here: perfect knowledge

  Next: limited sensor functions, where
  each car E sees own safety envelope and the size of other cars:

  $$\Omega_E(I, \mathcal{TS}) \equiv \text{if } I = E \text{ then } se(I, \mathcal{TS}) \text{ else } size(I) \text{ fi}.$$

  Done for motorways in [HLOR11].

- Link to car dynamics

- Proof theory of MLSL: see [LH13] at ICTAC 2013, towards automatic verification

- Visual specification: S. Linker

Related Work

California PATH project: car platoons including lane change

- Lygeros et al. [LGS98]: sketch of safety proof taking car dynamics into accout, admitting *safe collisions*.

Lane Change Manoeuvres

- Platzer et al. [LPN11]: Quantified differential dynamic Logic QdL expresses car dynamics and creation of new cars.

Controller design for hybrid systems

- Raisch et al. [MRD03]: abstraction and refinement for hierarchical design of hybrid control systems.
- Van Schuppen et al. [HCvS06]: synthesis of control laws for piecewise-affine hybrid systems based on simplices.

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