Synthesis of Tissue Systems

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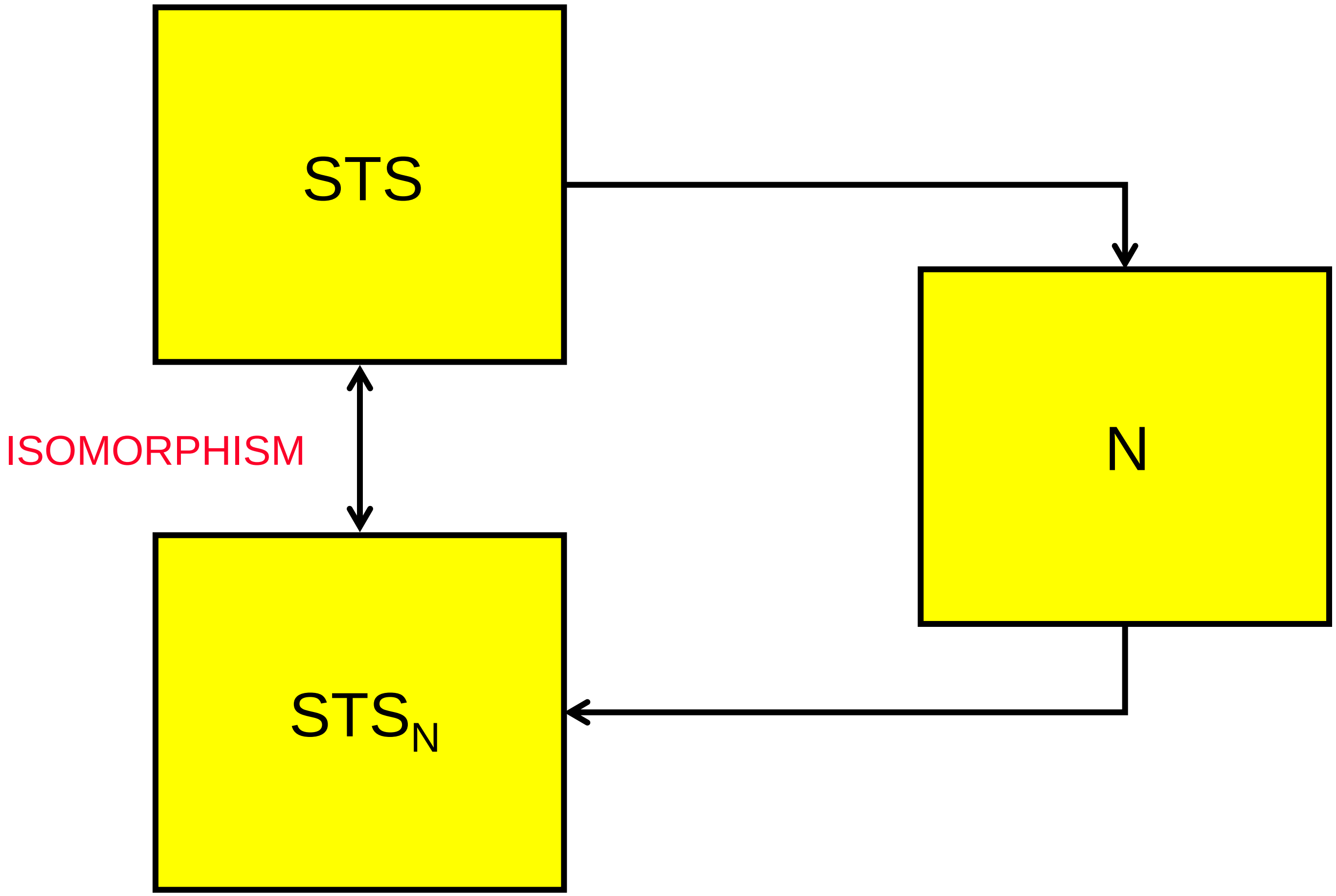
Synthesis technique: our focus

- Semantics based on steps: groups of transitions executed simultaneously
- Steps are constrained through firing policies: maximal parallelism, priorities, energy, etc
- Construction of Petri nets from step transition systems
- Foundations of such synthesis developed in

  [Ph.Darondeau, M.K, M.P-K, A.Yakovlev, 2008/09]

Relevant synthesis problem



PTL-nets: our nets with firing policy

PTL–net (PT-net with localities)

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PT-net (P,T,W,M_0)

+

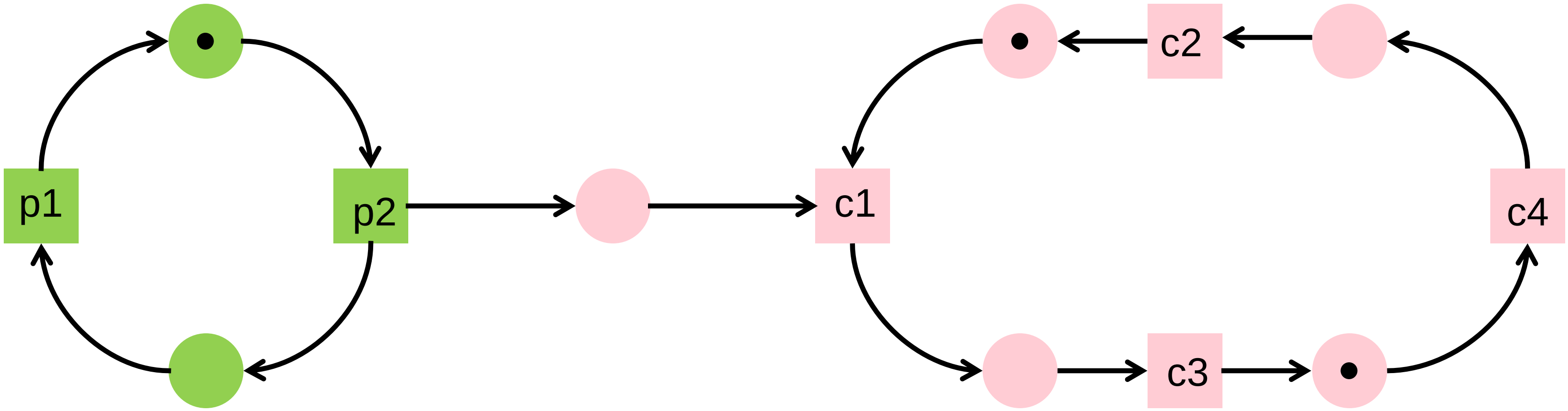
co-location relation ≏ for transitions and places

- If x ≏ z then x and z are co-located
- Each locality identifies a group of transitions executed in a maximally concurrent manner
- The behaviour is given by step transition system CRG

Locally maximal firing policy: lmax

Step sequences for one producer and two co-located consumers

- {p2}{c1} is *illegal*: lmax violated by {c1}
- {p2}{c1, c4} & {p2}{c1, c4, p1} are *legal*



Applications of PTL-nets

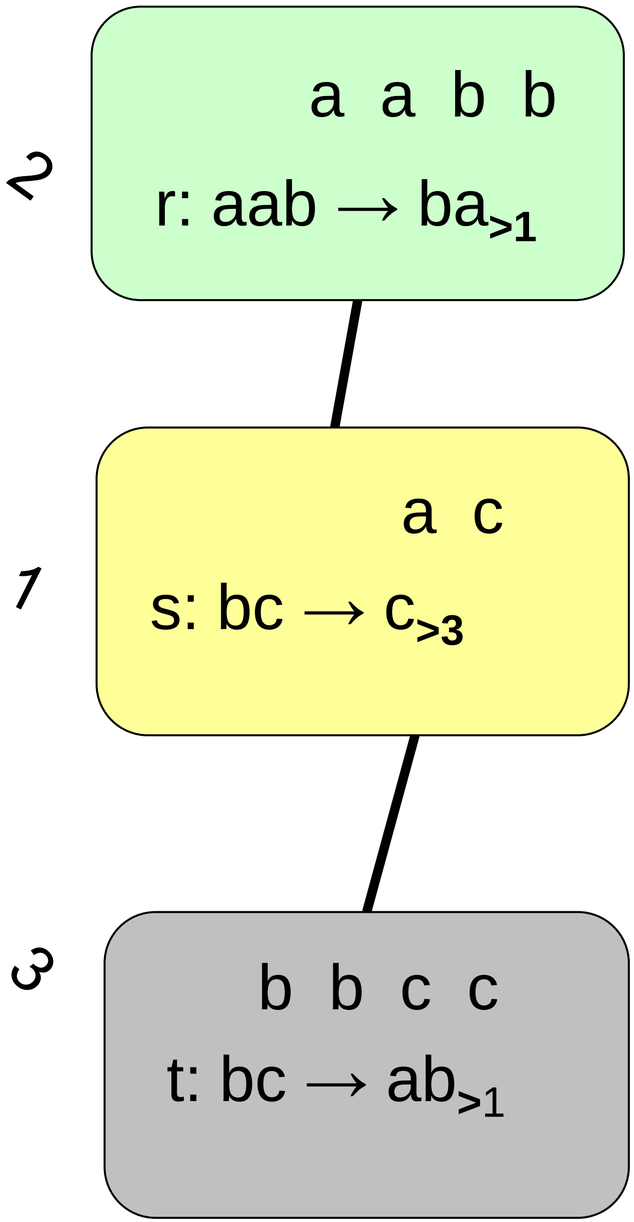
- PTL-nets can model globally asynchronous locally synchronous systems (GALS)

- Examples:
  - VLSI chips with multiple clocks for synchronisation of different subsets of gates
  - Membrane systems modelling cells inside which reactions are carried out in co-ordinated pulses
  - Tissue systems

Tissue systems

- Formal computational model inspired by compartments and functionality of living cells
- Biochemical reactions take place in compartments
- Compartments are determined by the structure of membranes (can be porous)
- Biochemical reactions represented by rewriting rules

Tissue systems



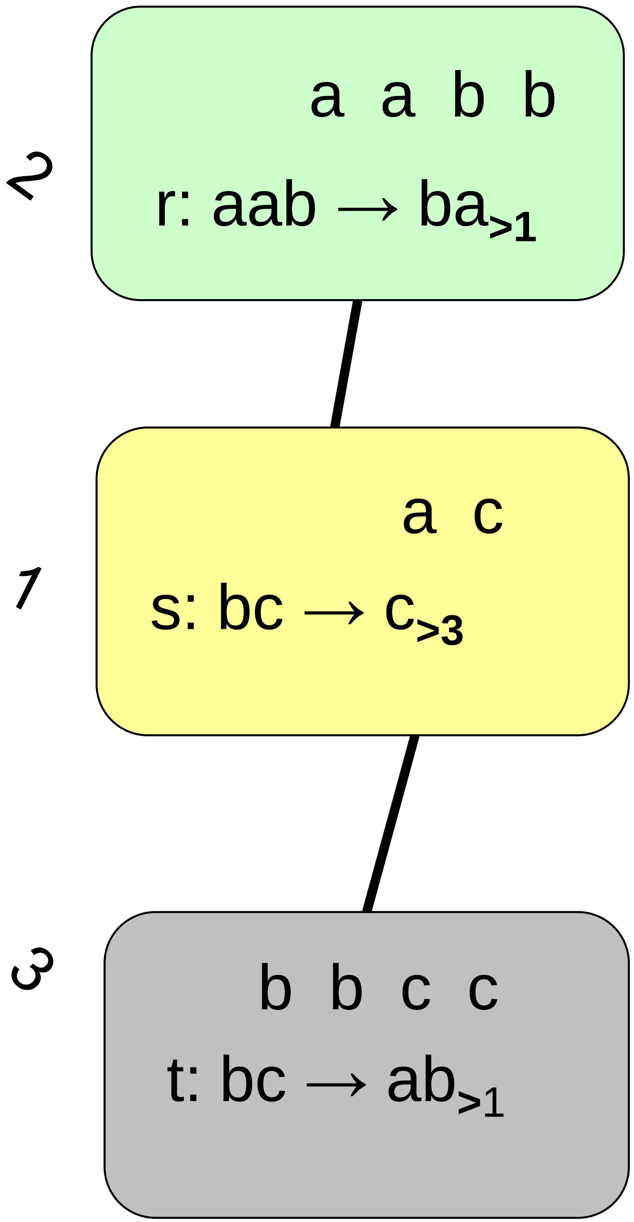
alphabet
tissue structure
initial objects
evolution rules
effect of individual occurrence

fixed structure

no exchange of objects with the external environment

...

Executed steps: multisets of rules

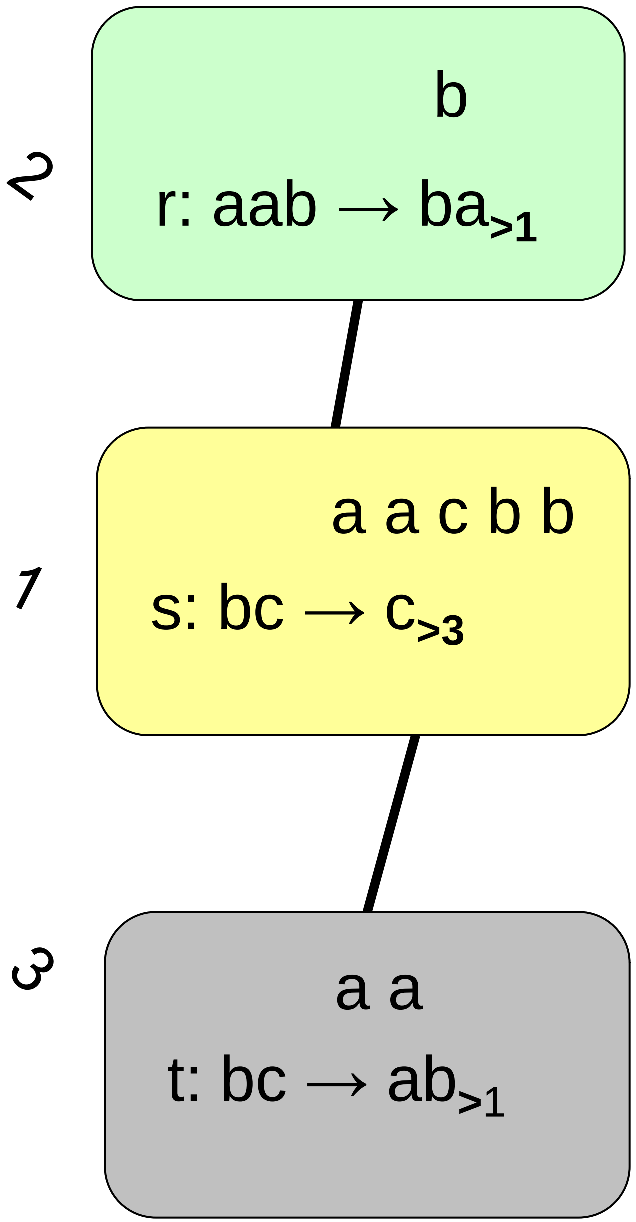


Finite and infinite sequences of executed steps

{r,t} is illegal

{r,t,t} is legal and leads to

Executed steps: multisets of rules



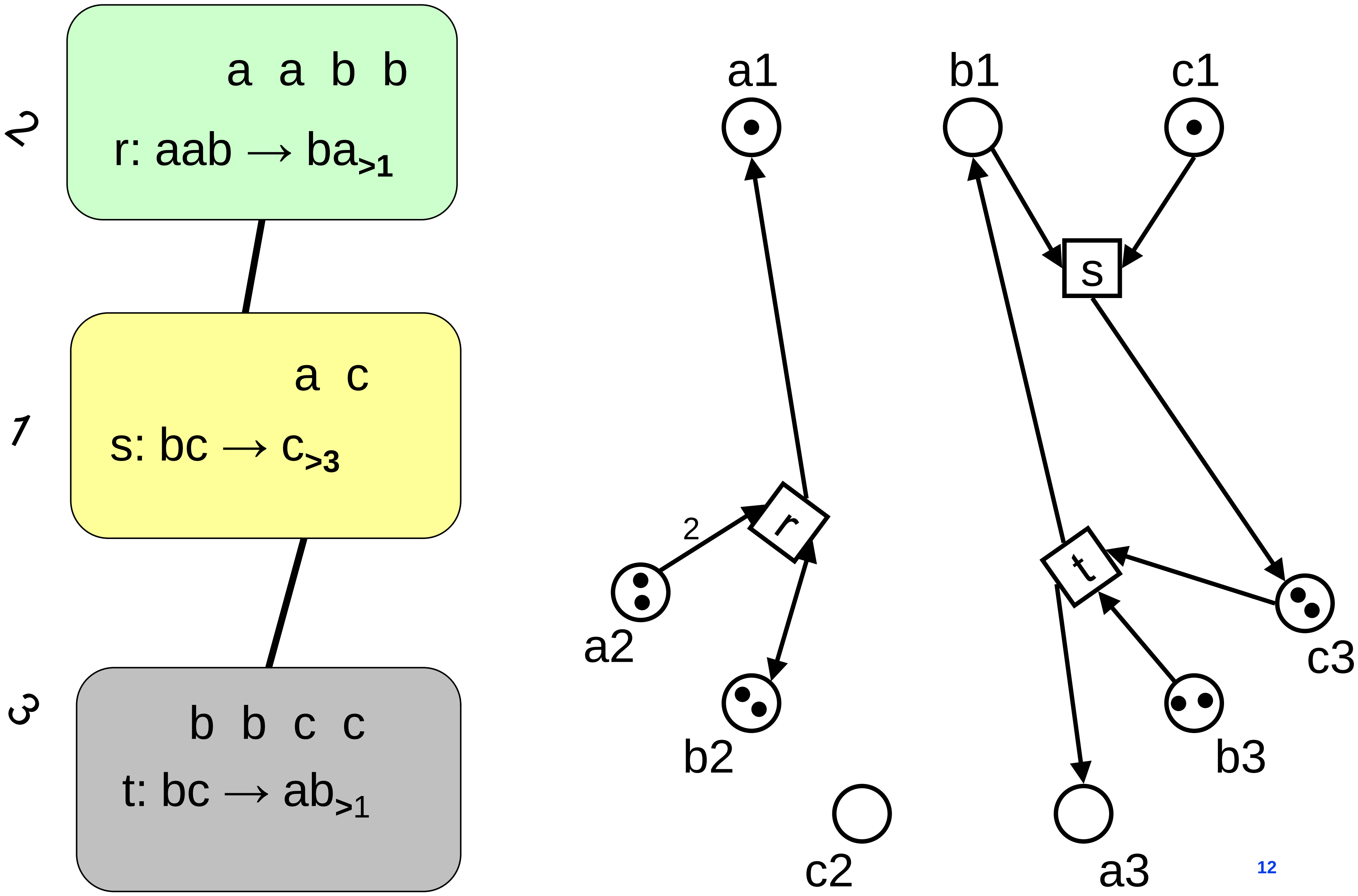
Finite and infinite sequences of executed steps

{r,t} is illegal

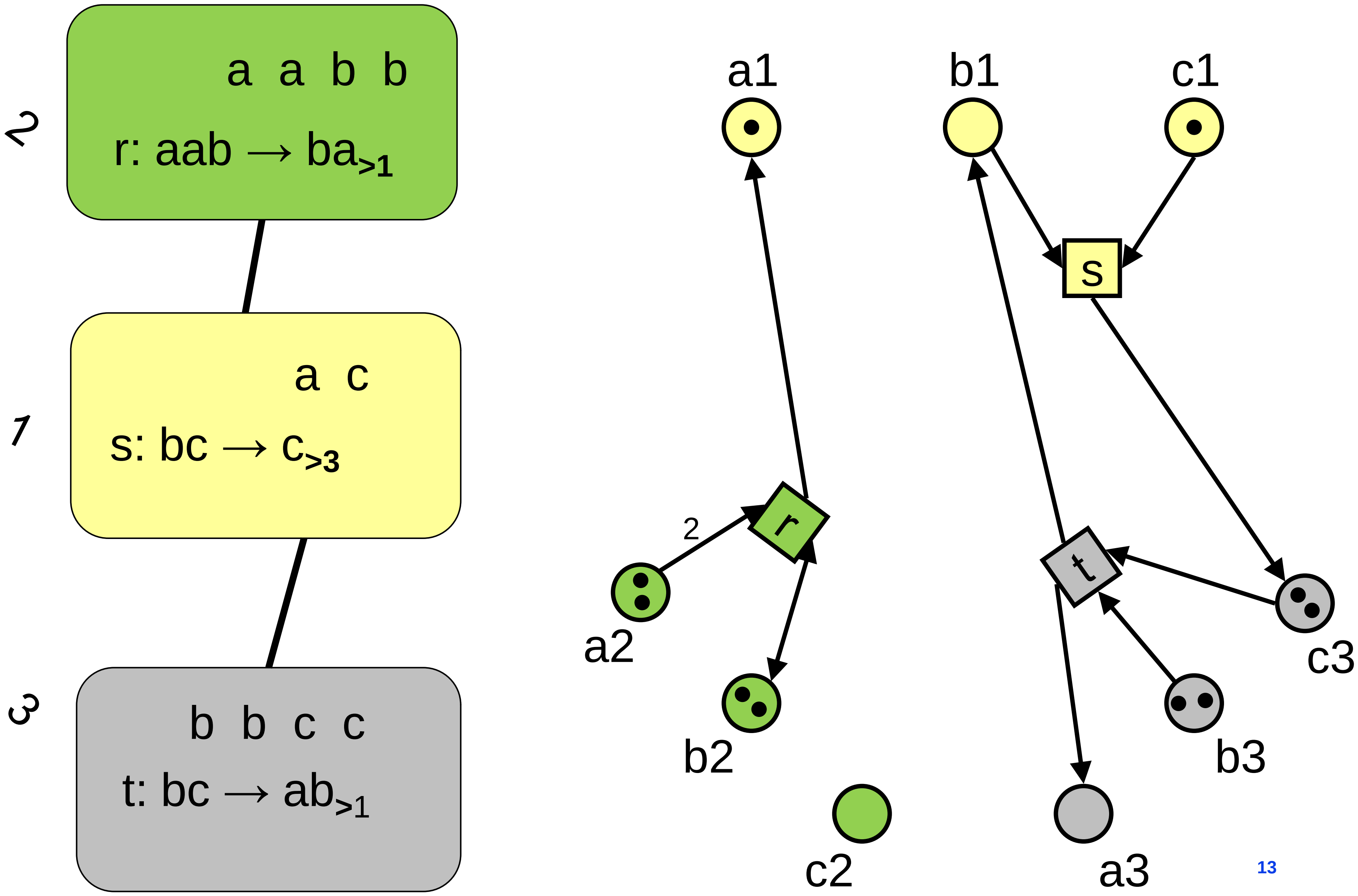
{r,t,t} is legal and leads to

Tissue systems and PTL-nets

Tissue system as PTL-net



Tissue system as PTL-net



Tissue system as PTL-net

PTL-net is spanned over the tissue structure:

- input only from the same locality
- output to the same locality or the neighbours

local resource	corresponds to	token in place
rule	corresponds to	transition

lmax executions of tissue system and the corresponding PTL-net generate isomorphic step transition systems

Synthesis of tissue systems

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synthesis of PTL-nets spanned over tissue structures

Step transition systems

- Behavioural model for PTL-nets
- A non-empty, finite set of states (Q)
- A finite set of transitions/arcs (A)
- An initial state (q_0)
- The arcs (which depict transitions) are labelled by multi-sets of (net) transitions executed concurrently
- $allSteps_q$ is the set of all steps labelling arcs outgoing from q
- $minSteps_q$ is the set of all non-empty steps from $allSteps_q$ which do not strictly include any other non-empty steps
- T_q is the set of all (net) transitions occurring in the steps of $allSteps_q$
- $\triangleq_q$ is the restriction of a co-location relation $\triangleq$ to $T_q x T_q$

Synthesis problem A

INPUT

finite step transition system STS with transitions T

tissue structure τ

co-location relation $\doteq$ for T

OUTPUT (if possible)

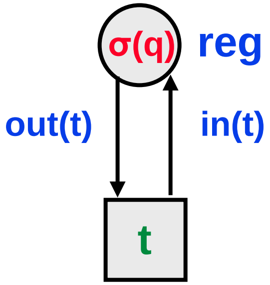
finite PTL-net N:

- N spanned over τ and respecting $\doteq$
- STS isomorphic to CRG(N)

Synthesis problem A

A region is

$$reg=(\sigma: Q \rightarrow \mathbb{N}, in: T \rightarrow \mathbb{N}, out: T \rightarrow \mathbb{N})$$



such that, for every arc (q,α,q') of STS

$$\sigma(q) \geq out(\alpha) \text{ and } \sigma(q') = \sigma(q) - out(\alpha) + in(\alpha)$$

reg with locality i_{reg} is compatible with tissue structure τ if:

- out(t)>0 implies that t and reg are co-located
- in(t)>0 implies that t and reg are co-located or located in neighbour localities

Only compatible regions can be places in synthesised net!

Synthesis problem A

- Regions are used to check the feasibility of the synthesis problem and to construct PTL-net

- Regions can be computed [Chernikova 1965] as integer solutions $p = x_0 \ldots x_m y_1 \ldots y_n z_1 \ldots z_n$ of the following system:

  $x_i \geq \alpha.z$
  $x_j = x_i + \alpha.(y-z)$ for all arcs (q_i,α,q_j) in STS

  where $\sigma(q_i) = x_i$ and $in(t_j) = y_j$ and $out(t_j) = z_j$

- Each compatible solution p of the system above can be expressed as a non-negative linear combination of some k basic compatible integer solutions (rays):

$$p = r_1.p^1 + \ldots + r_k.p^k$$

Synthesis problem A

For STS to be synthesisable it needs to satisfy:

- State separation checked for states q_i and q_j as follows:

  Is there any ray $x_0^b \ldots x_m^b y_1^b \ldots y_n^b z_1^b \ldots z_n^b$ such that $x_i^b \neq x_j^b$?

- Forward closure checked for every state q_i as follows:

  Is there a ray such that $x_i^b - \alpha . z^b < 0$?
  The answer Yes means: α is not a region enabled step at q_i

  We check whether the steps on outgoing arcs are exactly region enabled steps α for which there is no transition t (co-located with some transition from α) such that $\alpha + t$ is region enabled at q_i

What if the tissue structure is not known?

Synthesis problem B

INPUT

finite step transition system STS with transitions T

OUTPUT (if possible)

finite PTL-net N and tissue structure τ:

- N spanned over τ
- STS isomorphic to CRG(N)

What if the tissue structure is not known?

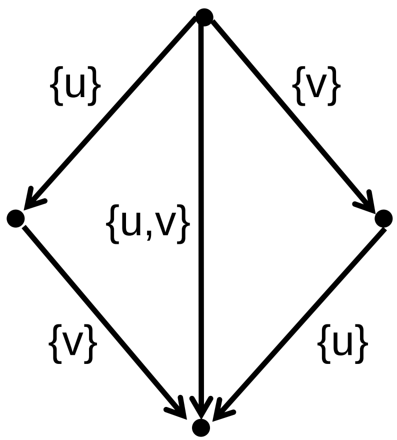
The number of co-location relations for n transitions is finite – Perhaps we can try them all ?   very expensive!

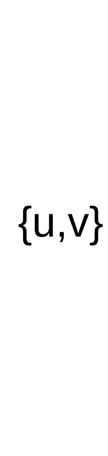
Perhaps we can deduce potential tissue structures and reduce the number of cases?

In any case we can assume that the tissue structure is a clique!

Can we deduce $\bumpeq$ from STS in general?

YES



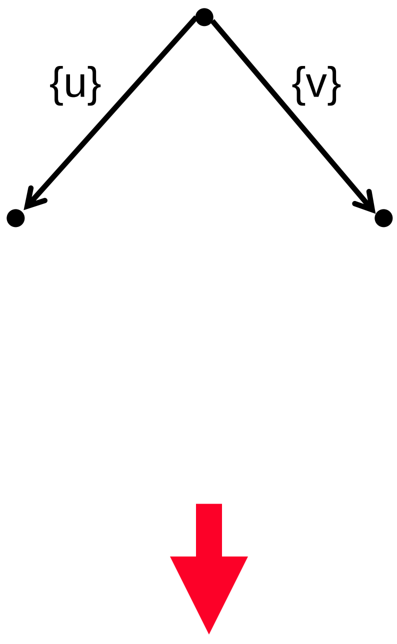


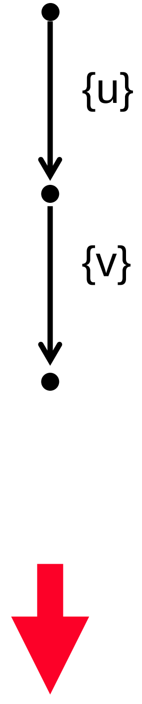
$u \not\bumpeq v$

$u \bumpeq v$

Can we deduce $\hat{=}$ from STS in general?

NO



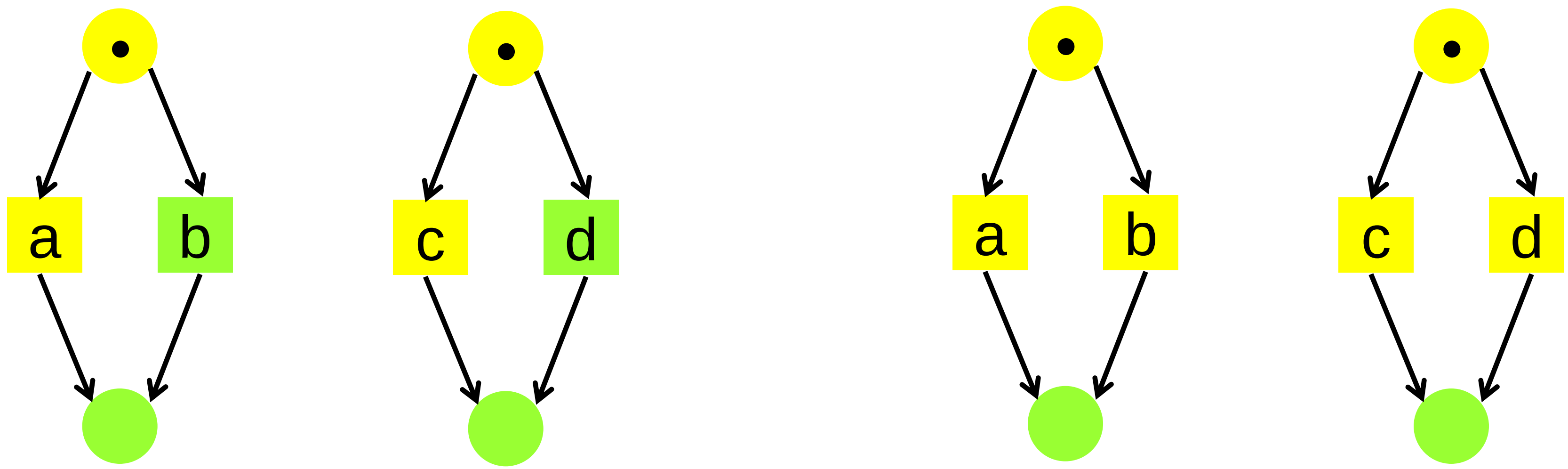


u ? v

u ? v

Can we deduce ≏ from STS in general?

PTL-nets generating the same STS



It is hard to guess co-location in the presence of conflicts!

But conflicts in PTL-nets spanned over tissue structures are restricted !

PTL-nets spanned over tissue structures

A key result for PTL-nets spanned over tissue structures

two transitions enabled at marking M are co-located

iff

either there is no step lmax-enabled at M containing them

or there is a minimal step lmax-enabled at M containing them

Hence a unique co-location relationship $\hat{=}_q$ exists for transitions occurring in steps labelling arcs outgoing from a node q of STS!

Synthesis Problem B

- Compute $\triangleq_q$ for all states q of STS
- Form the transitive closure $\triangleq_{ok}$ of their union
- Check whether each $\triangleq_q$ is equal to projected $\triangleq_{ok}$
  - No: Synthesis Problem B fails
  - Yes: run Synthesis Problem A for all $\triangleq$ containing $\triangleq_{ok}$
- If the state separation and forward closure are satisfied for any such $\triangleq$ then the construction succeeds
- We can further restrict ourselves to the largest (in terms of set inclusion) relations $\triangleq$
- Finding the largest $\triangleq$ relations is related to the minimum clique cover problem

Future work

- Criteria for reducing the number of relations ♎ one needs to consider

- Constraining the number and/or topology of connections in the synthesised tissue structure

- Other synthesis problems for tissue systems: scenarios, etc

Thank you!