On Properties of Higher-Order Languages

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Our Recent Research

(joint work with C. Broadbent, A. Igarashi, K. Matsuda, R. Sato, A. Shinohara, T. Terauchi, T. Tsukada, H. Unno, ...)

- Theory and Practice of Higher-Order Model Checking (model = higher-order grammar)
  - properties of higher-order grammars [LICS13]
  - higher-order model checking algorithms [JACM 13, FoSSaCS11,CSL13, ESOP13, APLAS13]
  - automated verification of higher-order programs [JACM 13, PLDI11,FLOPS12,PEPM13,POPL13]
  - data compression (generalization of grammar-based approach) [PEPM12]

Our Recent Research

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  - properties of higher-order grammars [LICS13]
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  - data compression (generalization of grammar-based approach) [PEPM12]

This Talk

- A survey on higher-order (formal) languages
  - what are higher-order languages?
  - solved/open problems
- Applications of λ-calculus and types to studies of higher-order languages
  - Pumping lemma for higher-order recursion schemes (HORS) [K, LICS13]
  - Context-sensitiveness (ongoing work)

Outline

- Background
  - What are higher-order languages and what are they for?
  - Some variations
    - deterministic/non-deterministic, safe/unsafe, OI/IO, word/tree
- Solved/open problems
  - decision problems
  - language hierarchy
- Applications of typed λ-calculus

Higher-Order Grammars
[Maslov74,Wand75,...]

- Extension of CFG where non-terminals take parameters (cf. macro grammar)

Example of order-1 grammar G_1

$S \to A\ e$

$A\ x \to x$

$A\ x \to a\ (A\ (b\ x))$

$S: o,\ A: o \to o$

$S \to A\ e \to a(A\ (b\ e)) \to a(b\ e)$

$L(G_1) = \{a^n b^n e \mid n \geq 0\}$

Higher-Order Grammars

- Extension of CFG where non-terminals take parameters

Example of order-2 grammar G_2	
$S \to A\ b$	$T\ f\ x \to f\ (f\ x)$
$A\ f \to f\ e$	$A\ f \to a\ (A\ (T\ f))$
S: o, A: (o→o)→o, T: (o→o)→o→o	

$$S \to A\ b \to a(A\ (T\ b)) \to^* a^n(A(T^n\ b)))$$

$$\to a^n(T^n\ b\ e) \quad \to^* a^n(b^{2^n} e)$$

$$L(G_2) = \{a^n b^{2^n} e \mid n \geq 0\}$$

Why Higher-Order Languages?

- **Semantics of programs**
  ("recursive program schemes" [Park68,Nivat72,...]**)**
- **Natural extension of Chomsky hierarchy**
  [Wand74,Damm82,..] **(order-0 = regular,**
  order-1 = context-free, order-2 = indexed)
- **Verification of higher-order programs**
  (higher-order grammars as natural models of
  functional programs [Knapik+02, Ong06, K09,...]**)**
  - **generalization of model checking approach to program verification (order-0 = finite state m.c., order-1 = pushdown m.c.)**

Classification of model checking

	model	corresponding inclusion problem (*)	software model checkers
finite state m.c.	automata, regular languages	regular $\subseteq$ regular	BLAST (for C)
pushdown m.c.	pushdown, context-free grammars	context-free $\subseteq$ regular	SLAM (for C)
higher-order m.c. [Knapik+02] [Ong06]	higher-order pushdown, higher-order grammars	higher-order $\subseteq$ regular	MoCHi (for ML) [K+ PLDI11]

(*) infinite words/trees may be considered

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Deterministic vs Non-Deterministic Grammars

- **Deterministic** (aka higher-order recursion schemes):
  - **exactly one rule** for each non-terminal
  - generates a **single** (possibly infinite) tree
    (word, if the arities of terminal symbols are at most 1)
  - models of higher-order model checking
    [Knapik+ 02,Ong06]
- **Non-deterministic:**
  - **an arbitrary number of rules** for each non-terminal
  - generates a **language of** (usually finite) trees
    (words, if the arities of terminal symbols are at most 1)
  - further classification based on evaluation order
    (OI and IO [Damm82])

Safe vs Unsafe Grammar

- safe (“derived types” [Damm82])
  - The arguments of non-terminals are sorted in the decreasing order of their type-theoretic orders
    - ✓ $((o\to o)\to o) \to (o\to o) \to (o\to o\to o) \to o \to o$

      order-2 order-1 order-0

order(o) = 0
order($\tau_1 \to \tau_2$)=max(order(τ_1)+1, order(τ_2))

Safe vs Unsafe Grammar

- safe ("derived types" [Damm82])
  - The arguments of non-terminals are sorted in the decreasing order of their type-theoretic orders

✓ $\underbrace{((o \to o) \to o)}_{\text{order-2}} \to \underbrace{(o \to o)}_{\text{order-1}} \to \underbrace{(o \to o \to o)}_{\text{order-1}} \to \underbrace{o}_{\text{order-0}} \to o$

✗ $\underbrace{(o \to o)}_{\text{order-1}} \to \underbrace{o}_{\text{order-0}} \to \underbrace{(o \to o)}_{\text{order-1}} \to o$

$$\mathrm{order}(o) = 0$$
$$\mathrm{order}(\tau_1 \to \tau_2) = \max(\mathrm{order}(\tau_1) + 1, \mathrm{order}(\tau_2))$$

Safe vs Unsafe Grammar

- safe
  - The arguments of non-terminals are sorted in the decreasing order of type-theoretic orders
    - ✓ $((o\to o)\to o) \to (o\to o) \to (o\to o\to o) \to o \to o$
    - ✗ $(o \to o) \to o \to (o \to o) \to o$
  - Arguments of the same order must be passed at the same time

Non-example:
S -> F h a a.   F z x y -> f (F (F z y) y (z x)) x.
(S:o, F: $(o\to o)\to o\to o \to o$, f:$o\to o\to o$, h:$o\to o$, a:o)

Safe vs Unsafe Grammar

- safe
  - The arguments of non-terminals are sorted in the decreasing order of type-theoretic orders
    - ✓ $((o \to o) \to o) \to (o \to o) \to (o \to o \to o) \to o \to o$
    - ✗ $(o \to o) \to o \to (o \to o) \to o$
  - Arguments of the same order must be passed at the same time
  - Targets of earlier studies [Damm82, Knapik+02,...]
  - Equivalent to higher-order pushdown automata [Maslov74,Knapik+02,...]
- unsafe
  - Without safety restriction
  - Targets of recent studies [Ong06,K09,...]
  - Equivalent to collapsible pushdown automata [Hague+08]

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  - language hierarchy
- Applications of typed λ-calculus

Some decision problems

- **Model Checking**
  - Input: deterministic HO grammar (HORS) G
    regular (or MSO-definable) language R
    (of infinite trees)
  - Output: Tree(G) $\in$ R?
- **Language inclusion**
  - Input: non-deterministic HO grammar G,
    regular language R (of finite words/trees)
  - Output: L(G)$\subseteq$ R?
- **Equivalence**
  - Input: deterministic HO grammars (HORS) G_1, G_2
  - Output: Tree(G_1) = Tree(G_2)?

Problem Status

	safe	unsafe
model checking	decidable [Knapik+02]	decidable [Ong06]
inclusion	decidable [Damm82?]	decidable
equivalence	open (for order ≥ 2)	open (for order ≥ 2)

Language hierarchies

- Language/tree classes
  - $LANG_n$ = {L(G) | G: order-n n.d. word grammar}
  - $TREE_n$ = {Tree(G) | G: order-n HORS}
- Some questions about language hierarchy
  - strictness of word language hierarchy
    $LANG_n \neq LANG_{n+1}$ for all n?
  - strictness of tree hierarchy
    $TREE_n \neq TREE_{n+1}$ for all n?
  - context-sensitiveness
    Is L(G) context-sensitive for all G?
  - Is safety a genuine restriction?
    $SafeLANG_n = UnsafeLANG_n$? $SafeTREE_n = UnsafeTREE_n$?

Problem Status

	safe	unsafe
strictness of language hierarchy	yes [Engelfriet91]	open
strictness of tree hierarchy	yes [Damm82?]	yes [Kartzow&Parys12]
context sensitiveness	yes [Inaba&Maneth08]	open (for order ≥ 3)
safe trees = unsafe trees?	no [Parys 12]	
safe languages = unsafe languages?	open	

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- Background
- Solved/open problems
- Our approach based on typed λ-calculus
  - motivation
  - pumping lemma
  - towards context-sensitiveness

Motivation

- Many results have been obtained through higher-order pushdown systems/transducers
  -> non-intuitive, complex proofs
- Simpler, more direct reasoning about grammars seems possible through λ-calculus and types

	safe	unsafe
strictness of language hierarchy	yes pushdown [Engelfriet91]	open
strictness of tree hierarchy	yes [Damm82?]	yes pushdown [Kartzow&Parys12]
context sensitiveness	yes pushdown [Inaba&Maneth08]	open (for order $\geq$ 3)

Motivation

- Many results have been obtained through higher-order pushdown systems/trandsducers
  -> non-intuitive, complex proofs
- Simpler, more direct reasoning about grammars seems possible through λ-calculus and types
  - Demonstration through:
    - pumping lemma (for deterministic case) and application to strictness of tree hierarchy
    - context-sensitiveness (ongoing work, with only preliminary result)

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  - pumping lemma for deterministic HO tree grammar (HORS)
    - background
    - statement of the lemma
    - proof sketch
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Pumping Lemmas

- State properties about "repeated structures" generated by grammars/automata

  e.g. Pumping lemma for CFL:

  "Any sufficiently long word $s \in L$ can be decomposed to

  $s = uvwxy$ (with $vx \neq \varepsilon$)

  and $uv^iwx^iy \in L$ for every $i \geq 0$"

- Used for separation of language classes

  e.g. $L = \{a^nb^nc^n \mid n \geq 0\}$ is not a CFL.

  If L were CFL, then for sufficiently large n,

  $a^nb^nc^n = uvwxy$ and $uv^iwx^iy \in L$ for every $i \geq 0$,

  but this is impossible.

Pumping lemmas for "higher-order" grammars/PDA

- Pumping lemma for indexed languages [Hayashi 73]
- Pumping lemmas for HPDS/CPDS [Parys 12; Kartzow&Parys 12]
  - strictness of hierarchy of trees/graphs generated by CPDS/HORS [Kartzow&Parys 12]
  - separation between HPDS and CPDS (or "safe" vs "unsafe" trees) [Parys 12]

Proofs are:

- complex (at least for non-experts on CPDS)
- indirect (for reasoning about HORS)

(cf. proof of pumping lemma for CFL)

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Higher-Order Recursion Scheme (HORS)

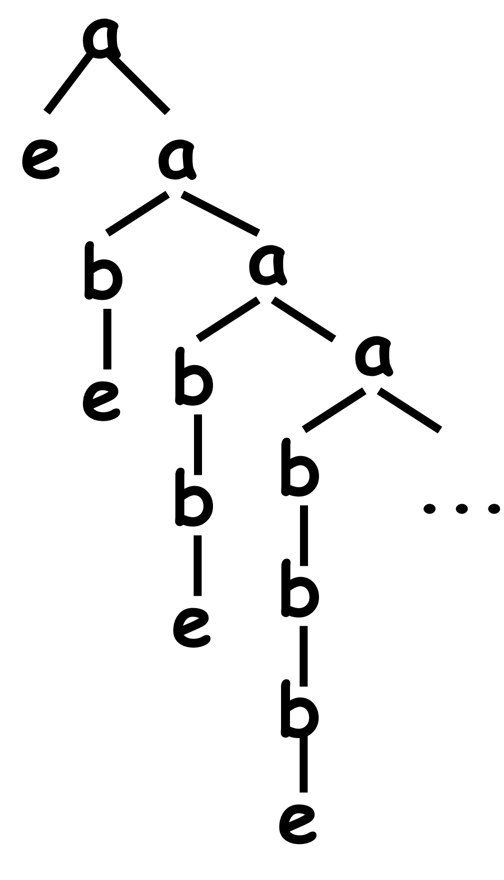
- Simply-typed, deterministic, higher-order grammar for an infinite tree

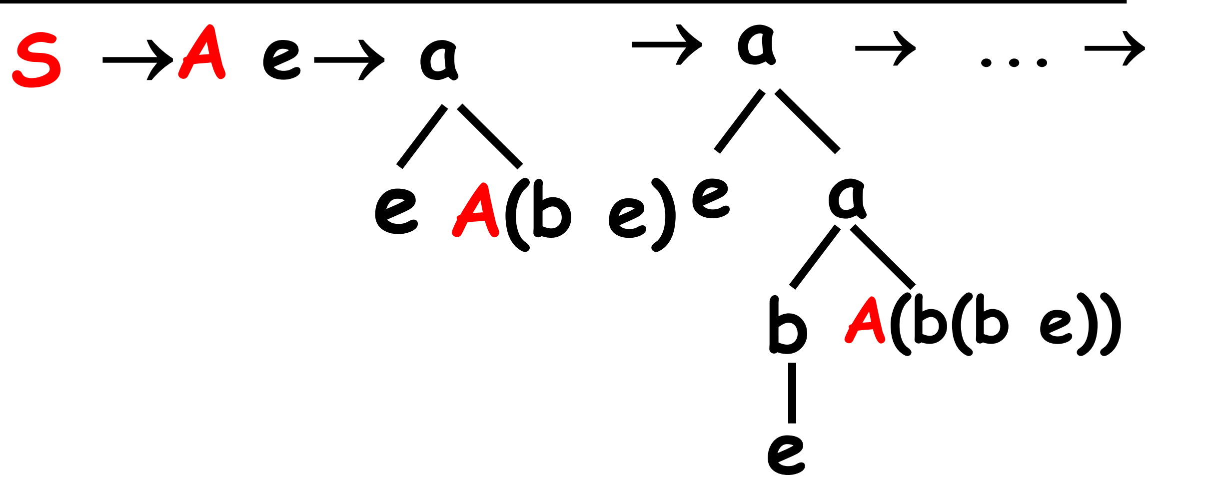
Order-1 HORS

S → A e

A x → a x (A (b x))

S: o, A: o→ o





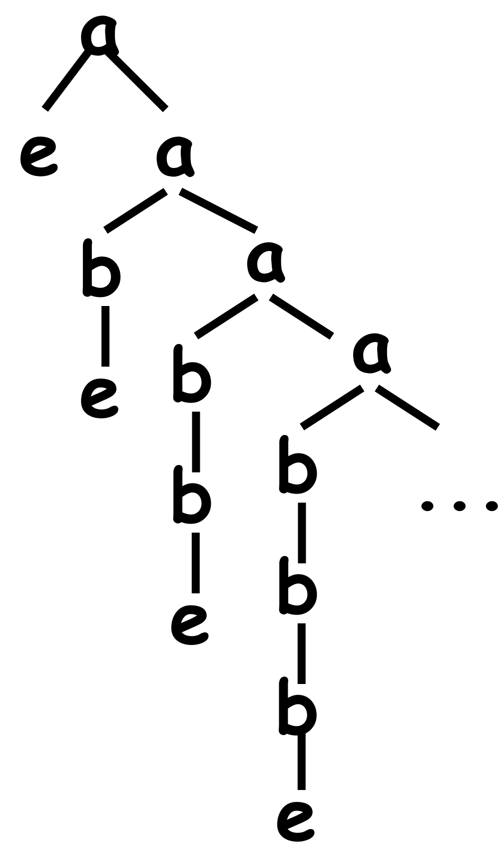
HORS as Labeled Transition System
[Carayol&Serre, LICS12]

$S \rightarrow A\ e$

$A\ x \rightarrow a\ x\ (A(b\ x))$

$$S \xrightarrow{\varepsilon} A\ e \xrightarrow{\varepsilon} a\ e\ (A\ (b\ e)) \xrightarrow{(a,2)} A\ (b\ e)$$

The second branch of node a has been chosen.



HORS as Labeled Transition System
[Carayol&Serre, LICS12]

$S \rightarrow A\ e$

$A\ x \rightarrow a\ x\ (A(b\ x))$

$S \xrightarrow{\varepsilon} A\ e$

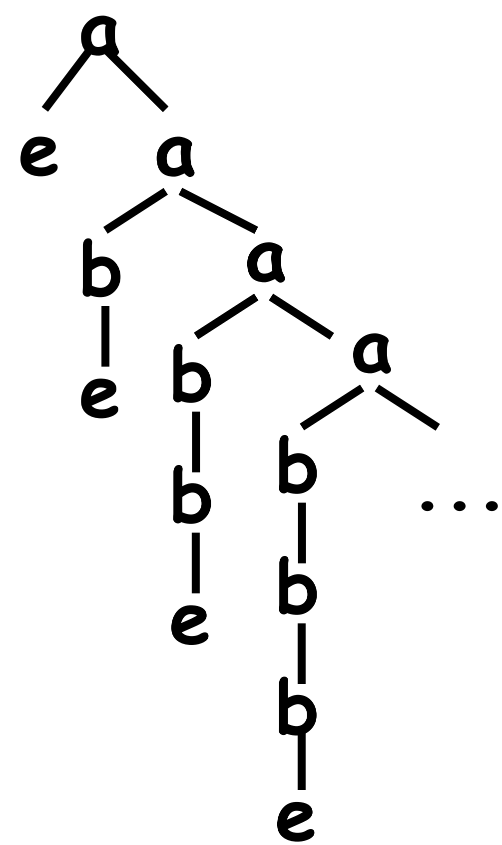
$\xrightarrow{\varepsilon} a\ e\ (A\ (b\ e))$

$\xrightarrow{(a,2)} A\ (b\ e)$

$\xrightarrow{\varepsilon} a\ (b\ e)\ (A\ (b\ (b\ e)))$

$\xrightarrow{(a,1)} b\ e$

$\xrightarrow{(b,1)} e$

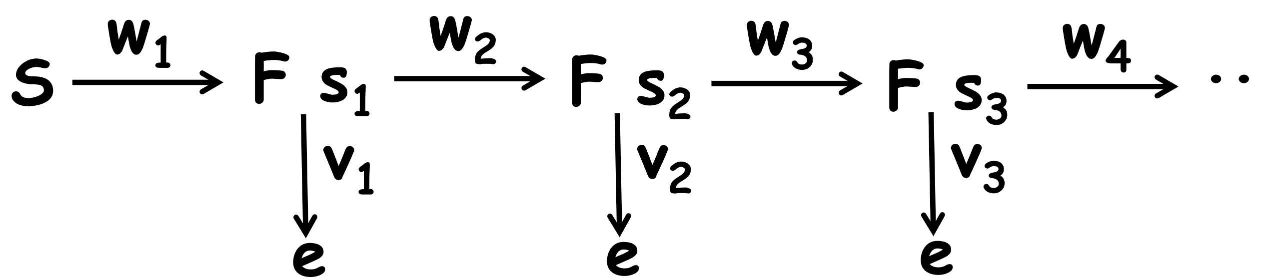


Pumping Lemma

$\forall G$: order-n HORS. $\exists c,d$.

$S \xrightarrow{w} e$ and $|w| > \exp_{n-1}(c)$ imply:

(i)



(ii) $|u_m| \leq \exp_{n-1}((m+1)c^2)$ for $u_m = w_1 \ldots w_m v_m$

(iii) $u_m \neq u_{m'}$ if $|m-m'| \geq d$

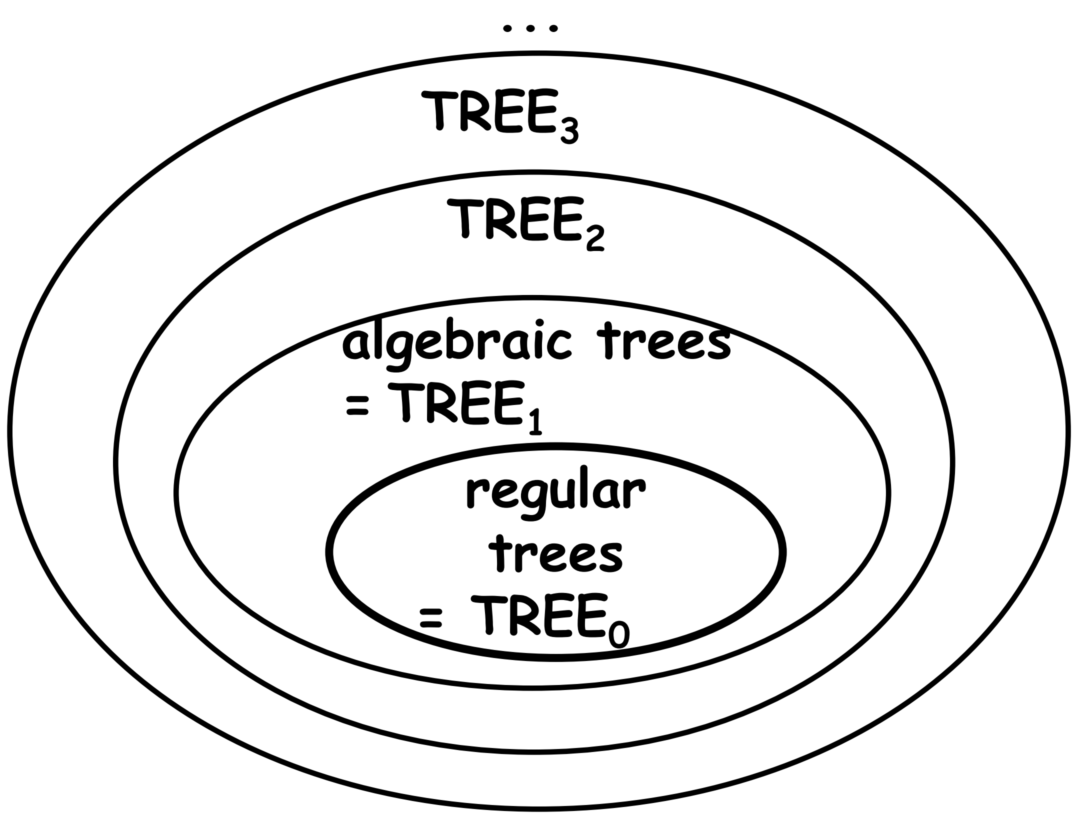
$\exp_n(x) = \underbrace{2^{2^{\cdot^{\cdot^{2^x}}}}}_{n}$

Strictness of HORS Tree Hierarchy

TREE_n = { Tree(G) | G: an order-n HORS}

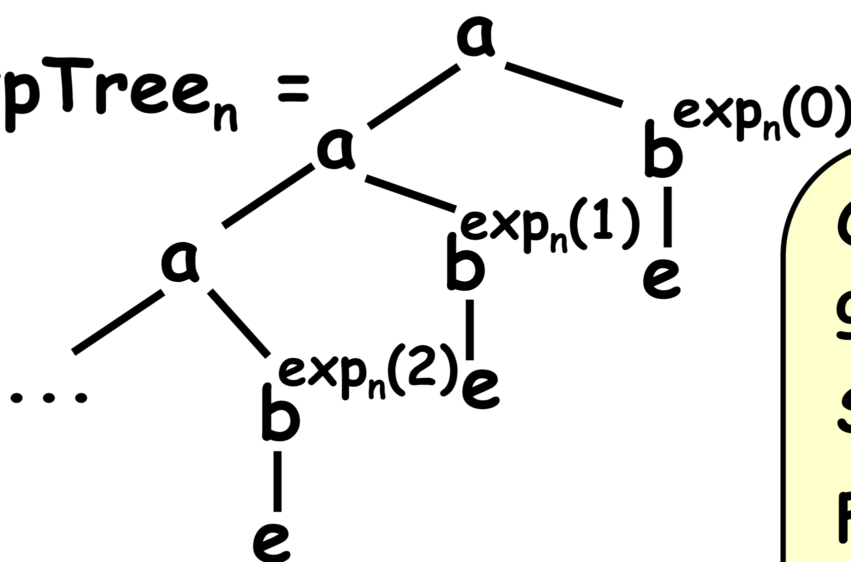
Theorem [Kartzow&Parys 12] :

$\mathrm{TREE}_0 \subsetneq \mathrm{TREE}_1 \subsetneq \ldots \subsetneq \mathrm{TREE}_n \subsetneq \mathrm{TREE}_{n+1} \subsetneq \ldots$



Witness of $\mathrm{TREE}_n \subsetneq \mathrm{TREE}_{n+1}$

$\mathrm{ExpTree}_n$ =



Order-(n+1) HORS that generates $\mathrm{ExpTree}_n$:

$S \rightarrow F\ T_{n-1}$

$F\ f \rightarrow a\ (F\ (T_n\ f))$
$\qquad (f\ T_{n-2}\ \ldots\ T_1\ b\ e)$

$T_k\ f\ x \rightarrow f(f\ x)$
$\qquad$ (for $k=1,\ldots,n$)

$$S \rightarrow F\ T_{n-1} \xrightarrow{(a,1)^k} F\ (T^k{}_n\ T_{n-1}) \xrightarrow{(a,2)} T^k{}_n\ T_{n-1} \ldots T_1\ b\ e \xrightarrow{(b,1)^{exp_n(k)}} e$$

$\text{ExpTree}_n \notin \text{TREE}_n$

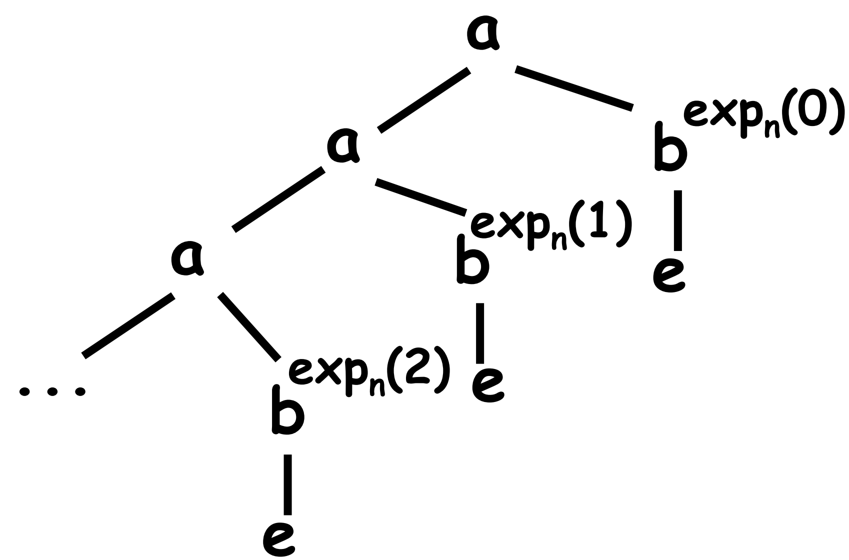
Suppose $\text{ExpTree}_n \in \text{TREE}_n$.

$\exists\ u_1, u_{1+d}, u_{1+2d}, \ldots$

(i) $S \xrightarrow{u_{1+kd}} e$

(ii) $|u_{1+kd}| \leq \exp_{n-1}((kd+2)c^2)$

(iii) $u_{1+kd} \neq u_{1+jd}$ if $k \neq j$.



$S \xrightarrow{w} e$ and $|w| > \exp_{n-1}(c)$ imply:

(i) $S \xrightarrow{w_1} F\ s_1 \xrightarrow{w_2} F\ s_2 \xrightarrow{w_3} F\ s_3 \xrightarrow{w_4} \cdots$, with $F\ s_1 \xrightarrow{v_1} e$, $F\ s_2 \xrightarrow{v_2} e$, $F\ s_3 \xrightarrow{v_3} e$

(ii) $|u_m| \leq \exp_{n-1}((m+1)c^2)$ for $u_m = w_1 \ldots w_m v_m$

(iii) $u_m \neq u_{m'}$ if $|m-m'| \geq d$

$ExpTree_n \notin TREE_n$

Suppose $ExpTree_n \in TREE_n$.

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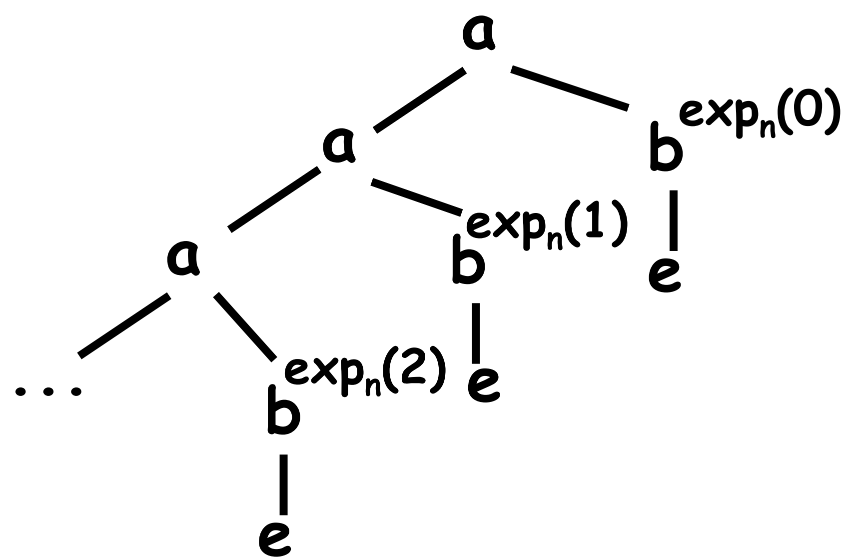


$exp_{n-1}((kd+2)c^2)$

$\geq \max(|u_1|,\ |u_{1+d}|, \ldots,\ |u_{1+kd}|)$

$\geq$ the length of $(k+1)$-th shortest path

$= k+2+exp_n(k)$

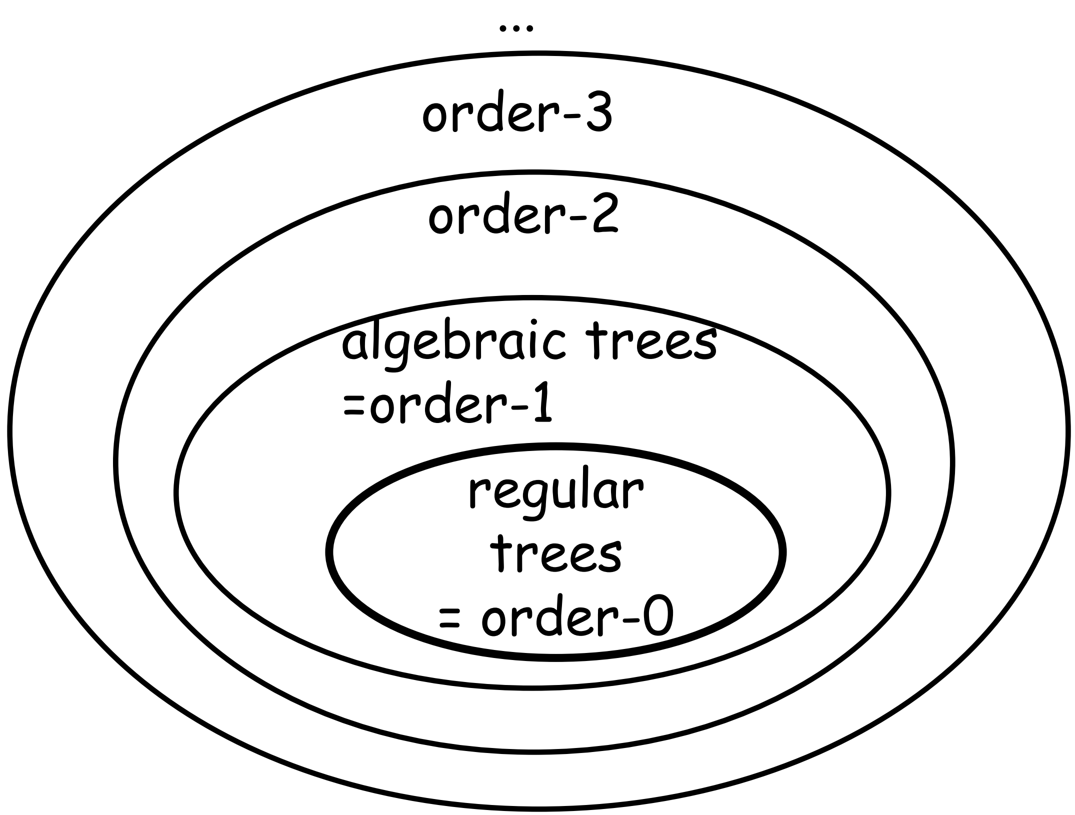
Contradiction for large k

Strictness of HORS Tree Hierarchy

TREE_n = { Tree(G) | G: an order-n HORS}

Theorem [Kartzow&Parys 12] :

$\mathrm{TREE}_0 \subsetneq \mathrm{TREE}_1 \subsetneq \ldots \subsetneq \mathrm{TREE}_n \subsetneq \mathrm{TREE}_{n+1} \subsetneq \ldots$



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    - statement of the lemma and application
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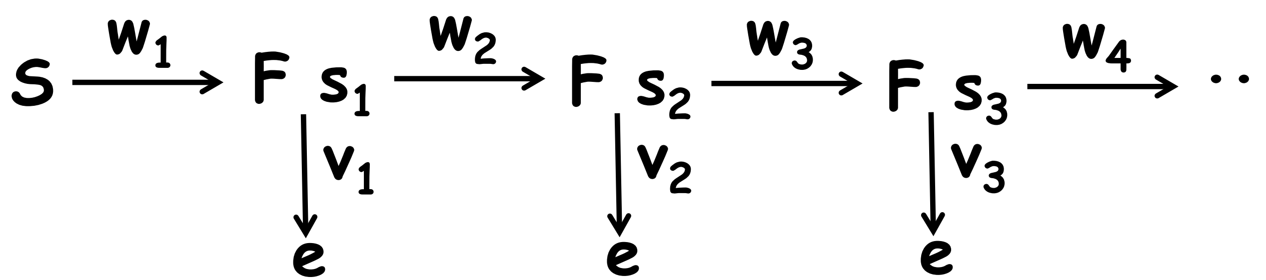
How to prove pumping lemma?

$\forall G$: order-n HORS. $\exists c,d$.

$S \xrightarrow{w} e$ and $|w| > \exp_{n-1}(c)$ imply:

(i)



(ii) $|u_m| \leq \exp_{n-1}((m+1)c^2)$ for $u_m = w_1 \ldots w_m v_m$

(iii) $u_m \neq u_{m'}$ if $|m-m'| \geq d$

$\exp_n(x) = 2^{2^{\cdot^{\cdot^{2^x}}}}$ (n 2's)

Pumping Lemma for CFL

"Any sufficiently long word $s \in L$ can be decomposed to
$s = uvwxy$ (with $vx \neq \varepsilon$)
and $uv^iwx^iy \in L$ for every $i \geq 0$"

Proof.

The derivation of s must contain repeated occurrences of a non-terminal F:

$S \to^* uFy \to^* uvFxy \to^* uvwxy$ (=s).

By repeating the part $F \to^* vFx$,

$S \to^* uFy \to^* uvFxy \to^* uvvFxxy \to^* uv^iFx^iy \to^* uv^iwx^iy$

Pumping for HORS?

- Sufficiently long transition sequence must contain repeated occurrences of non-terminal in the head position?

$$S \xrightarrow{u} F\ s_1 \xrightarrow{w} F\ s_2 \xrightarrow{v} e$$

Yes!

- Can the part "$F\ s_1 \rightarrow F\ s_2$" be pumped??

Not necessarily:

For $G = \{S \rightarrow F(F\ e),\ F\ x \rightarrow x\}$,

$S \rightarrow F(F\ e) \rightarrow F\ e \rightarrow e$

but the part "$F(F\ e) \rightarrow F\ e$" cannot be repeated!

Conditions for Pumping F $s_1 \to$ F s_2?

- F should be obtained by unfolding F
  - × F(F e) → F e where F x → x
  - ✓ F e $\to^{(a,1)}$ F (c e) where F x → a (F (c x))

  Sufficient?

  No.

  For G={S→F I, F f→f(F K), I x→x, K x→e},

  S → F I → I(F K) → F K,

  but F K → K(F K) → e $\not\to^*$ F ...

  because I and K have different behaviors!

  (I uses the argument but K ignores it)

Conditions for Pumping F $s_1 \to$ F s_2?

- F should be obtained by unfolding F
  - × F(F e) → F e where F x → x
  - ✓ F e $\to^{(a,1)}$ F (c e) where F x → a (F (c x))
- s_2 should have the same type as s_1

  For G={S→F I, F f→f(F K), I x→x, K x→e},

  S → F I → I(F K) → F K,

  but I: r→r and K: T→r

Use an argument

Ignore an argument

Intersection Types for Expressing Reduction Behavior

τ (types) ::=

r (terms that reduce to e)

$(\tau_1 \wedge \ldots \wedge \tau_k) \to \tau$ (functions that use an argument as a value of types $\tau_1, \ldots, \tau_k$ and return a value of type τ)

Types	Examples	Non-examples
$r \to r$	$\lambda x.x$, $\lambda x.a\ x\ e$	$\lambda x.e$
$T \to r$	$\lambda x.e$, $\lambda x.a\ x\ e$	$\lambda x.x$
$(r \to r) \to r$	$\lambda f.f\ e$, $\lambda f.f(f\ e)$	$\lambda f.e$

(inaccurate) Key Lemma

If

(i) F $s_1 \rightarrow^* $ F s_2

(ii) F comes from F

(iii) F and F have the same type τ,

then

(1) F $s_2 \rightarrow^* $ F s_3 for some s_3

(2) F comes from F

(3) F has type τ

Proof Sketch of Pumping Lemma

1. A sufficiently long transition sequence is of the form (for a sufficiently large k > #types):

$$S \xrightarrow{u_1} F^{\tau_1} t_1 \xrightarrow{u_2} F^{\tau_2} t_2 \xrightarrow{u_3} \ldots \xrightarrow{u_k} F^{\tau_k} t_k \xrightarrow{u_{k+1}} e$$

2. Assign an intersection type to each F.

3. Pick i and j such that $\tau_i = \tau_j$ and "pump" the part $F\ t_i \xrightarrow{u_{i+1} \ldots u_j} F\ t_j$

$S \xrightarrow{w} e$ and $|w| > \exp_{n-1}(c)$ imply:

(i) $S \xrightarrow{w_1} F\ s_1 \xrightarrow{w_2} F\ s_2 \xrightarrow{w_3} F\ s_3 \xrightarrow{w_4} \cdots$, with $F\ s_1 \xrightarrow{v_1} e$, $F\ s_2 \xrightarrow{v_2} e$, $F\ s_3 \xrightarrow{v_3} e$

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3. Pick i and j such that $\tau_i = \tau_j$ and "pump" the part $F\ t_i \xrightarrow{u_{i+1} \ldots u_j} F\ t_j$ and obtain:

$$S \xrightarrow{w_1} F\ s_1 \xrightarrow{w_2} F\ s_2 \xrightarrow{w_3} F\ s_3 \xrightarrow{w_4} \cdots$$

$$F\ s_1 \xrightarrow{v_1} e \qquad F\ s_2 \xrightarrow{v_2} e \qquad F\ s_3 \xrightarrow{v_3} e$$

Proof Sketch

...

3. Pick i and j such that $\tau_i = \tau_j$ and "pump" the part $F\ t_i \xrightarrow{u_{i+1}\ldots u_j} F\ t_j$ and obtain:

$$\begin{array}{ccccccccc} S & \xrightarrow{w_1} & F\ s_1 & \xrightarrow{w_2} & F\ s_2 & \xrightarrow{w_3} & F\ s_3 & \xrightarrow{w_4} & \cdots \\ & & \downarrow v_1 & & \downarrow v_2 & & \downarrow v_3 & & \\ & & e & & e & & e & & \end{array}$$

4. To obtain the bound $|w_1\ldots w_m v_m| \leq \exp_{n-1}((m+1)c^2)$, simulate $S \xrightarrow{w_1\ldots w_m v_m} e$

   by: $t \xrightarrow{w_1\ldots w_m v_m} e$

   for a λ-term t (obtained by unfolding S). $|w_1\ldots w_m v_m|$ is bounded by the size of β-normal form of t [Beckmann 01]

Pumping Lemma for HORS: summary

- The same reasoning as for context-free languages is possible, with a help of types
  - A sufficiently long transition sequence must contain repeated occurrences of the same non-terminal
  - The part where the same non-terminal is used as the same type can be pumped
  - The length of pumped words can be bounded by using the standard result on the size of β-normal form of simply typed λ-terms

Pumping Lemma for Word Language Grammar?

$S \xrightarrow{w} e$ and $|w| > \exp_{n-1}(c)$ imply:

(i) $$\begin{array}{ccccccccc} S & \xrightarrow{w_1} & F\ s_1 & \xrightarrow{w_2} & F\ s_2 & \xrightarrow{w_3} & F\ s_3 & \xrightarrow{w_4} & \cdots \\ & & \downarrow v_1 & & \downarrow v_2 & & \downarrow v_3 & & \\ & & e & & e & & e & & \end{array}$$

(ii) $|u_m| \leq \exp_{n-1}((m+1)c^2)$ for $u_m = w_1 \ldots w_m v_m$

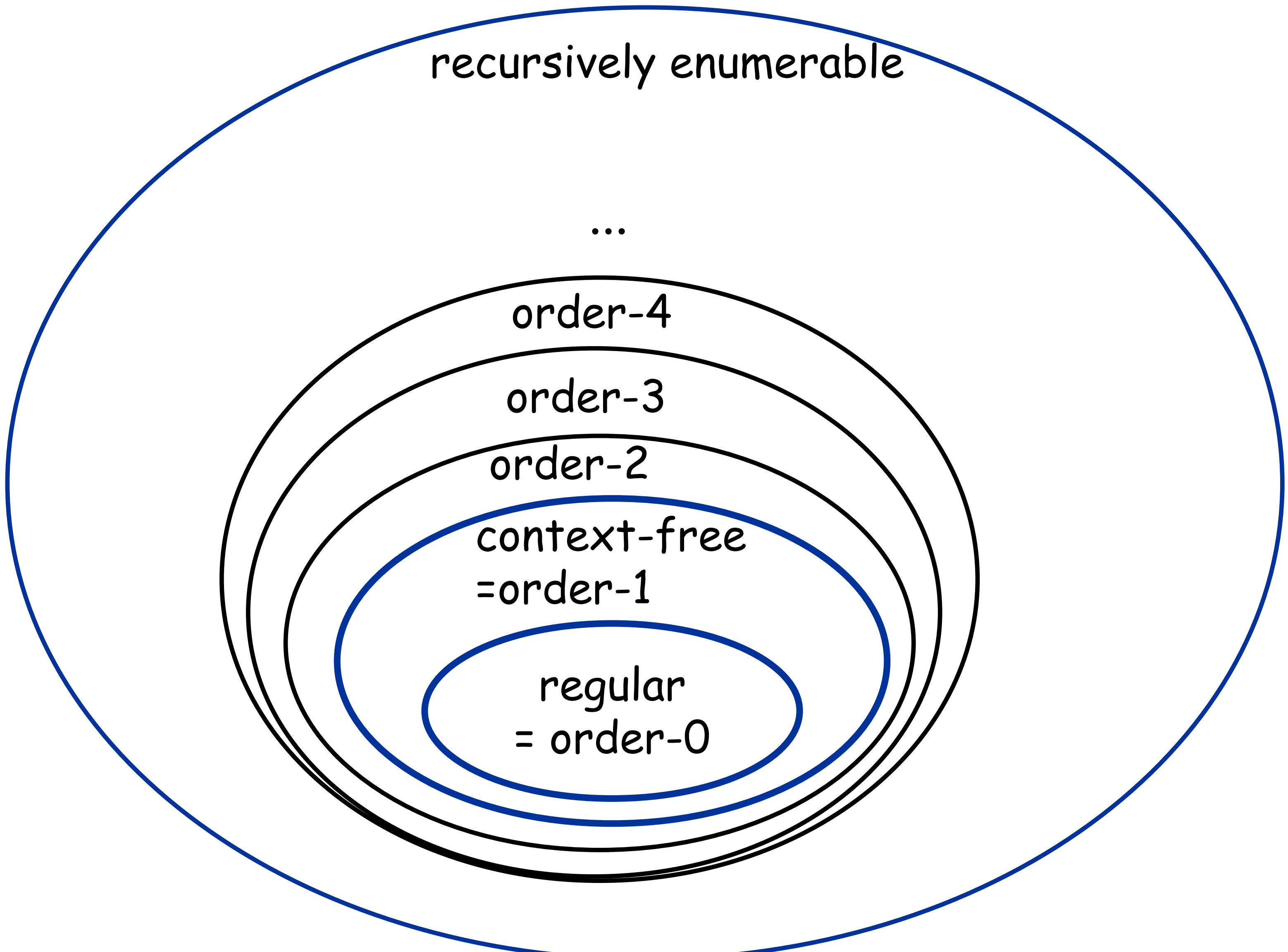
(iii) $u_m \neq u_{m'}$ if $|m-m'| \geq d$

→ A further twist is required

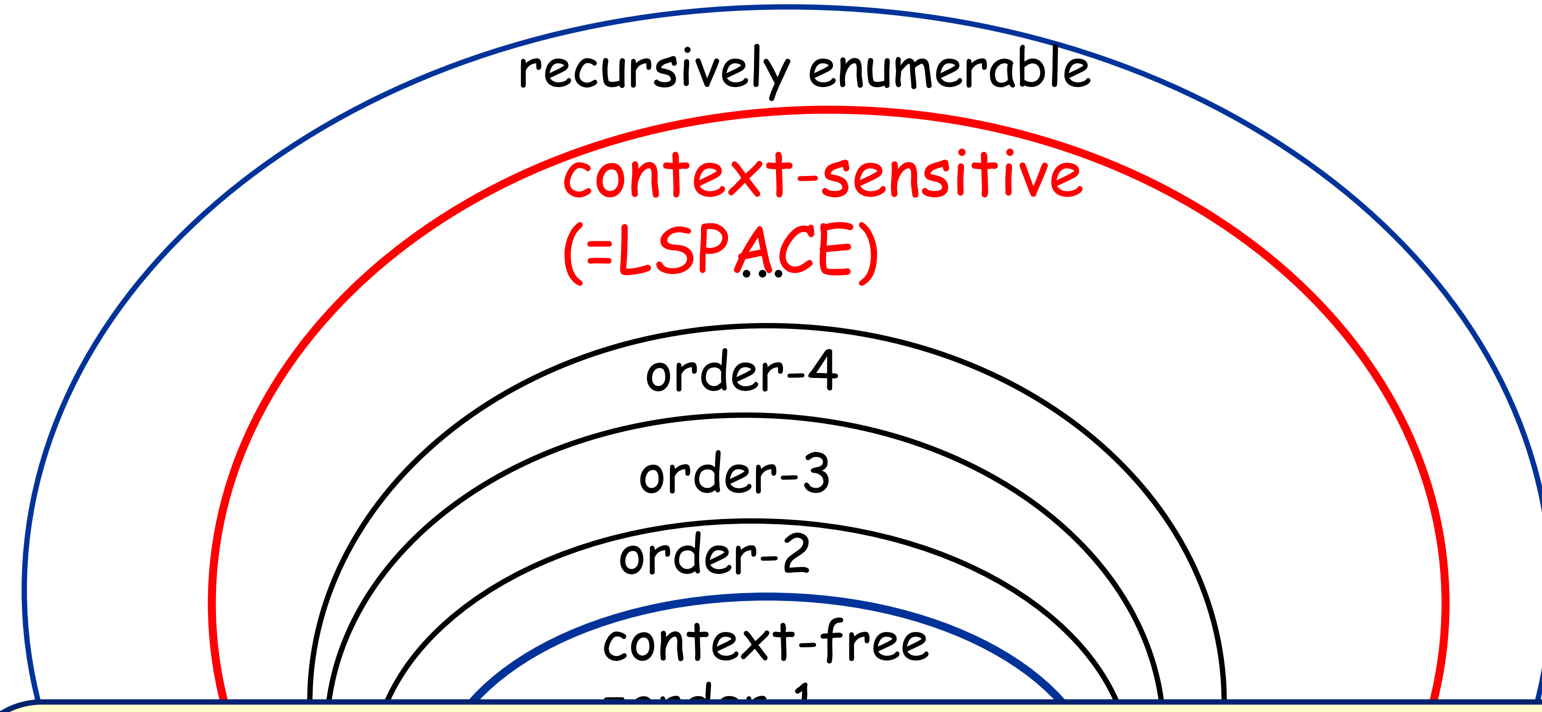
Outline

- Background
- Solved/open problems
- Applications of typed λ-calculus
  - motivation
  - pumping lemma
  - towards context-sensitiveness of unsafe languages (ongoing work with Kazuhiro Inaba and Takeshi Tsukada)

Chomsky hierarchy and HO languages



Chomsky hierarchy and HO languages



Theorem [Inaba&Maneth 08]
Safe order-n word languages are context-sensitive for every n
(The proof goes through decomposition of higher-order transducers.)

Context-sensitiveness of Context-free languages

- Eliminate ε-generating rules

S -> aAb
A -> ε   A -> aAb

↓

S -> aAb   S -> ab
A -> ε   A -> aAb   A -> ab

The normalized grammar has only monotonically increasing production sequences

($S \rightarrow t_1 \rightarrow t_2 \rightarrow \ldots \rightarrow w$ implies $|S| \leq |t_1| \leq |t_2| \leq \ldots \leq |w|$)

$\Rightarrow$ membership is NLINSPACE

Context-sensitiveness of Order-1 tree grammar?

- What should be removed to ensure the monotonicity of production sequence?
  - Redundant arguments

$S \to F\ e$	$F\ x \to e$	$F\ x \to F\ (a\ x)$

$S \to F\ e \to F\ (a\ e) \to^* F\ (a^n\ e) \to e$

$S \to F\ e$	$F\ x \to e$	$F\ x \to F\ (a\ x)$

cf. type-based useless code elimination [Damiani&Prost 96, K 2000]

Context-sensitiveness of Order-1 tree grammar?

- What should be removed to ensure the monotonicity of production sequence?
  - Redundant arguments

$S \to F\ e \qquad F\ x \to e \qquad F\ x \to F\ (a\ x)$

  - Identity functions

$S \to F\ e \qquad F\ x \to x \qquad F\ x \to F\ (F\ x)$

$S \to F\ e \to F\ (F\ e) \to F^n\ e \to F\ e \to e$

$S \to e \mid F\ e \qquad F\ x \to x \qquad F\ x \to F\ (F\ x) \mid F\ x$

Context-sensitiveness of Order-2 tree grammar?

- What should be removed to ensure the monotonicity of production sequence?
  - Redundant arguments
  - Identity functions
  - Permutators (+α)

$S \to F\ a \qquad F\ x \to x\ c\ d \qquad F\ x \to F\ (G\ x)$
$G\ x\ y\ z \to x\ z\ y$

$S \to F\ a \to F\ (G\ a) \to^* F\ (G^{2n}\ a) \to (G^{2n}\ a)\ c\ d$
$\to G^{2n-1}\ a\ d\ c \to G^{2n-2}\ a\ c\ d \to^*\ a\ c\ d$

$S \to F\ a \qquad F\ x \to x\ c\ d \qquad F\ x \to F_G\ x$
$F_G\ x \to x\ d\ c \qquad F_G\ x \to F\ x \qquad G\ x\ y\ z \to x\ z\ y$

Open Problems (for proving context-sensitiveness)

- What should be removed to ensure the monotonicity of production sequence for grammars of arbitrary orders?
  - Removing hereditary permutators is necessary, but not sufficient
- Is there a systematic transformation that removes them?

A positive answer would imply context-sensitiveness of (unsafe) higher-order languages

Summary

- A survey of properties of higher-order languages
  - Many problems have been solved for safe languages, but open for unsafe ones
- λ-calculus and types seem to be a promising approach to studies of unsafe languages
  - simpler proof of strictness of tree hierarchy
  - work is under way for proving context-sensitiveness of HO languages
  - "safety" is natural for HPDS,
    but "unsafety" is more natural for λ-calculus